Observation of the Multifractal Spectrum at the Termination Shock by Voyager 1

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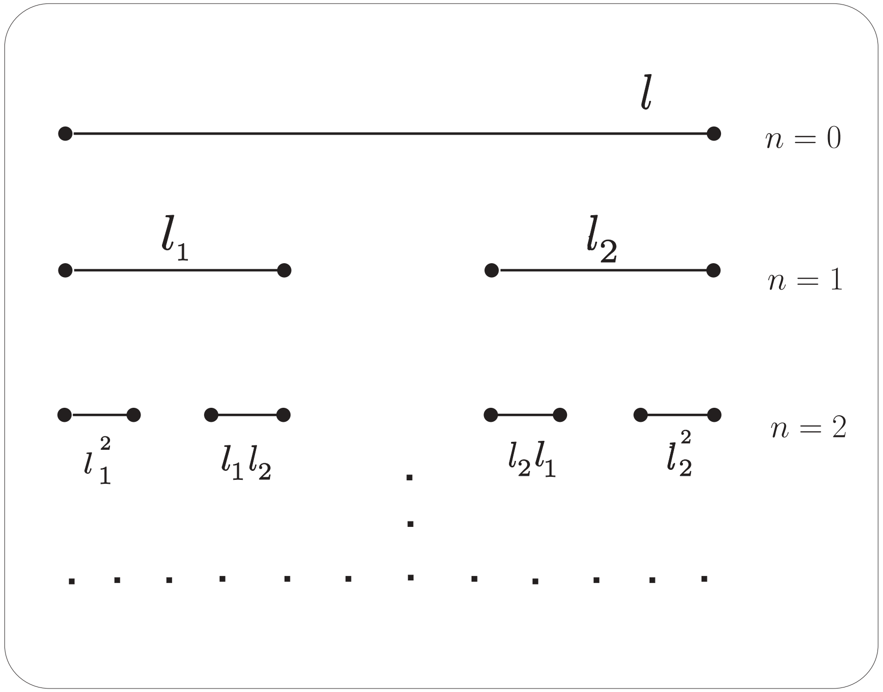
Prologue

A **fractal** is a rough or fragmented geometrical object that can be subdivided in parts, each of which is (at least approximately) a reduced-size copy of the whole. Fractals are generally *self-similar* and independent of scale (fractal dimension).

A **multifractal** is a set of intertwined fractals. Self-similarity of multifractals is scale dependent (spectrum of dimensions). A deviation from a strict self-similarity is also called **intermittency**.



Two-scale **Cantor** set.

Importance of Multifractality

Starting from seminal works of Kolmogorov (1941) and Kraichnan (1965) many authors have attempted to recover the observed scaling laws, by using multifractal phenomenological models of turbulence describing distribution of the energy flux between cascading eddies at various scales (Meneveau and Sreenivasan, 1987, Carbone, 1993, Frisch, 1995).

In particular, multifractal scaling of this flux in solar wind turbulence using Helios (plasma) data in the inner heliosphere has been analyzed by March et al. (1996). It is known that fluctuations of the solar magnetic fields may also exhibit multifractal scaling laws. The multifractal spectrum has been investigated using magnetic field data measured *in situ* by Voyager in the outer heliosphere up to large distances from the Sun (Burlaga, 1991, 1995, 2004) and even in the heliosheath (Burlaga and Ness, 2010; Burlaga et al., 2006).

To quantify scaling of solar wind turbulence we have developed a generalized two-scale weighted Cantor set model using the partition technique (Macek 2007; Macek and Szczepaniak, 2008), which leads to complementary information about the multifractal nature of the fluctuations as the rank-ordered multifractal analysis (cf. Lamy et al., 2010).

We have investigated the spectrum of generalized dimensions and the corresponding multifractal singularity spectrum depending on one probability measure parameter and two rescaling parameters. In this way we have looked at the inhomogeneous rate of the transfer of the energy flux indicating multifractal and intermittent behavior of solar wind turbulence. In particular, we have studied in detail fluctuations of the velocity of the flow of the solar wind, as measured in the inner heliosphere by Helios (Macek and Szczepaniak, 2008), Advanced Composition Explorer (ACE) (Szczepaniak and Macek, 2008), and Voyager in the outer heliosphere (Macek and Wawrzaszek, 2009), including Ulysses observations at high heliospheric latitudes (Wawrzaszek and Macek, 2010).

In December 2004 at distances of 94 AU from the Sun Voyager 1 crossed the termination heliospheric shock separating the Solar System plasma from the surrounding heliosheath and entered the subsonic solar wind, where it encountered quite unusual conditions. It is worth noting that magnetic fields in the heliosheath are normally distributed in contrast to the lognormal distribution in the outer heliosphere (Burlaga et al., 2005). It has also appeared that the magnetic field in the near heliosheath, at $\sim$95 AU, has a multifractal structure (Burlaga et al., 2006).

The results of the generalized dimensions and the multifractal spectrum obtained using the Voyager 1 data of the magnetic field strength have been discussed at distances of 83.4–85.9 AU from the Sun, i.e., before the termination shock crossing (Burlaga, 2004), and in the heliosheath at 94.2–97.2 AU (Burlaga et al., 2006) and 108.5–112.1 AU (Burlaga and Ness, 2010), correspondingly. The comparison of the weighted one-scale and two-scale Cantor set models and other formulae fitting the experimental data have been presented in Figures 3–6 of the paper by (Macek and Wawrzaszek, 2010), where the dependence on the various parameters of these models and the resulting degrees of multifractality and asymmetry have also been thoroughly discussed. In particular, space filling turbulence has been recovered (Burlaga et al., 1993).

The aim of this study is to examine the question of scaling properties of intermittent solar wind turbulence using our weighted two-scale Cantor set model at a wide range of the heliospheric distances focusing on the solar cycle variations. In particular, we show that the degree of multifractality modulated by the solar activity is also decreasing with distance: before shock crossing is greater than that in the heliosheath. Moreover, we demonstrate that the multifractal spectrum is asymmetric before shock crossing, in contrast to the nearly symmetric spectrum observed in the heliosheath; the solar wind in the outer heliosphere may exhibit strong asymmetric scaling.

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Fractal

A measure (volume) V of a set as a function of size l

$$V(l) \sim l^{D_{\mathrm{F}}}$$

The number of elements of size l needed to cover the set

$$N(l) \sim l^{-D_{\mathrm{F}}}$$

The fractal dimension

$$D_{\mathrm{F}} = \lim_{l \to 0} \frac{\ln N(l)}{\ln 1/l}$$

Multifractal

A (probability) measure versus singularity strength, α

$$p_i(l) \propto l^{\alpha_i}$$

The number of elements in a small range from α to $\alpha + \mathrm{d}\alpha$

$$N_l(\alpha) \sim l^{-f(\alpha)}$$

The multifractal singularity spectrum

$$f(\alpha) = \lim_{\varepsilon \to 0} \lim_{l \to 0} \frac{\ln[N_l(\alpha+\varepsilon) - N_l(\alpha-\varepsilon)]}{\ln 1/l}$$

The generalized dimension

$$D_q = \frac{1}{q-1} \lim_{l \to 0} \frac{\ln \sum_{k=1}^{N} (p_k)^q}{\ln l}$$

Multifractal Characteristics

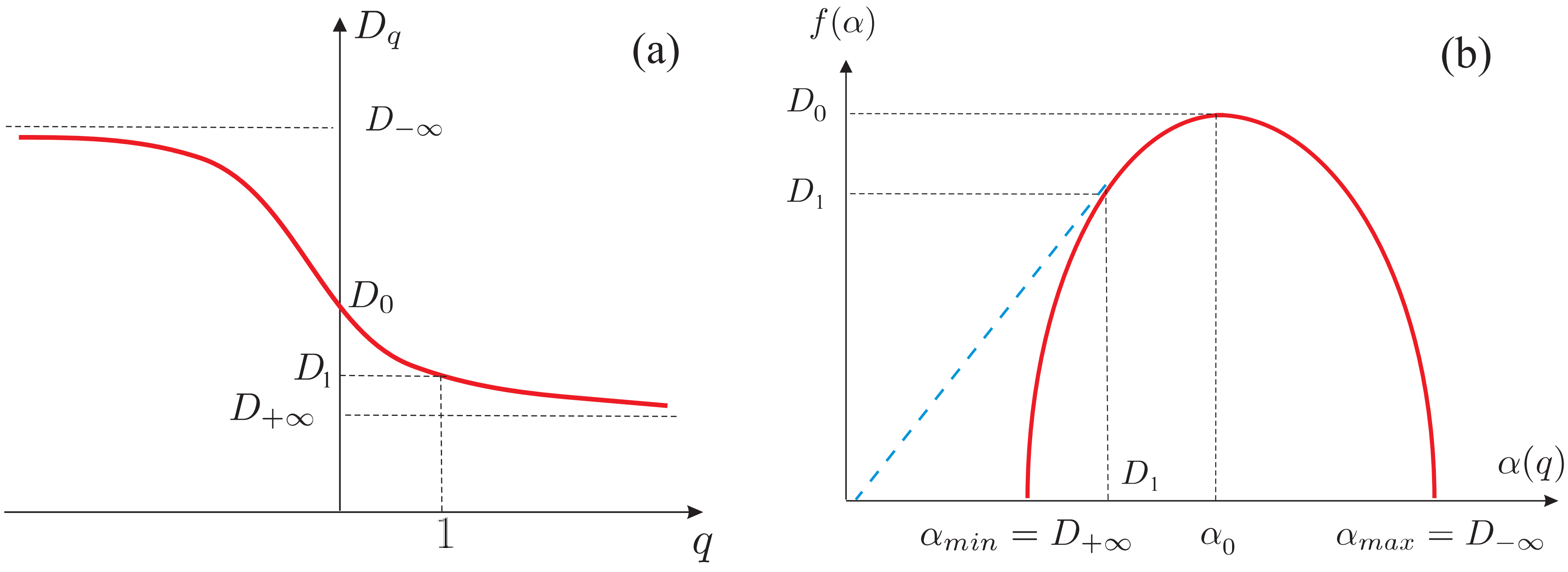


Fig. 1. (a) The generalized dimensions D_q as a function of any real q, $-\infty < q < \infty$, and (b) the singularity multifractal spectrum $f(\alpha)$ versus the singularity strength α with some general properties: (1) the maximum value of $f(\alpha)$ is D_0; (2) $f(D_1) = D_1$; and (3) the line joining the origin to the point on the $f(\alpha)$ curve where $\alpha = D_1$ is tangent to the curve (Ott *et al.*, 1994).

Generalized Scaling Property

The generalized dimensions are important characteristics of *complex* dynamical systems; they quantify multifractality of a given system (Ott, 1993). In the case of turbulence cascade these generalized measures are related to inhomogeneity with which the energy is distributed between different eddies (Meneveau and Sreenivasan, 1991). In this way they provide information about dynamics of multiplicative process of cascading eddies. Here high positive values of $q > 1$ emphasize regions of intense fluctuations larger than the average, while negative values of q accentuate fluctuations lower than the average (cf. Burlaga 1995).

Using ($\sum {p_i}^q \equiv \langle {p_i}^{q-1} \rangle_{\mathrm{av}}$) a generalized average probability measure

$$\bar{\mu}(q,l) \equiv \sqrt[q-1]{\langle (p_i)^{q-1} \rangle_{\mathrm{av}}} \tag{1}$$

we can identify D_q as scaling of the measure with size l

$$\bar{\mu}(q,l) \propto l^{D_q} \tag{2}$$

Hence, the slopes of the logarithm of $\bar{\mu}(q,l)$ of Eq. (2) versus $\log l$ (normalized) provides

$$D_q = \lim_{l \to 0} \frac{\log \bar{\mu}(q,l)}{\log l} \tag{3}$$

Measures and Multifractality

Similarly, we define a one-parameter q family of (normalized) generalized pseudoprobability measures (Chhabra and Jensen, 1989; Chhabra *et al.*, 1989)

$$\mu_i(q,l) \equiv \frac{p_i^q(l)}{\sum_{i=1}^{N} p_i^q(l)} \tag{4}$$

Now, with an associated fractal dimension index $f_i(q,l) \equiv \log \mu_i(q,l)/\log l$ for a given q the multifractal singularity spectrum of dimensions is defined directly as the average taken with respect to the measure $\mu(q,l)$ in Eq. (4) denoted by $\langle \ldots \rangle$

$$f(q) \equiv \lim_{l \to 0} \sum_{i=1}^{N} \mu_i(q,l)\; f_i(q,l) = \lim_{l \to 0} \frac{\langle \log \mu_i(q,l) \rangle}{\log(l)} \tag{5}$$

and the corresponding average value of the singularity strength is given by
(Chhabra and Jensen, 1987)

$$\alpha(q) \equiv \lim_{l \to 0} \sum_{i=1}^{N} \mu_i(q,l)\; \alpha_i(l) = \lim_{l \to 0} \frac{\langle \log p_i(l) \rangle}{\log(l)}. \tag{6}$$

Methods of Data Analysis

Magnetic Field Strength Fluctuations and Generalized Measures

Given the normalized time series $B(t_i)$, where $i = 1, \ldots, N = 2^n$ (we take n = 8), to each interval of temporal scale Δt (using $\Delta t = 2^k$, with $k = 0, 1, \ldots, n$) we associate some probability measure

$$p(x_j, l) \equiv \frac{1}{N} \sum_{i=1+(j-1)\Delta t}^{j\Delta t} B(t_i) = p_j(l), \tag{7}$$

where $j = 2^{n-k}$, i.e., calculated by using the successive (daily) average values $\langle B(t_i, \Delta t)\rangle$ of $B(t_i)$ between t_i and $t_i + \Delta t$. At a position $x = v_{\rm sw} t$, at time t, where $v_{\rm sw}$ is the average solar wind speed, this quantity can be interpreted as a probability that the magnetic flux is transferred to a segment of a spatial scale $l = v_{\rm sw}\Delta t$ (Taylor's hypothesis).

The average value of the qth moment of the magnetic field strength B should scale as

$$\langle B^q(l)\rangle \sim l^{\gamma(q)}, \tag{8}$$

with the exponent $\gamma(q) = (q-1)(D_q - 1)$ as shown by Burlaga et al. (1995).

Mutifractal Models for Turbulence

$$p_1 + p_2 = 1$$

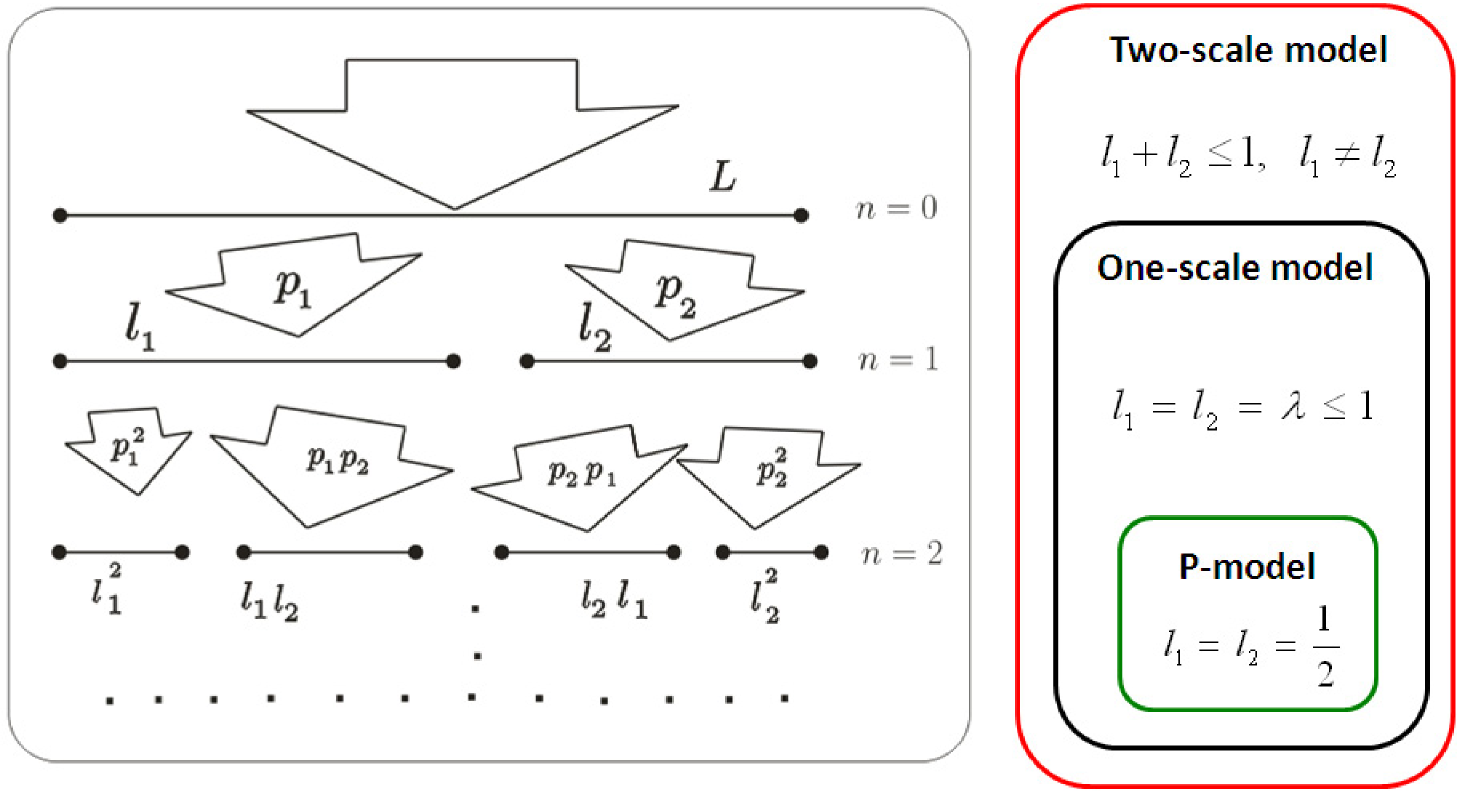


Fig. 1. Generalized two-scale Cantor set model for turbulence (Macek, 2007).

Solutions

Transcendental equation (for $n \to \infty$)

$$\frac{p^q}{l_1^{\tau(q)}} + \frac{(1-p)^q}{l_2^{\tau(q)}} = 1 \tag{9}$$

For $l_1 = l_2 = \lambda$ and any q in Eq. (9) one has for the generalized dimensions

$$\tau(q) \equiv (q-1)D_q = \frac{\ln[p^q + (1-p)^q]}{\ln \lambda}. \tag{10}$$

Space filling turbulence ($\lambda = 1/2$):
the multifractal cascade $p-$model for fully developed turbulence,
the generalized weighted Cantor set (Meneveau and Sreenivasan, 1987).
The usual middle one-third Cantor set (without any multifractality):
$p = 1/2$ and $\lambda = 1/3$.

Degree of Multifractality and Asymmetry

The difference of the maximum and minimum dimension (the least dense and most dense points in the phase space) is given, e.g., by Macek (2006, 2007)

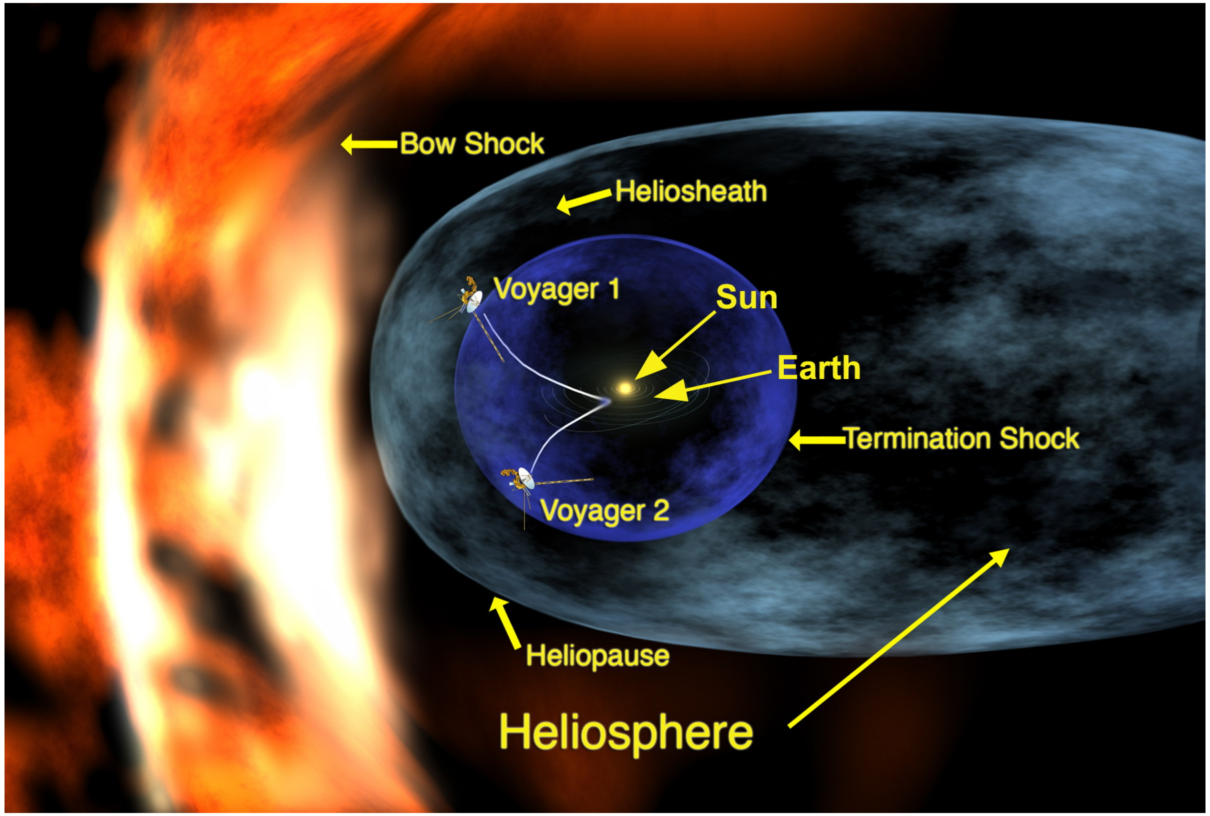
$$\Delta \equiv \alpha_{\max} - \alpha_{\min} = D_{-\infty} - D_{\infty} = \left| \frac{\log(1-p)}{\log l_2} - \frac{\log(p)}{\log l_1} \right|. \tag{11}$$

In the limit $p \to 0$ this difference rises to infinity (degree of multifractality).

The degree of multifractality Δ is simply related to the deviation from a simple self-similarity. That is why Δ is also a measure of intermittency, which is in contrast to self-similarity (Frisch, 1995, chapter 8).

Using the value of the strength of singularity α_0 at which the singularity spectrum has its maximum $f(\alpha_0) = 1$ we define a measure of asymmetry by

$$A \equiv \frac{\alpha_0 - \alpha_{\min}}{\alpha_{\max} - \alpha_0}. \tag{12}$$



Schematic of the Heliospheric Boundaries.

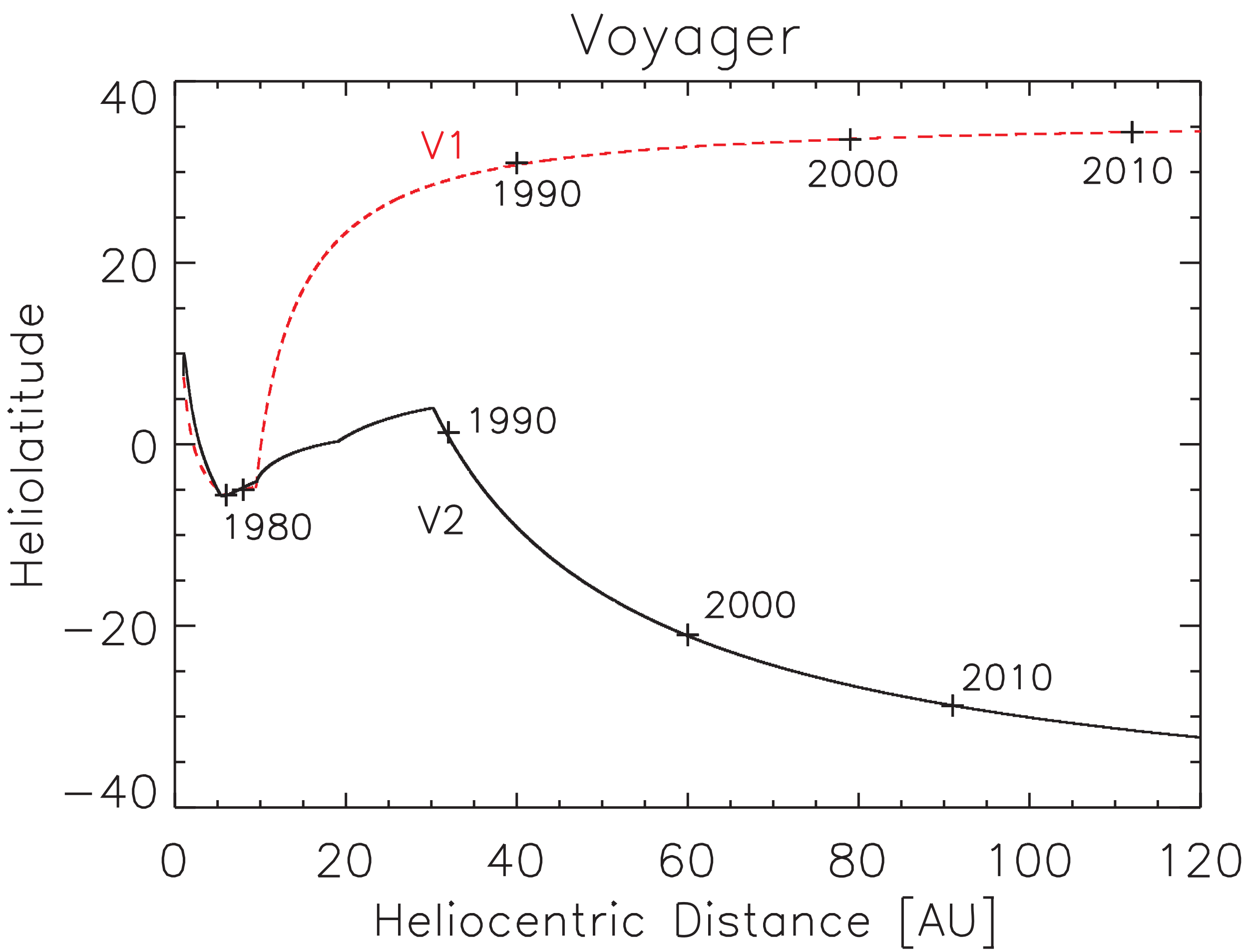


Fig. 2. The heliospheric distances from the Sun and the heliographic latitudes during each year of the Voyager mission. Voyager 1 and 2 spacecraft are located above and below the solar equatorial plane, respectively.

Voyager Spacecraft

7 – 60 AU (1980 – 1995)
70 – 90 AU (1999 – 2003)
95 – 107 AU (2005 – 2008)

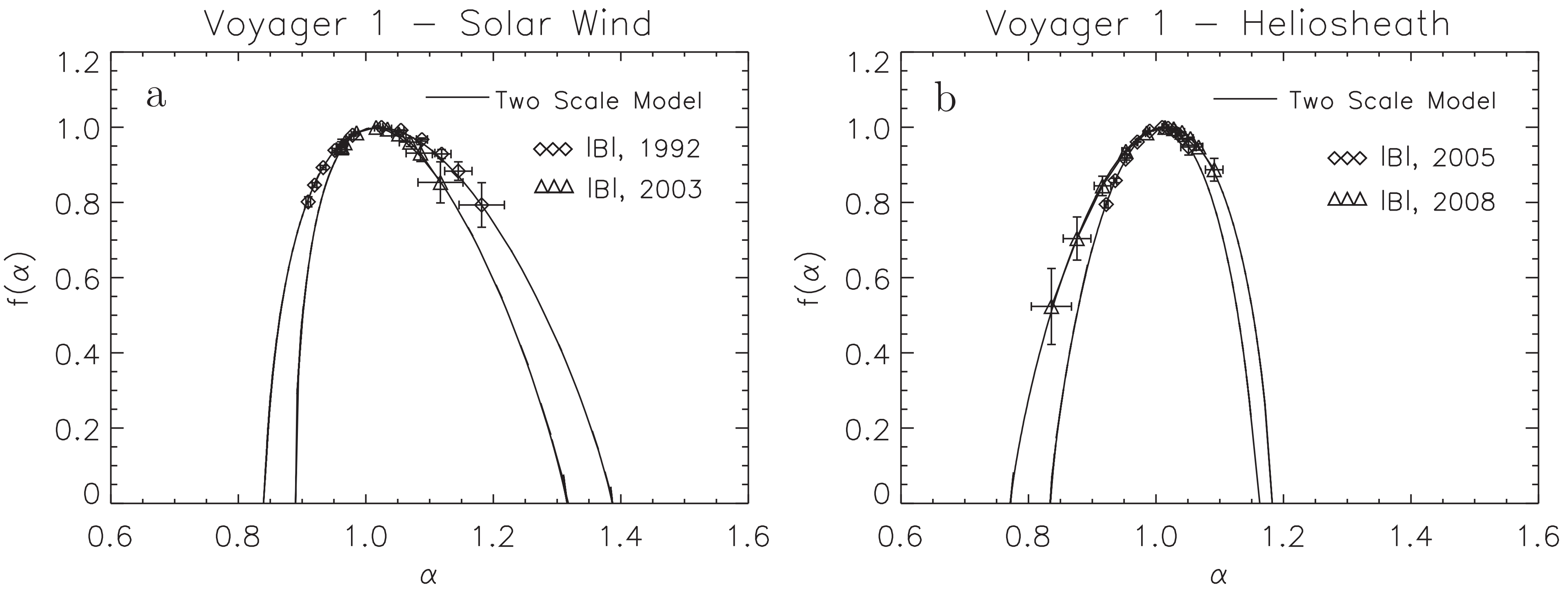


Fig. 1. The multifractal singularity spectrum of the magnetic fields observed by Voyager 1 (a) in the solar wind near 50 and 90 AU (1992, diamonds, and 2003, triangles) and (b) in the heliosheath near 95 and 105 AU (2005, diamonds, and 2008, triangles) together with a fit to the two-scale model (solid curve), suggesting change of the symmetry of the spectrum at the termination shock (Macek et al. 2011).

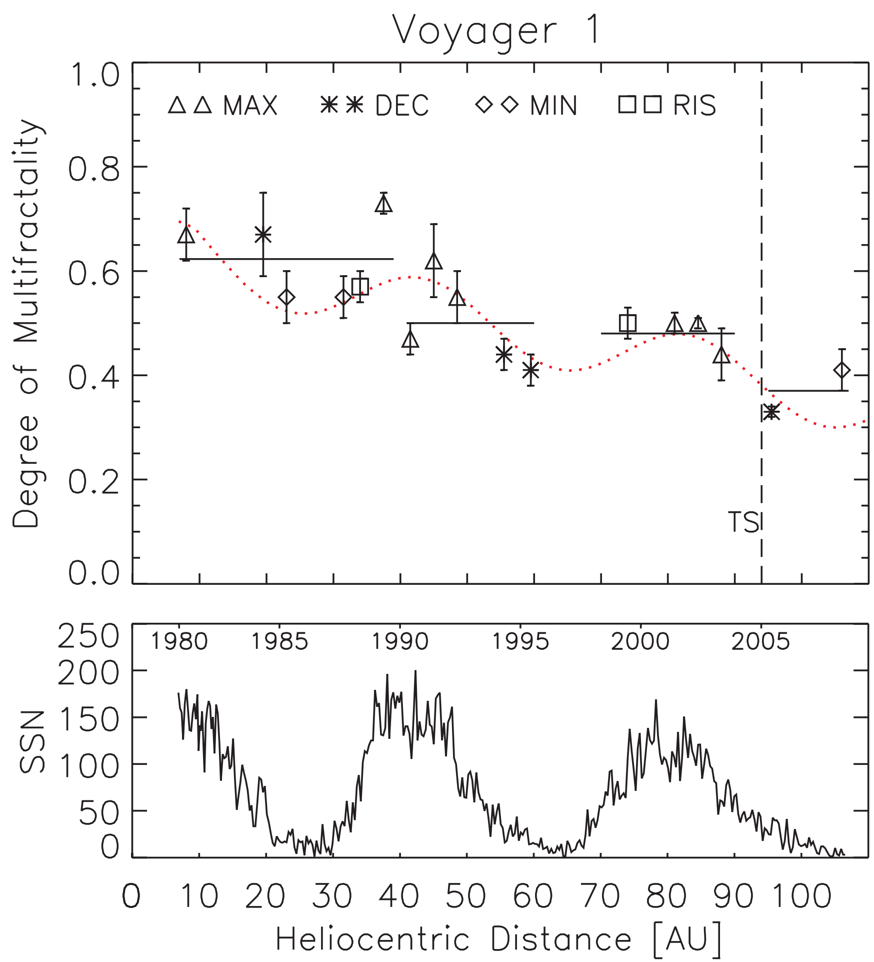


Fig. 2. The degree of multifractality Δ in the heliosphere versus the heliospheric distances compared to a periodically decreasing function (dotted) during solar minimum (MIN) and solar maximum (MAX), declining (DEC) and rising (RIS) phases of solar cycles, with the corresponding averages shown by continuous lines. The crossing of the termination shock (TS) by Voyager 1 is marked by a vertical dashed line. Below is shown the Sunspot Number (SSN) during years 1980–2008 (Macek et al. 2011).

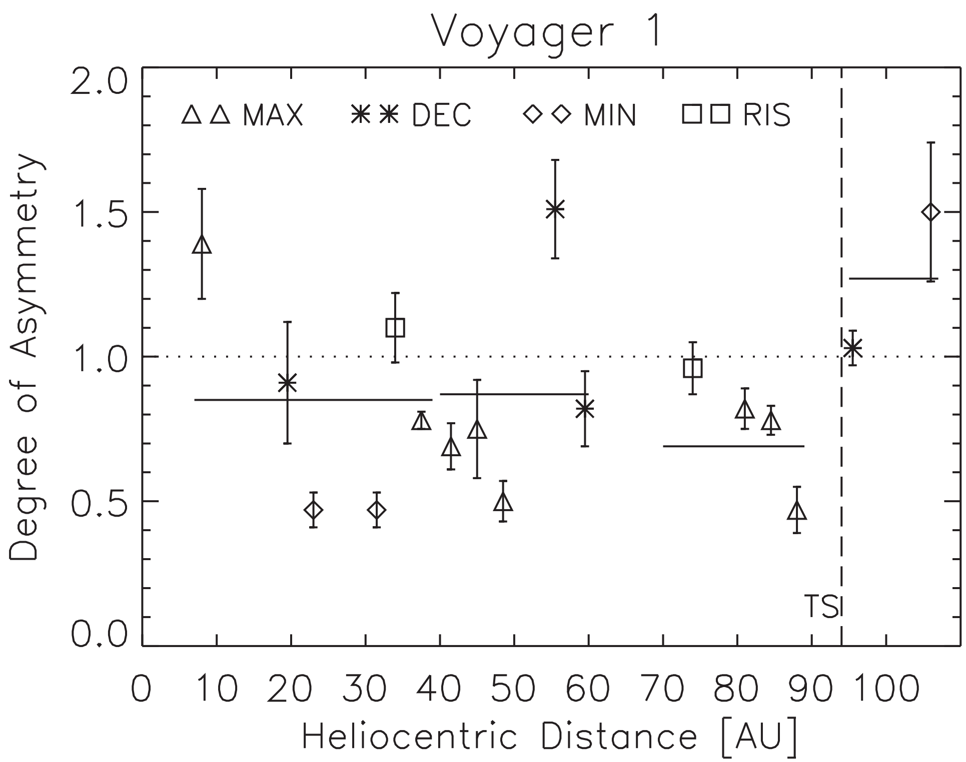


Fig. 3. The degree of asymmetry A of the multifractal spectrum in the heliosphere as a function of the heliospheric distance during solar minimum (MIN) and solar maximum (MAX), declining (DEC) and rising (RIS) phases of solar cycles with the corresponding averages denoted by continuous lines; the value $A = 1$ (dotted) corresponds to the one-scale symmetric model. The crossing of the termination shock (TS) by Voyager 1 is marked by a vertical dashed line (Macek et al. 2011).

Termination shock ⇒ Heliosheath

	7 − 40 **AU**	40 − 60 **AU**	70 − 90 **AU**	95 − 107 **AU**
Burlaga	$\mathbf{\Delta} = 0.64$ $\mathbf{A} = 0.69$	$\mathbf{\Delta} = 0.69$ $\mathbf{A} = 0.63$	$\mathbf{\Delta} = 0.69$ $\mathbf{A} = 0.63$	$\mathbf{\Delta} = 0.34$ $\mathbf{A} = 0.89$
Two-Scale Model	$\mathbf{\Delta} = 0.55 - 0.73$ $\mathbf{A} = 0.47 - 1.39$	$\mathbf{\Delta} = 0.41 - 0.62$ $\mathbf{A} = 0.51 - 1.51$	$\mathbf{\Delta} = 0.44 - 0.50$ $\mathbf{A} = 0.47 - 0.96$	$\mathbf{\Delta} = 0.33 - 0.41$ $\mathbf{A} = 1.03 - 1.51$

Heliopause ⇒

Figure 1: Degree of multifractality Δ and asymmetry A for the magnetic field strengths in the outer heliosphere and beyond the termination shock (Macek et al. 2011).

Conclusions

- Using our weighted two-scale Cantor set model, which is a convenient tool to investigate the asymmetry of the multifractal spectrum, we confirm the characteristic shape of the universal multifractal singularity spectrum. $f(\alpha)$ is a downward concave function of scaling indices α.
- For the first time we show that the degree of multifractality for magnetic field fluctuations of the solar wind falls steadily with the distance from the Sun and seems to be modulated by the solar activity.
- Moreover, we have considered the multifractal spectra of fluctuations of the interplanetary magnetic field strength before and after shock crossing by Voyager 1. In contrast to the right-skewed asymmetric spectrum with singularity strength $\alpha > 1$ inside the heliosphere, the spectrum becomes more left-skewed, $\alpha < 1$, or approximately symmetric after the shock crossing in the heliosheath, where the plasma is expected to be roughly in equilibrium in the transition to the interstellar medium.
- We also confirm the results obtained by Burlaga et al. (2006) that before the shock crossing, especially during solar maximum, turbulence is more multifractal than that in the heliosheath.

- In addition, outside the heliosphere during solar minimum the spectrum seems to be dominated by values of $\alpha < 1$. This is very interesting because it represents a first direct *in situ* information of interest in the astrophysical context beyond the heliosphere. In fact, a density of measure dominated by $\alpha < 1$ implies that the magnetic field in the very local interstellar medium is roughly confined in thin filaments of high magnetic density.
- That is the heliosheath is possibly more dominated by 'voids' of magnetic fields, thus implying that magnetic turbulence tends to be more 'passive' (cf. Frisch, 1995) in the very local interstellar medium.
- These informations, obtained *in situ* rather than through scintillations observations, are relevant in the context of interstellar turbulence confirming stellar formation modeling (e.g., Spangler et al. 2009), related to the presence of very localized intense magnetic field structures.

Epilogue

Within the complex dynamics of the solar wind's and interstellar medium fluctuating intermittent plasma parameters, there is a detectable, hidden order described by a Cantor set that exhibits a multifractal structure.

This means that the observed **intermittent** behavior of magnetic fluctuations in the heliosphere and the very local interstellar medium may result from intrinsic *nonlinear* dynamics rather than from random external forces.

Thank you!