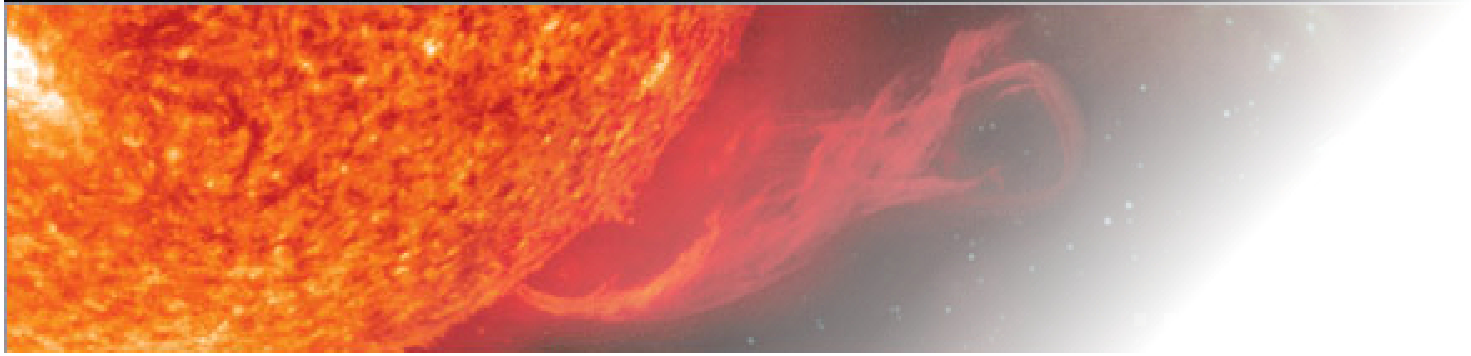

Multifractal Scaling of the Solar Wind Turbulence at High Latitudes

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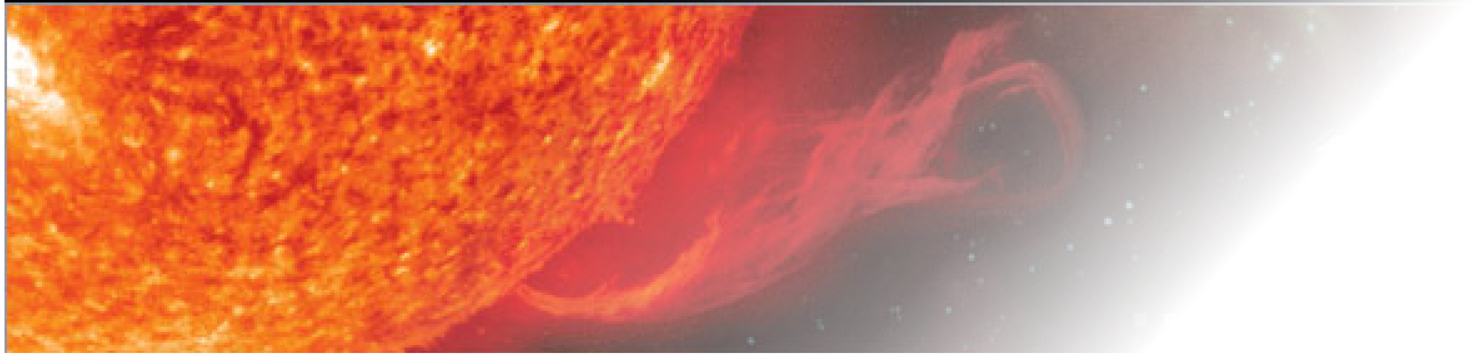
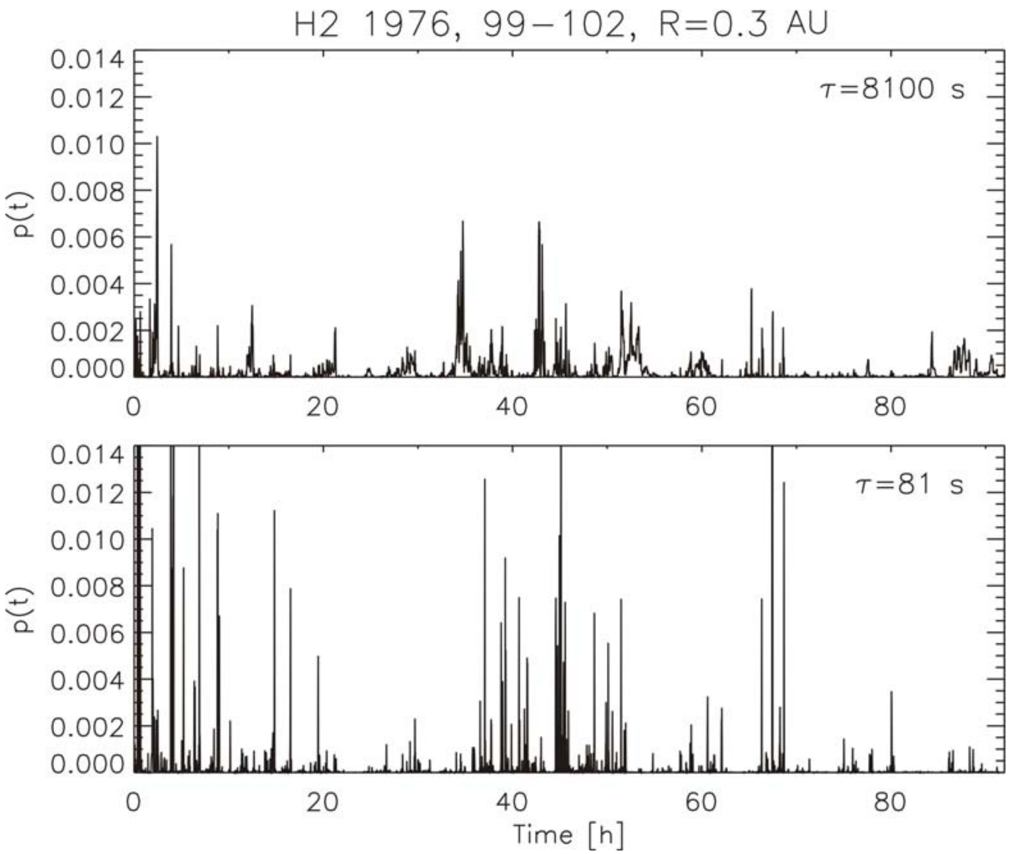


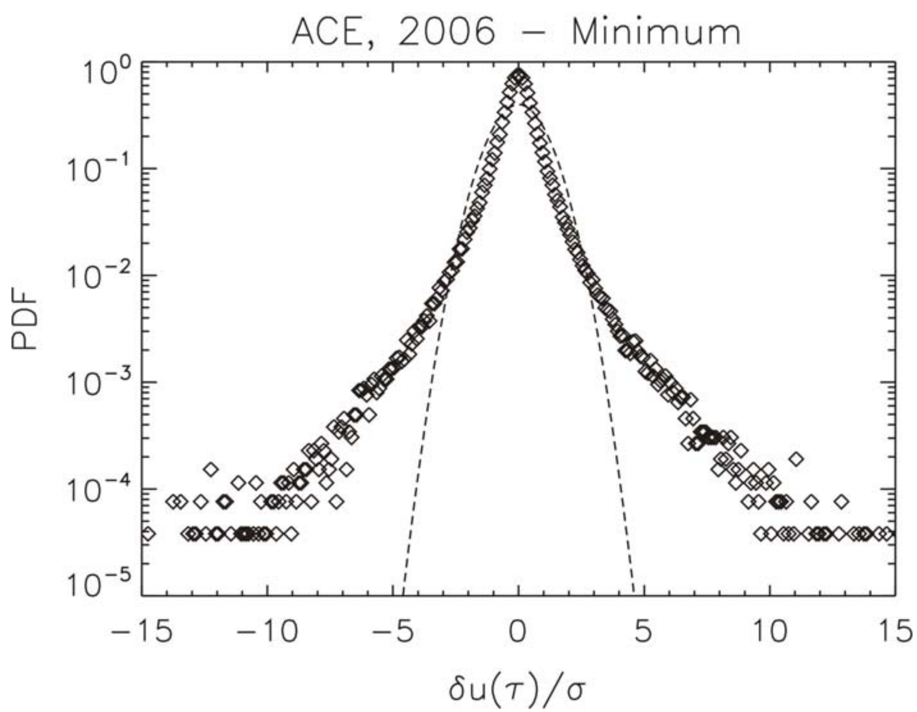
Outline

- Intemittency
- Multifractal Methods
- Solar Wind Data
- Results
  - Generalized Dimensions
  - Singularity Spectra
- Conclusions

Intermittency



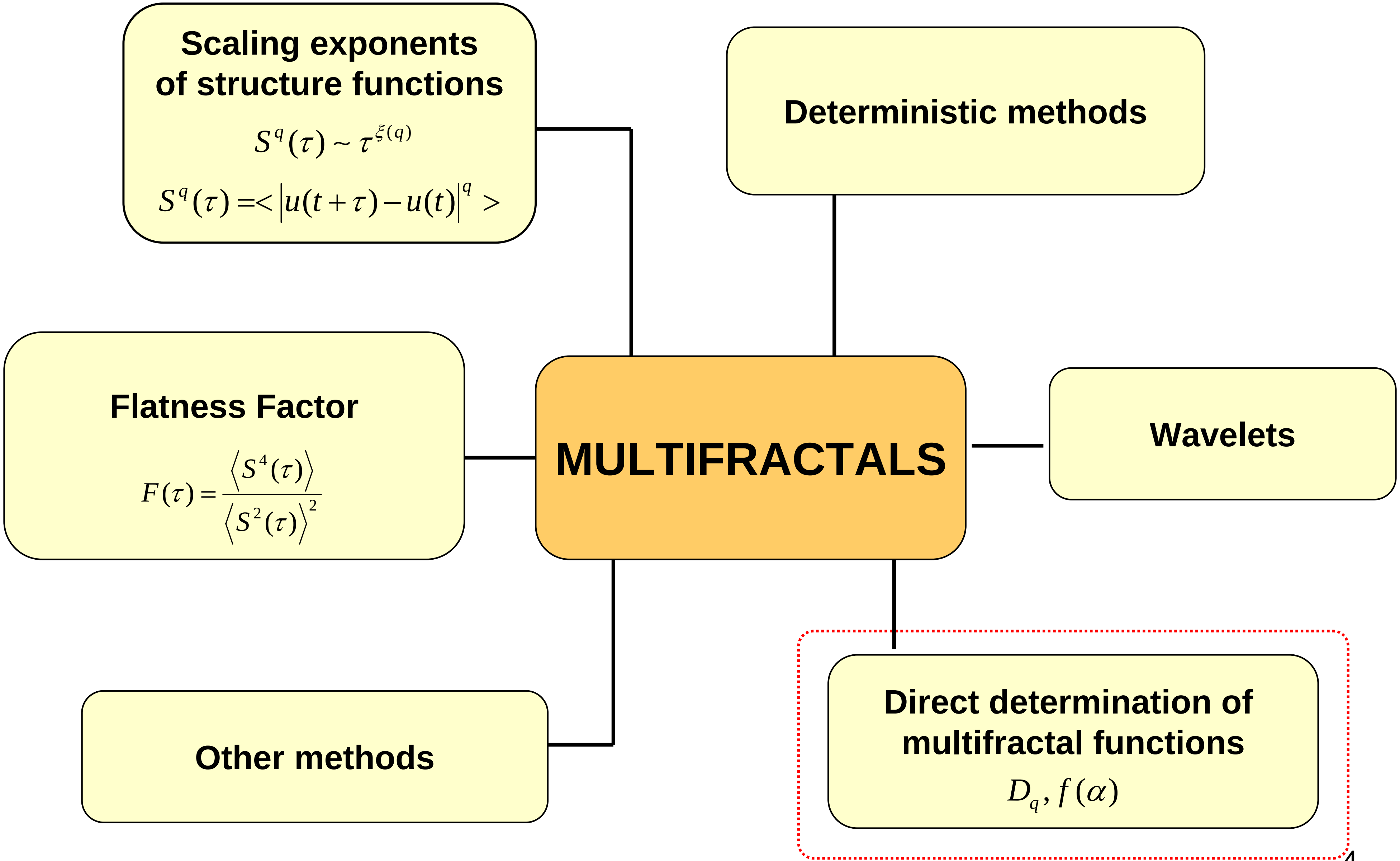
Nonuniform energy transfer rate between different eddies [Obhukov, 1962]



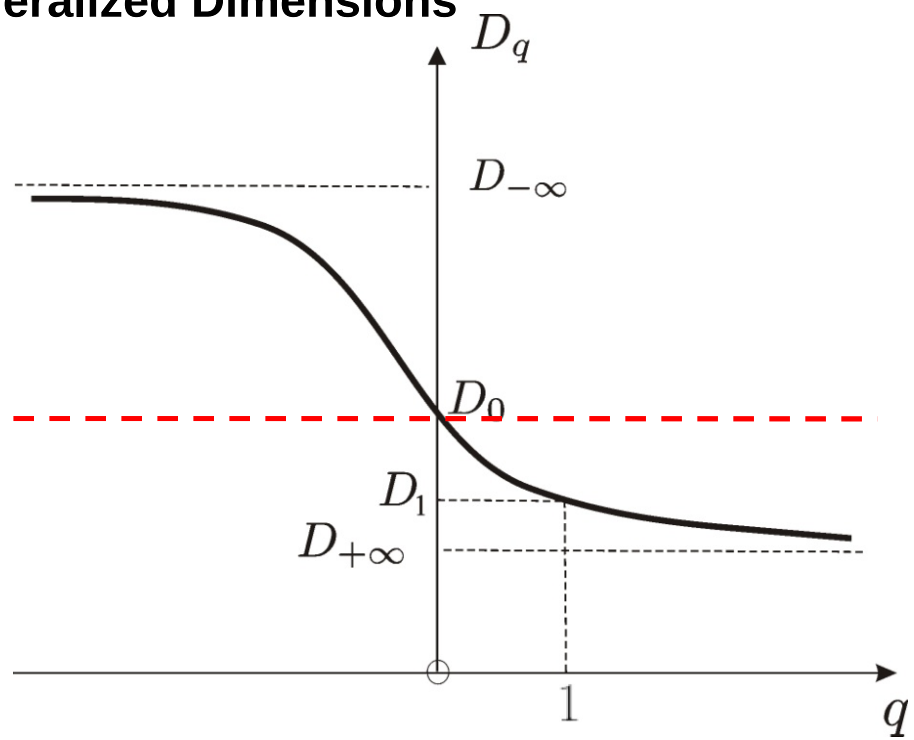
Intermittency reveals a deviation from normal distribution.

We need multifractal description

Approaches to study multifractality



Generalized Dimensions

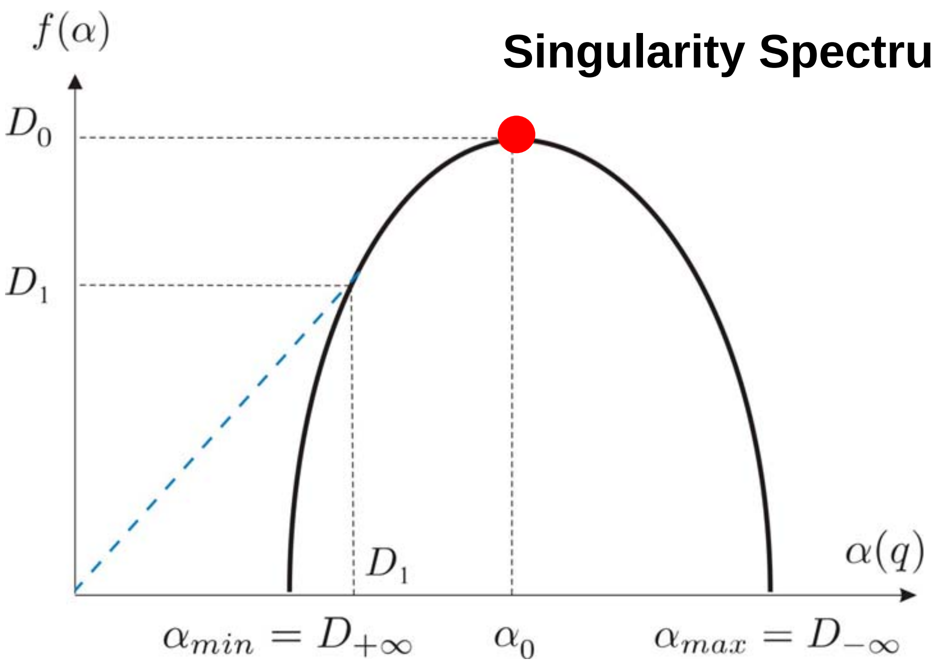


Multifractal Formalism

Monofractal $D_q = D$

Multifractal $D_q = D_q(q)$

Singularity Spectrum



Degree of Multifractality

$$\Delta = D_{-\infty} - D_{+\infty} = \alpha_{\max} - \alpha_{\min}$$

Degree of Asymmetry

$$A = \frac{\alpha_0 - \alpha_{\min}}{\alpha_{\max} - \alpha_0}$$

Method

Measure $$p_i(l) = \frac{\varepsilon_i(l)}{\sum_{i=1}^{N} \varepsilon_i(l)}$$

1 → **2**

2 Generalized Dimensions

Generalized average probability measure

$$\overline{\mu}(q,l) \equiv \sqrt[q-1]{\left\langle (p_i)^{q-1} \right\rangle_{\text{av}}}$$

$$D_q(l) = \frac{\log_{10} \overline{\mu}(q,l)}{\log_{10} l}$$

3 Generalized pseudoprobability measure

$$\mu_i(q,l) = \frac{p_i^{\,q}(l)}{\sum_{i=1}^{N} p_i^{\,q}(l)}$$

Singularity Spectrum

$$\alpha(q) = \lim_{l \to 0} \frac{\log_{10} \sum_{i=1}^{N} \mu_i(q,l) \log_{10} p_i(l)}{\log_{10} l}$$

$$f(q) = \lim_{l \to 0} \frac{\log_{10} \sum_{i=1}^{N} \mu_i(q,l) \log_{10} \mu_i(q,l)}{\log_{10} l}$$

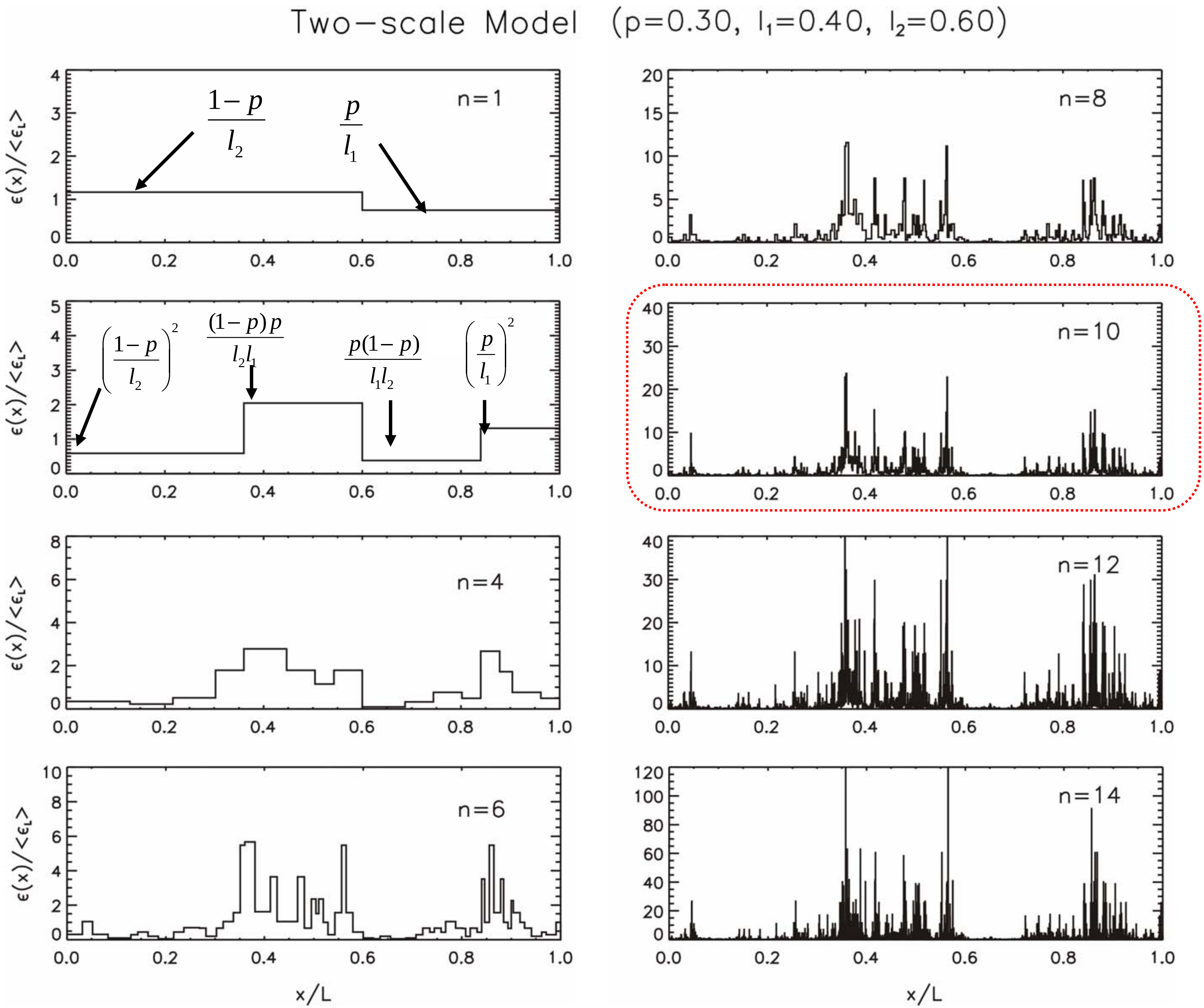
4 Spectrum Verification

$$\alpha = \frac{d}{dq}[(q-1)D_q]$$

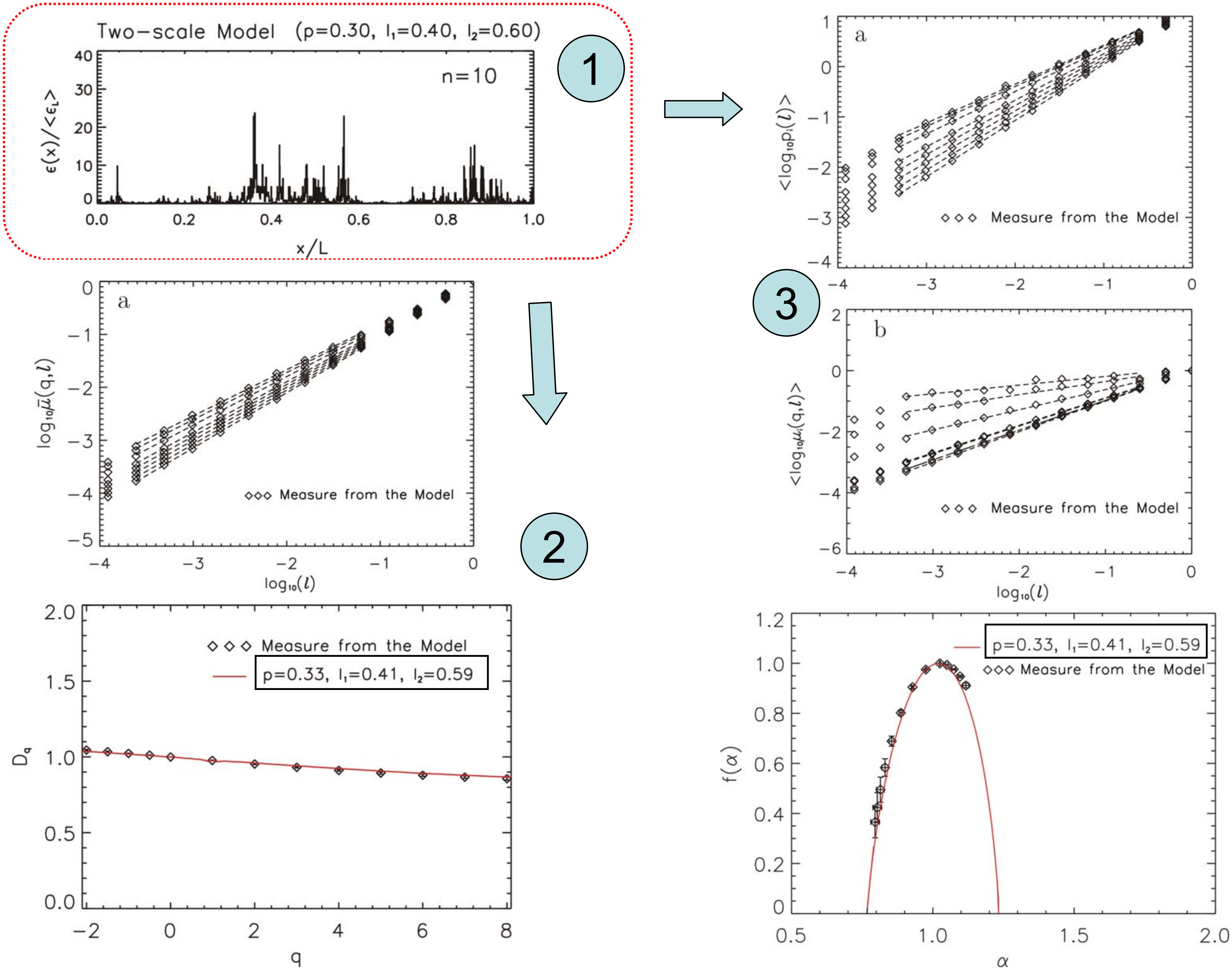
$$f(\alpha) = q\alpha - (q-1)D_q$$

Chhabra, Meneveau, Jensen,Sreenivasan [1989]

Halsey et al. [1986]



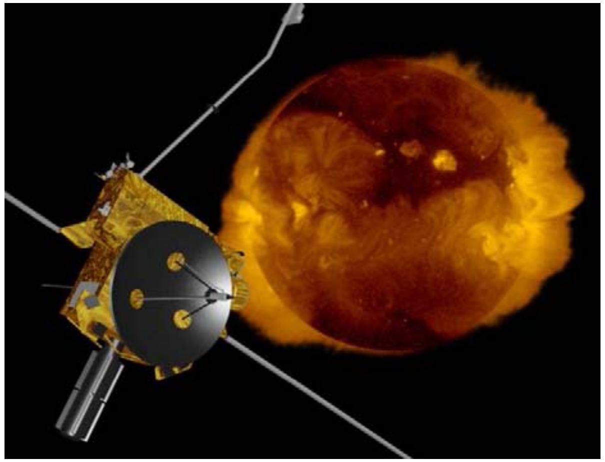
Two-scale Model (p=0.30, l₁=0.40, l₂=0.60)
ε(x)/<ε_L>
x/L
n=1
n=4
n=6
n=8
n=10
n=12
n=14
1−p / l₂
p / l₁
((1−p)/l₂)²
(1−p)p / l₂l₁
p(1−p) / l₁l₂
(p/l₁)²



Two-scale Model (p=0.30, l₁=0.40, l₂=0.60)
n=10
ε(x)/<ε_L>
x/L
1
2
3
a
log₁₀μ̄(q,l)
log₁₀(l)
Measure from the Model
<log₁₀p_i(l)>
b
<log₁₀μ_i(q,l)>
D_q
q
p=0.33, l₁=0.41, l₂=0.59
f(α)
α

Solar Wind Data

Ulysses (1990-2009)



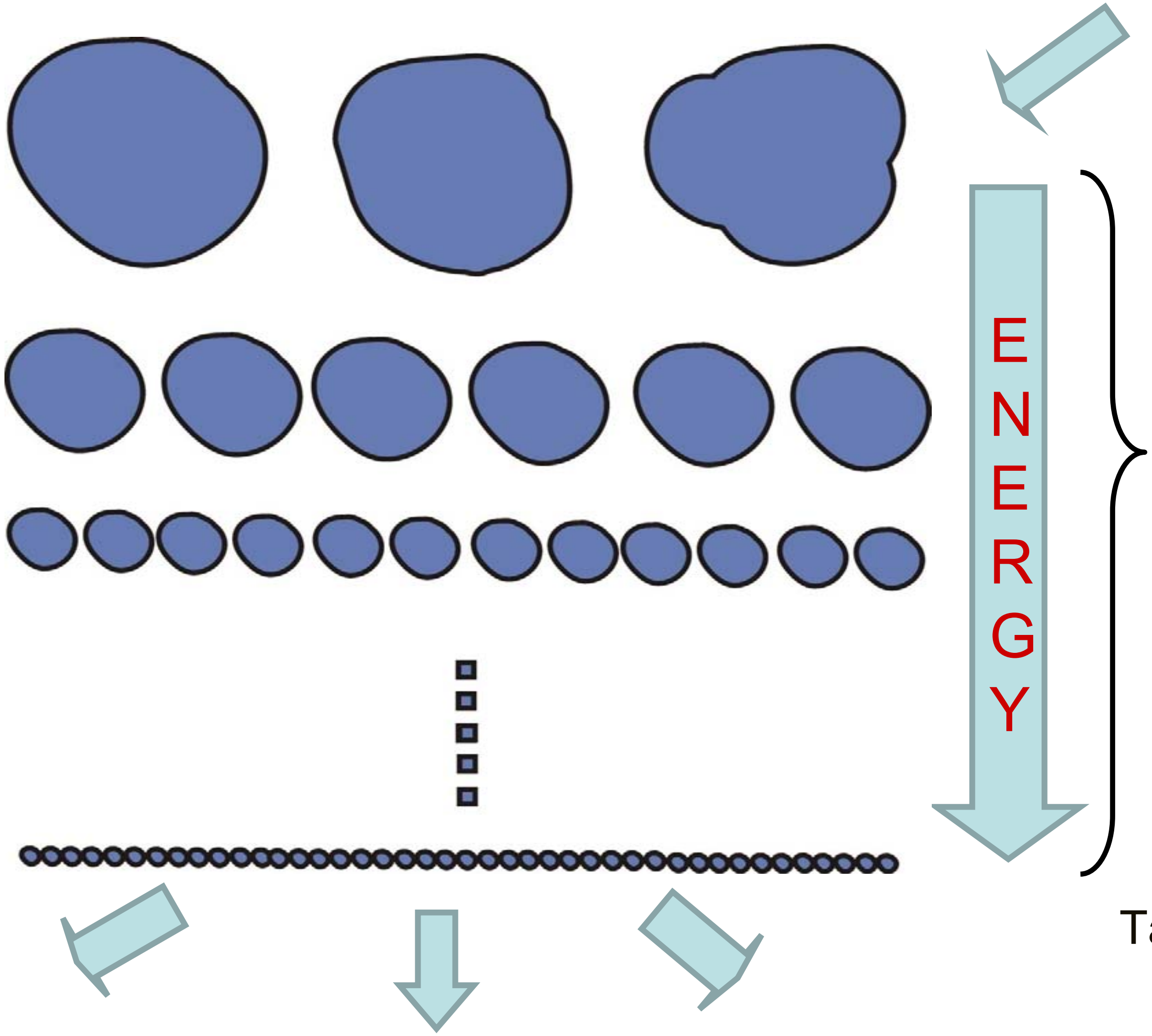
Phase of solar cycle	Solar minimum
Type of solar wind	In seven cases: fast wind In one case: slow wind
Solar wind parameter	Radial velocity
Sampling time	242 s≈ 4 min
Sample length	12 days

Year	Days	Heliographic Latitude	Heliocentric Distance
1997*	68 - 79	$+14° \div +15°$	4.9 AU
1995	104 - 115	$+32° \div +40°$	1.4 AU
1996	39 - 50	$+47° \div +48°$	3.3 AU
1995	181 - 192	$+74° \div +78°$	1.8 – 1.9 AU
1995	199 - 210	$+79° \div +80°$	1.9 – 2.0 AU
2007	150 - 161	$-40° \div -47°$	1.6 AU
1994	344 - 355	$-50° \div -56°$	1.6 – 1.7 AU
2006	317 - 328	$-69° \div -71°$	2.8 – 2.9 AU

Asterisk (*) denotes the case of the slow wind.

Energy Transfer Rate

Energy-containing scale



Dissipation range

Inertial range

Energy transfer rate:

$$\varepsilon(x,l) \sim \frac{|\upsilon_x(x+l)-\upsilon_x(x)|^3}{l}$$

A probability that energy is transfered to an eddy of size *l* in the turbulence cascade

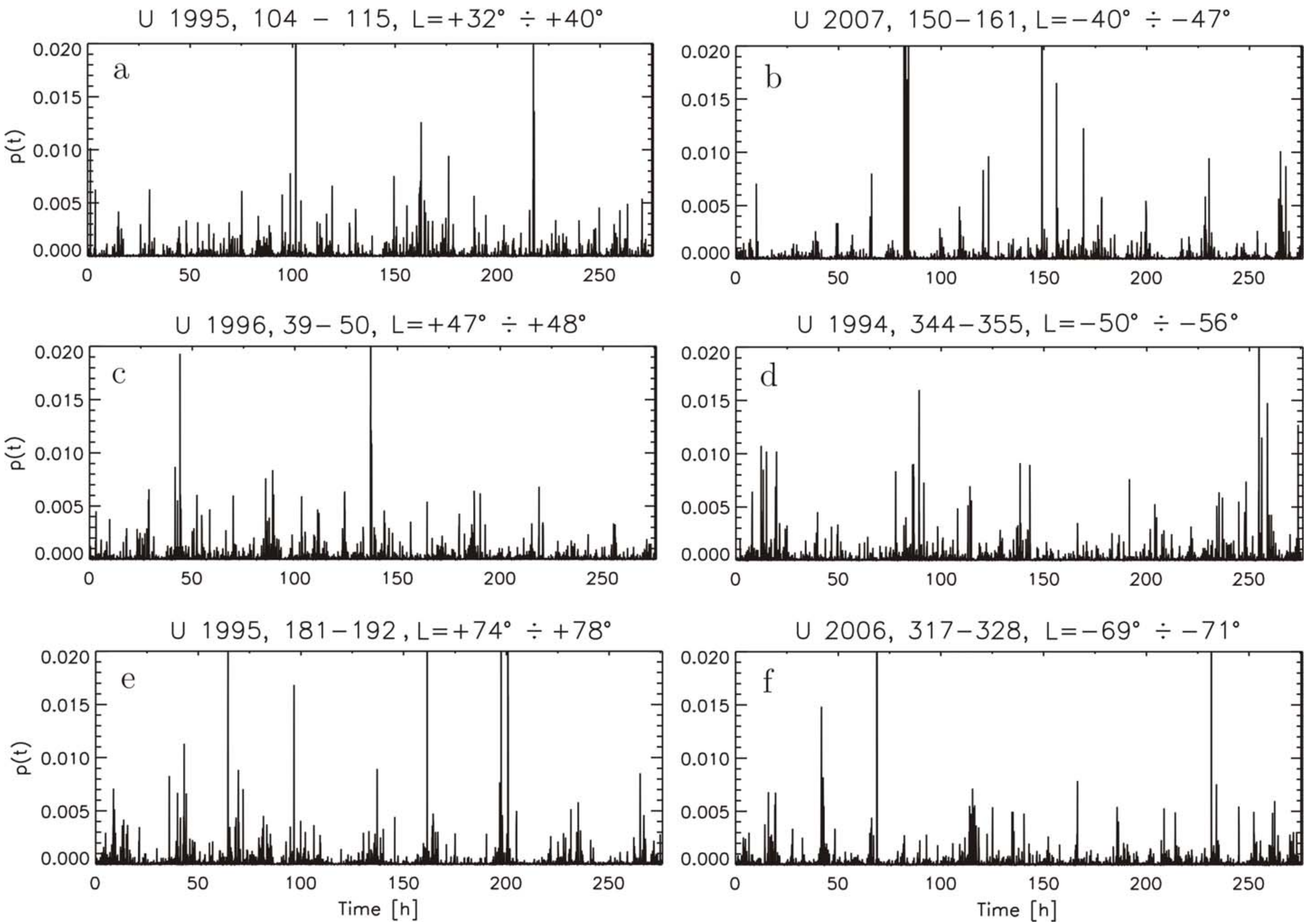
$$p(x_i,l)=\frac{\varepsilon(x_i,l)}{\sum_{i=1}^{N}\varepsilon(x_i,l)}=p_i(l)$$

Taylor's hypothesis: $x=\upsilon_{sw}t,\ l=\upsilon_{sw}\Delta t$

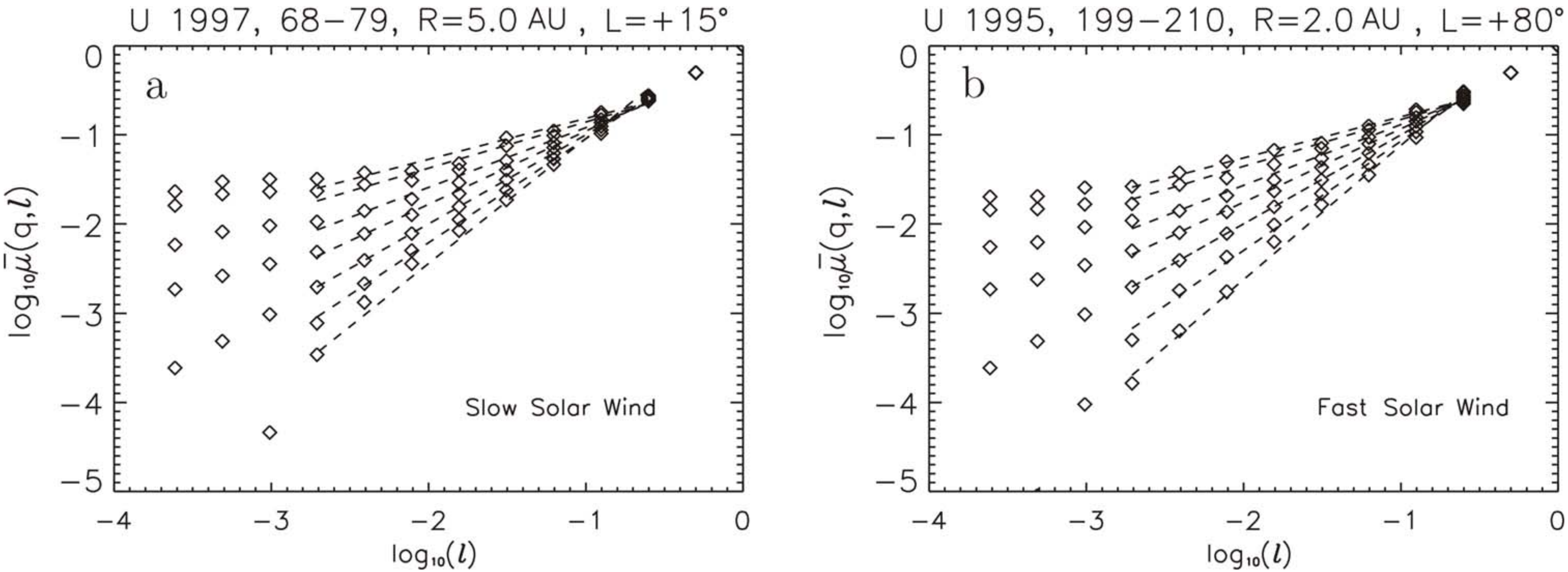
$$p(t_i,\Delta t)=\frac{\varepsilon(t_i,\Delta t)}{\sum_{i=1}^{N}\varepsilon(t_i,\Delta t)}$$

Energy Transfer Rate

$\Delta t = 242\ s$



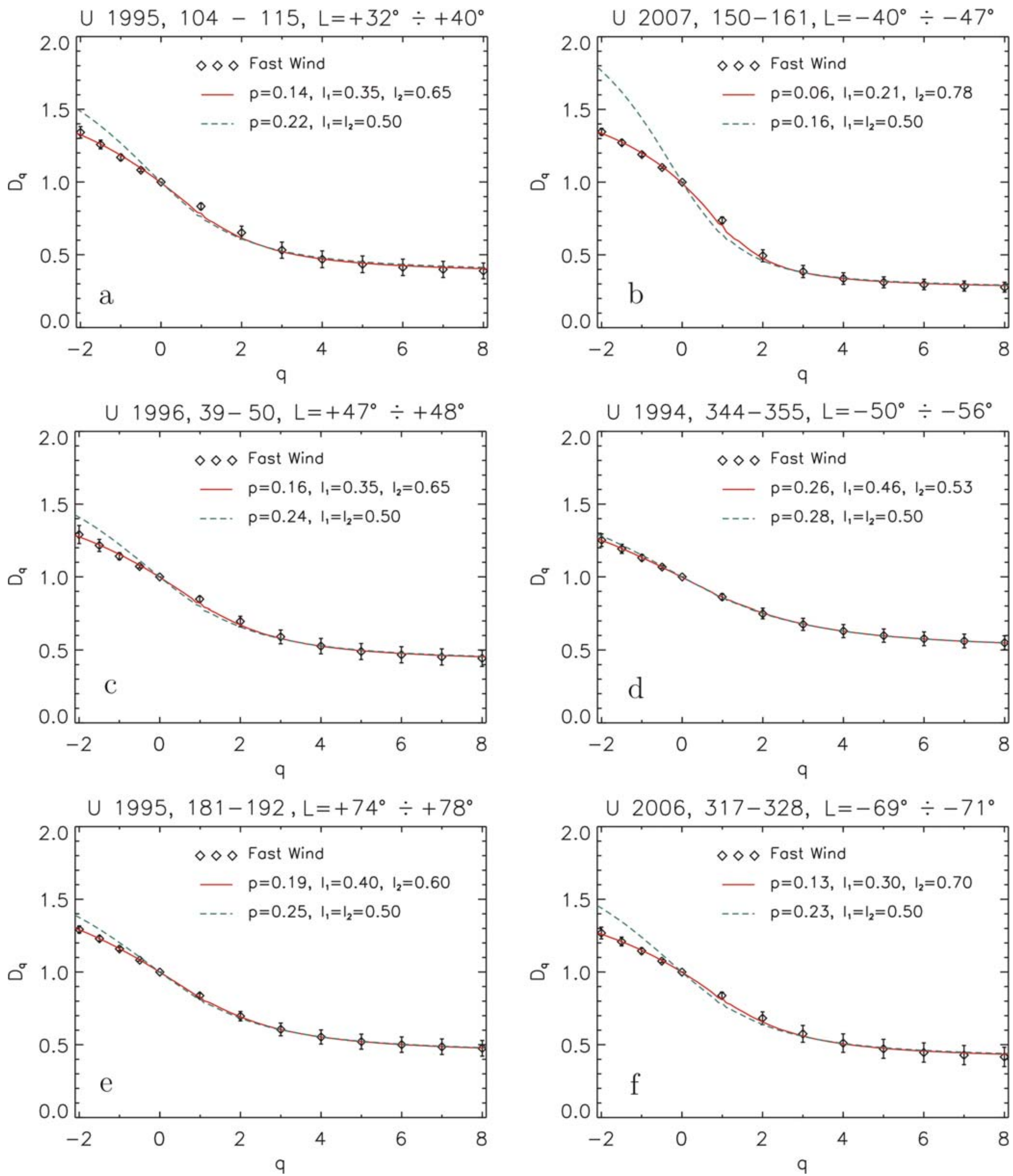
Multifractal Scaling



RANGE OF SCALING: $\Delta t_{\min} = 648 \text{ s}, \Delta \mathrm{t}_{\max} = 11.5 \text{ h}$

Figure: Plots of the generalized average probability of cascading eddies $\log_{10} \overline{\mu}(q,l)$ versus $\log_{10} l$ for the following values of *q:* 6,,4,2,1,0,-1,-2. These results are obtained using data measured by Ulysses during solar minimum (1995-1997) at 15° and 80° (diamonds) for the (a) slow and (b) fast solar wind, correspondingly.

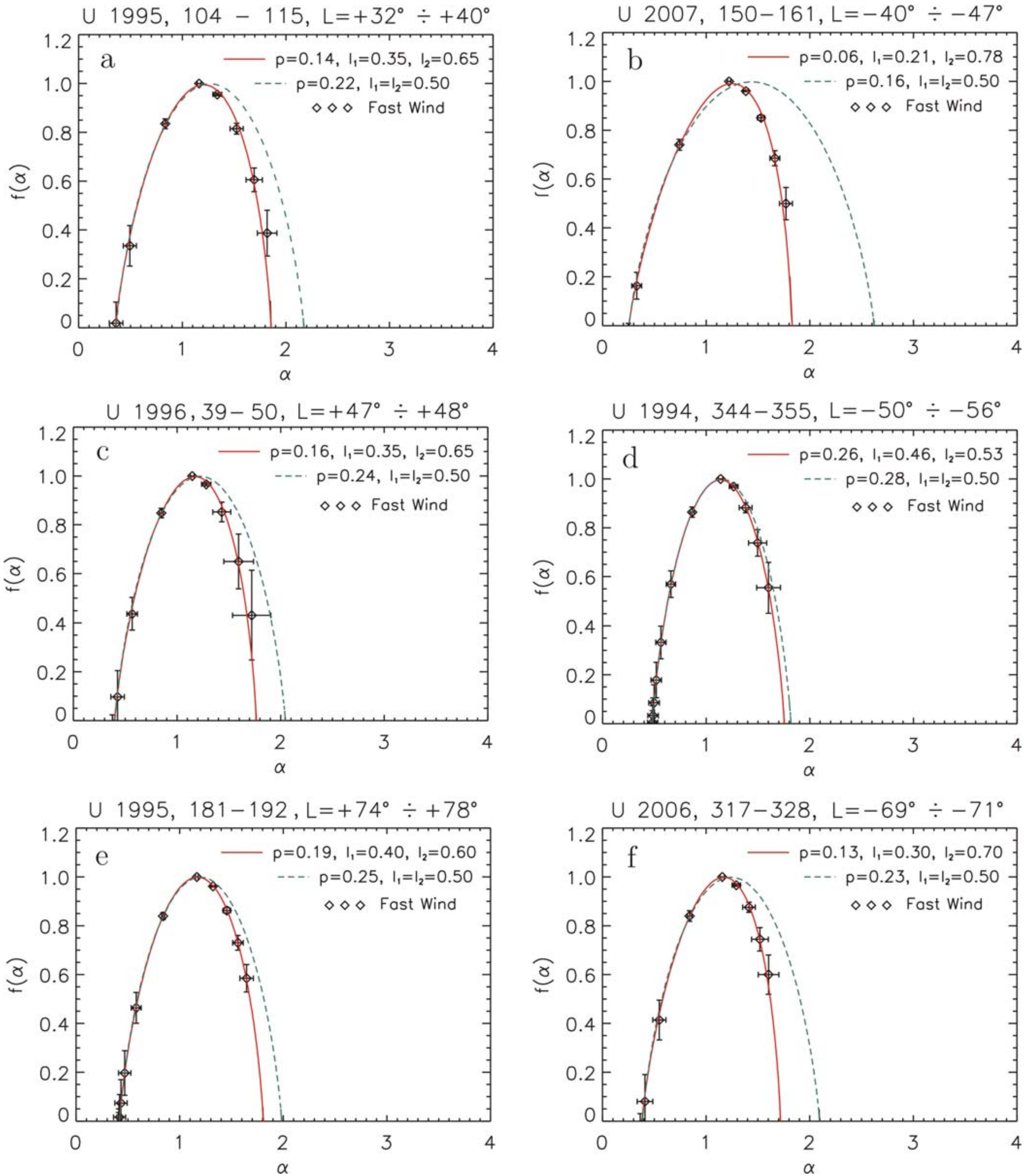
Generalized Dimensions



$$0.16 < p < 0.28$$

Wawrzaszek and Macek, JGR, A15176 (2010), in press.

Singularity Spectra



Wawrzaszek and Macek, JGR, A15176 (2010), in press.

Generalized Dimensions and Singularity Spectra

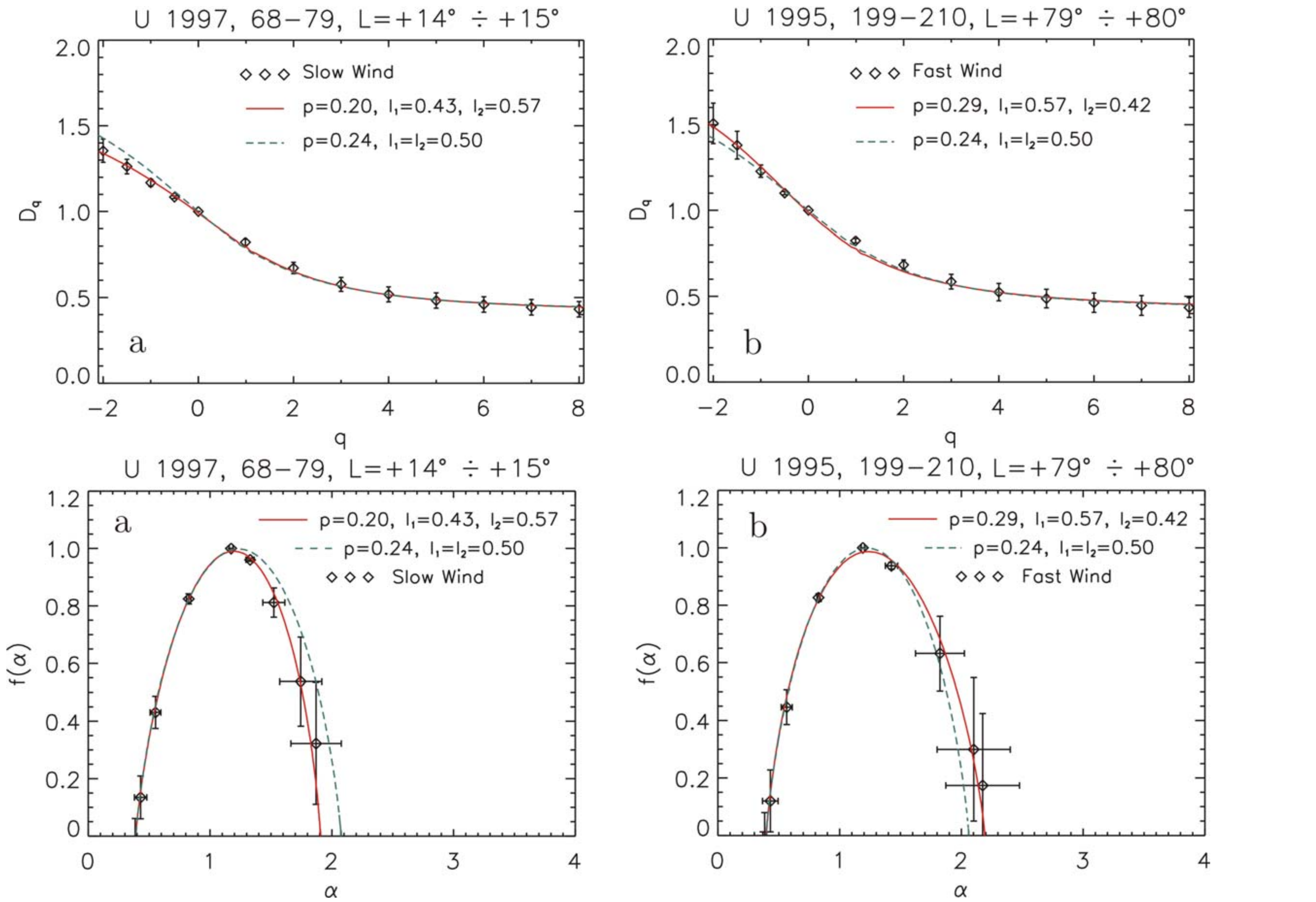


Figure : Generalized dimensions and singularity spectra for the slow (at 15°) and fast (at 80°) solar wind during solar minimum (1995-1997).

Multifractality - Latitudinal Dependence

Heliographic Latitude	Heliocentric Distance	Multifractality Δ	Asymmetry A
$+14^\circ \div +15^\circ$ (1997)*	4.9 AU	1.52 ± 0.22	1.14 ± 0.30
$+32^\circ \div +40^\circ$ (1995)	1.4 AU	1.50 ± 0.17	1.10 ± 0.20
$+47^\circ \div +48^\circ$ (1996)	3.3 AU	1.37 ± 0.18	1.21 ± 0.32
$+74^\circ \div +78^\circ$ (1995)	1.8 – 1.9 AU	1.39 ± 0.07	1.17 ± 0.12
$+79^\circ \div +80^\circ$ (1995)	1.9 – 2.0 AU	1.80 ± 0.28	0.80 ± 0.29
$-40^\circ \div -47^\circ$ (2007)	1.6 AU	1.57 ± 0.07	1.53 ± 0.20
$-50^\circ \div -56^\circ$ (1994)	1.6 – 1.7 AU	1.27 ± 0.12	1.07 ± 0.20
$-69^\circ \div -71^\circ$ (2006)	2.8 – 2.9 AU	1.34 ± 0.10	1.36 ± 0.25

Asterisk (*) denotes the case of the slow wind.

Multifractality - Latitudinal Dependence

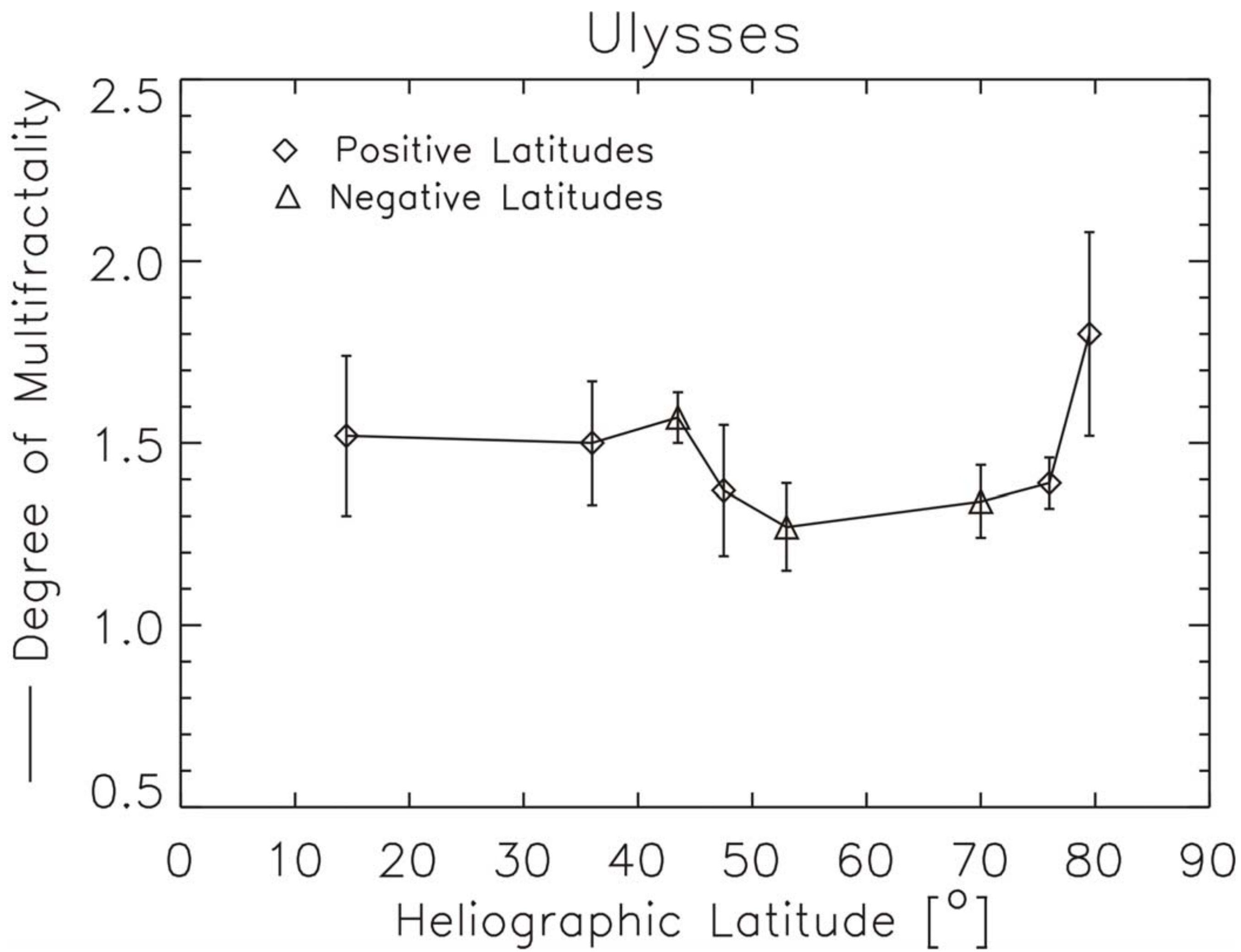


Figure: Degree of multifractality Δ (continuous line) for the slow (at 15°) and fast (above 15°) solar wind during solar minimum (1994-1996, 2006-2007) in dependence on heliographic latitude below (triangles) and above (diamonds) the ecliptic.

Asymmetry - Latitudinal Dependence

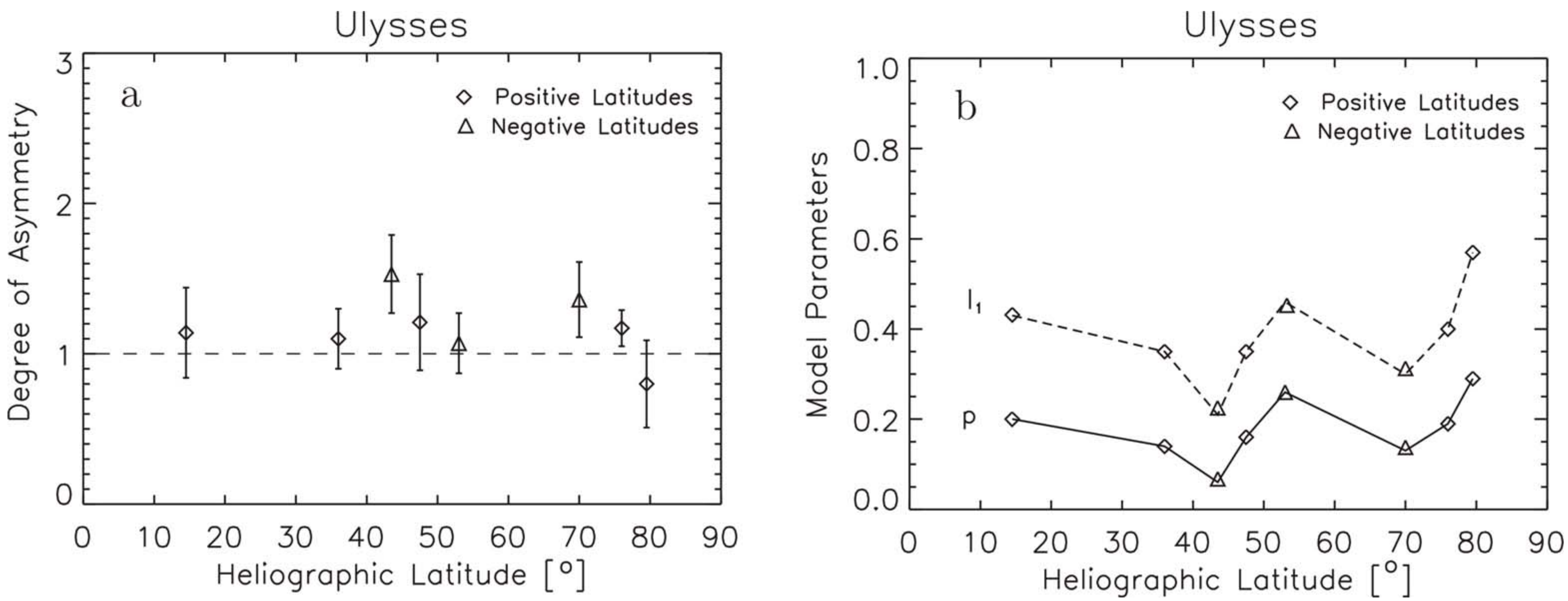


Figure: (a) Degree of asymmetry (*A*) and (b) change of two-scale model parameters *p* (diamonds) and l_1 (asterisk) in dependence on heliographic latitude during solar minimum (1994-1996, 2006-2007).

Conclusions

We have studied the inhomogeneous rate of the transfer of the energy flux indicating multifractal and intermittent behavior of solar wind turbulence out of the ecliptic plane. In particular:

• We have identified somewhat smaller degree of multifractality at high latitudes during solar minimum for the fast solar wind, where turbulence evolves more slowly as compared with that at the ecliptic.

• We have shown that the degree of multifractality and asymmetry of the fast solar wind exhibit latitudinal dependence with some symmetry with respect to the ecliptic plane.

• The multifractal singularity spectra out of ecliptic plane become roughly symmetric.

Conclusions

• The minimum intermittency is observed at mid-latitudes and is possibly related to the transition from the region where the interaction of the fast and slow streams takes place to a more homogeneous region of the pure fast solar wind.

Basically, the multifractal spectra for solar wind are consistent with the two-scale model for both positive and negative q, but rather with different scaling parameters for sizes of eddies in the ecliptic, while in most cases the usual p model is sufficient to reproduce the spectrum of the fast solar wind out of the ecliptic plane.


Thank you for your attention