Time Series Multifractal Analysis of Turbulent Magnetized Plasmas in the Solar System

Wiesław M. Macek

Space Research Centre, Polish Academy of Sciences,

Bartycka 18 A, 00-716 Warsaw, Poland;

Faculty of Mathematics and Natural Sciences, Cardinal Stefan Wyszyński University,

Wóycickiego 1/3, 01-938 Warsaw, Poland;

e-mail: macek@cbk.waw.pl, http://www.cbk.waw.pl/∼macek

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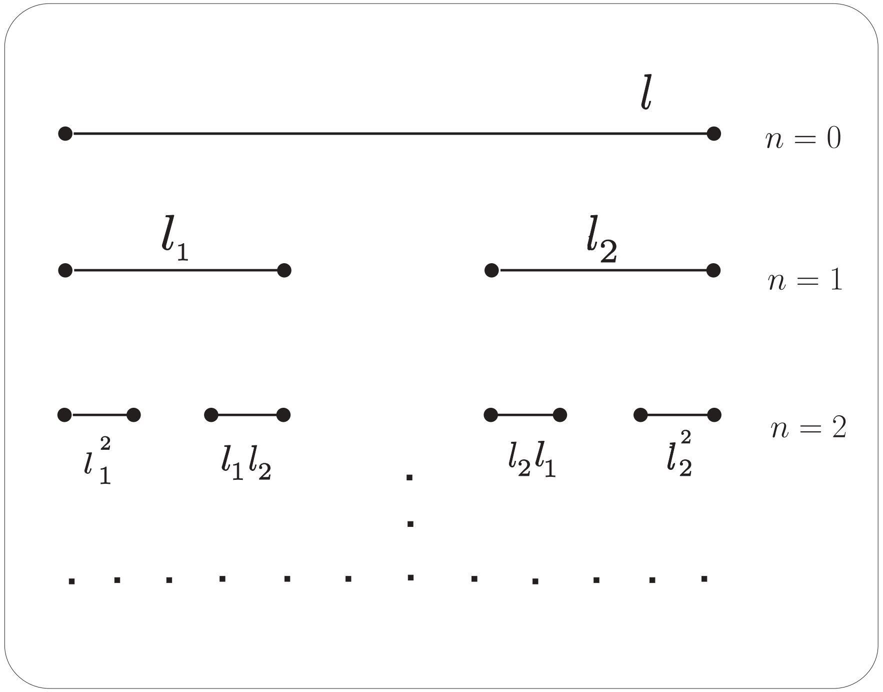
Prologue

A **fractal** is a rough or fragmented geometrical object that can be subdivided in parts, each of which is (at least approximately) a reduced-size copy of the whole. Fractals are generally *self-similar* and independent of scale (fractal dimension).

A **multifractal** is a set of intertwined fractals. Self-similarity of multifractals is scale dependent (spectrum of dimensions). A deviation from a strict self-similarity is also called **intermittency**.



Two-scale **Cantor** set.

Importance of Multifractality

Turbulent behavior of the solar wind plasma with the embedded magnetic field may exhibit some unexpected regularity decribed by multifractal scaling laws. The multifractal spectrum of this complex system has been investigated using magnetic field data measured *in situ* by Voyager in the outer heliosphere up to large distances from the Sun (Burlaga, 1991, 1995, 2004) and even in the heliosheath (Burlaga and Ness, 2010, 2012; Burlaga et al., 2006). In addition, Multifractal scaling of the energy flux in solar wind turbulence using Helios (plasma) data in the inner heliosphere has been analyzed by March et al. (1996).

To quantify scaling of solar wind turbulence we have developed a generalized two-scale weighted Cantor set model using the partition technique (Macek 2007; Macek and Szczepaniak, 2008), We have investigated the spectrum of generalized dimensions and the corresponding multifractal singularity spectrum depending on one probability measure parameter and two rescaling parameters. In this way we have looked at the inhomogeneous rate of the transfer of the energy flux indicating multifractal and intermittent behavior of solar wind turbulence. In particular, we have studied in detail fluctuations of the velocity of the flow of the solar wind, as measured in the inner heliosphere by Helios (Macek and Szczepaniak, 2008), Advanced Composition Explorer (ACE) (Szczepaniak and Macek, 2008), and Voyager in the outer heliosphere (Macek and Wawrzaszek, 2009), including Ulysses observations at high heliospheric latitudes (Wawrzaszek and Macek, 2010).

Voyager 1 crossed the termination heliospheric shock, which separates the Solar System plasma from the surrounding heliosheath, with the subsonic solar wind, on 16 December 2004 at heliocentric distances of 94 AU (at present its distance to the Sun is about 128 AU after crossing the heliopause; Strumik et al., 2013; Gurnett et al., 2013). Please note that (using the pressure balance) the distance to the nose of the heliopause has been estimated to be $\sim$120 AU (Macek, 1998). Later, in 2007 also Voyager 2 crossed the termination shock at least five times at distances of 84 AU (now is at 105 AU).

Variations of the magnetic field strength observed by Voyager 2 have also been analyzed including those prior and after crossing the termination shock up to distances of $\sim$90 AU in 2009 (from 2007.7 to 2009.4), see (Burlaga et al., 2008, 2009, 2010).

In our GRL letter (2011) we have shown that multifractal turbulence is modulated by the solar activity, and the degree of multifractality is decreasing with distance. We have further investigated the multifractal scaling for both Voyager 1 and 2 data that allowed us to infer more information about the heliospheric magnetic fields in both the northern and southern hemisphere, including the correlation with the solar cycle (Macek et al., 2012, Macek and Wawrzaszek, 2013).

In this paper we extend our analysis further in the heliosheath ahead of the heliopause.

In particular, we have confirmed that multifractal structure is modulated by the solar activity, but with some time delay, and the degree of multifractality is in fact decreasing with distance: before shock crossing is greater than that in the heliosheath.

Moreover, we have demonstrated that the multifractal spectrum is asymmetric before shock crossing, in contrast to the nearly symmetric spectrum observed in the heliosheath; the solar wind in the outer heliosphere may exhibit strong asymmetric scaling, where the spectrum is prevalently right-skewed.

The obtained delay between Voyager 1 and 2 can certainly be correlated with the evolution of the heliosphere, providing an additional support to some earlier independent claims that the solar wind termination shock itself is possibly asymmetric (Stone et al., 2008).

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Fractal

A measure (volume) V of a set as a function of size l

$$V(l) \sim l^{D_\mathrm{F}}$$

The number of elements of size l needed to cover the set

$$N(l) \sim l^{-D_\mathrm{F}}$$

The fractal dimension

$$D_\mathrm{F} = \lim_{l \to 0} \frac{\ln N(l)}{\ln 1/l}$$

Multifractal

A (probability) measure versus singularity strength, α

$$p_i(l) \propto l^{\alpha_i}$$

The number of elements in a small range from α to $\alpha + \mathrm{d}\alpha$

$$N_l(\alpha) \sim l^{-f(\alpha)}$$

The multifractal singularity spectrum

$$f(\alpha) = \lim_{\varepsilon \to 0} \ \lim_{l \to 0} \frac{\ln[N_l(\alpha+\varepsilon) - N_l(\alpha-\varepsilon)]}{\ln 1/l}$$

The generalized dimension

$$D_q = \frac{1}{q-1} \lim_{l \to 0} \frac{\ln \sum_{k=1}^{N} (p_k)^q}{\ln l}$$

Multifractal Characteristics

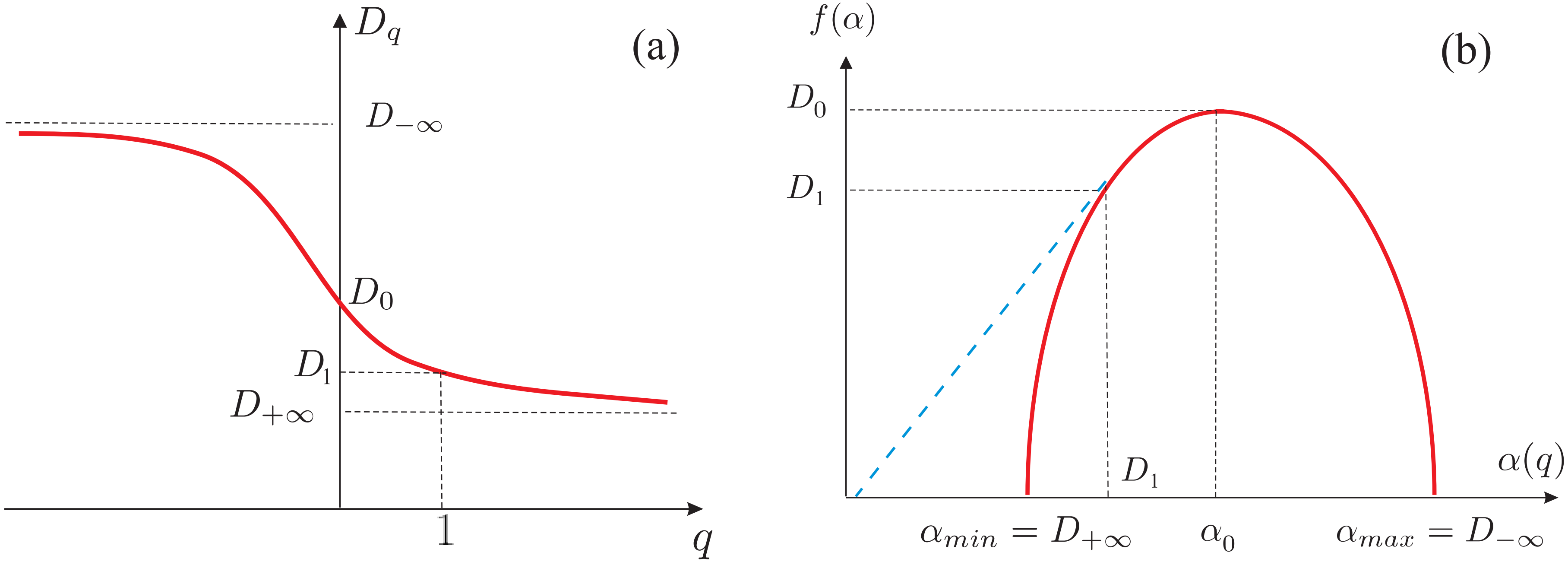


Fig. 1. (a) The generalized dimensions D_q as a function of any real q, $-\infty < q < \infty$, and (b) the singularity multifractal spectrum $f(\alpha)$ versus the singularity strength α with some general properties: (1) the maximum value of $f(\alpha)$ is D_0; (2) $f(D_1) = D_1$; and (3) the line joining the origin to the point on the $f(\alpha)$ curve where $\alpha = D_1$ is tangent to the curve (Ott *et al.*, 1994).

Measures and Multifractality

We define a one-parameter q family of (normalized) generalized pseudoprobability measures (Chhabra and Jensen, 1989; Chhabra *et al.*, 1989)

$$\mu_i(q,l) \equiv \frac{p_i^q(l)}{\sum_{i=1}^N p_i^q(l)} \tag{1}$$

Now, with an associated fractal dimension index $f_i(q,l) \equiv \log \mu_i(q,l)/\log l$ for a given q the multifractal singularity spectrum of dimensions is defined directly as the average taken with respect to the measure $\mu_i(q,l)$ in Eq. (1) denoted by $\langle \ldots \rangle$

$$f(q) \equiv \lim_{l \to 0} \sum_{i=1}^N \mu_i(q,l)\, f_i(q,l) = \lim_{l \to 0} \frac{\langle \log \mu_i(q,l) \rangle}{\log(l)} \tag{2}$$

and the corresponding average value of the singularity strength is given by
(Chhabra and Jensen, 1987)

$$\alpha(q) \equiv \lim_{l \to 0} \sum_{i=1}^N \mu_i(q,l)\, \alpha_i(l) = \lim_{l \to 0} \frac{\langle \log p_i(l) \rangle}{\log(l)}. \tag{3}$$

Methods of Data Analysis

Magnetic Field Strength Fluctuations and Generalized Measures

Given the normalized time series $B(t_i)$, where $i = 1, \ldots, N = 2^n$ (we take n = 8), to each interval of temporal scale Δt (using $\Delta t = 2^k$, with $k = 0, 1, \ldots, n$) we associate some probability measure

$$p(x_j, l) \equiv \frac{1}{N} \sum_{i=1+(j-1)\Delta t}^{j\Delta t} B(t_i) = p_j(l), \tag{4}$$

where $j = 2^{n-k}$, i.e., calculated by using the successive (daily) average values $\langle B(t_i, \Delta t)\rangle$ of $B(t_i)$ between t_i and $t_i + \Delta t$. At a position $x = v_{\rm sw} t$, at time t, where $v_{\rm sw}$ is the average solar wind speed, this quantity can be interpreted as a probability that the magnetic flux is transferred to a segment of a spatial scale $l = v_{\rm sw}\Delta t$ (Taylor's hypothesis).

The average value of the qth moment of the magnetic field strength B should scale as

$$\langle B^q(l)\rangle \sim l^{\gamma(q)}, \tag{5}$$

with the exponent $\gamma(q) = (q-1)(D_q - 1)$ as shown by Burlaga et al. (1995).

Mutifractal Models for Turbulence

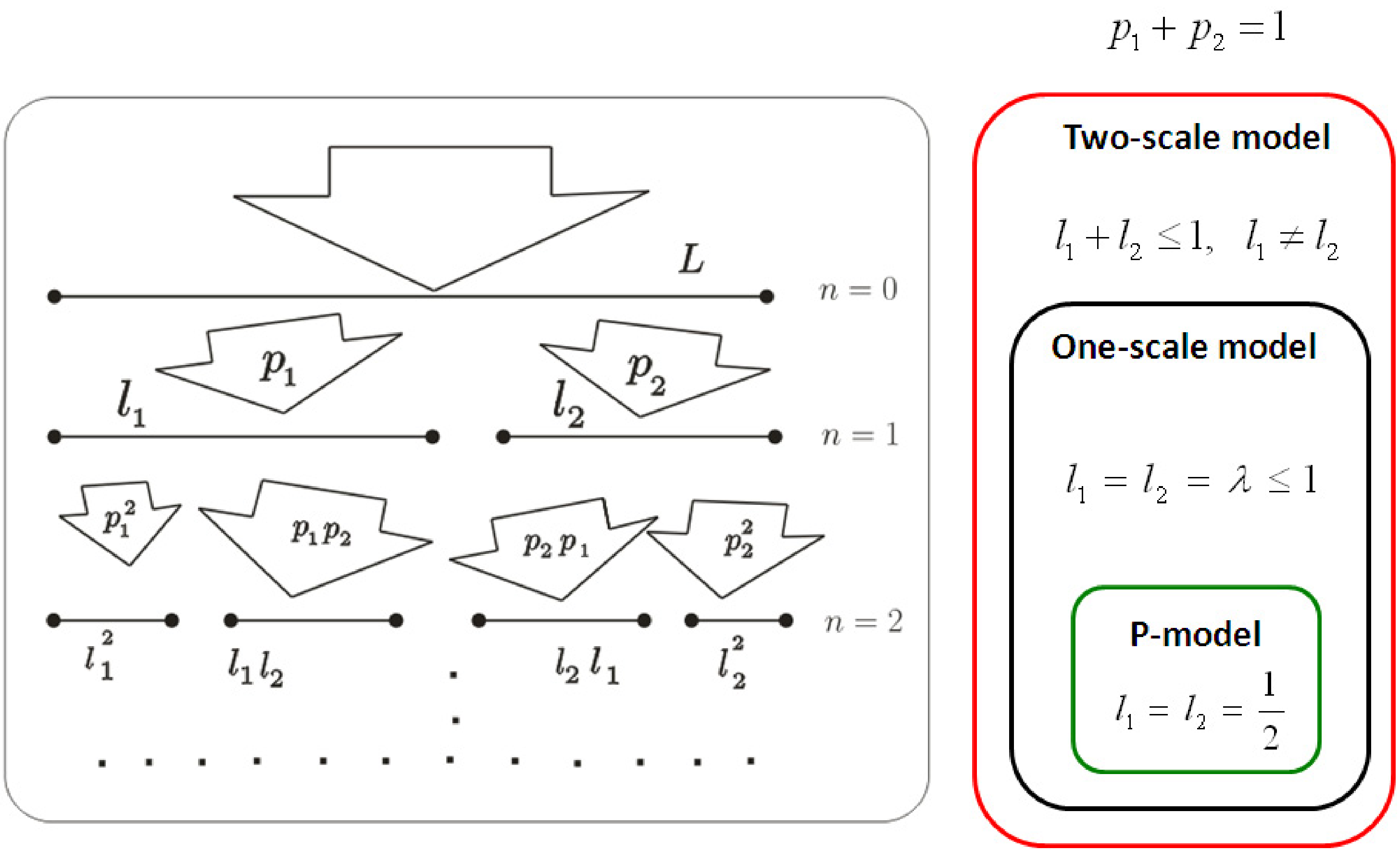


Fig. 1. Generalized two-scale Cantor set model for turbulence (Macek, 2007).

Solutions

Transcendental equation (for $n \to \infty$)

$$\frac{p^q}{l_1^{\tau(q)}} + \frac{(1-p)^q}{l_2^{\tau(q)}} = 1 \tag{6}$$

For $l_1 = l_2 = \lambda$ and any q in Eq. (6) one has for the generalized dimensions

$$\tau(q) \equiv (q-1)D_q = \frac{\ln[p^q + (1-p)^q]}{\ln \lambda}. \tag{7}$$

Space filling turbulence ($\lambda = 1/2$):
the multifractal cascade $p-$model for fully developed turbulence,
the generalized weighted Cantor set (Meneveau and Sreenivasan, 1987).
The usual middle one-third Cantor set (without any multifractality):
$p = 1/2$ and $\lambda = 1/3$.

Degree of Multifractality and Asymmetry

The difference of the maximum and minimum dimension (the least dense and most dense points in the phase space) is given, e.g., by Macek (2006, 2007)

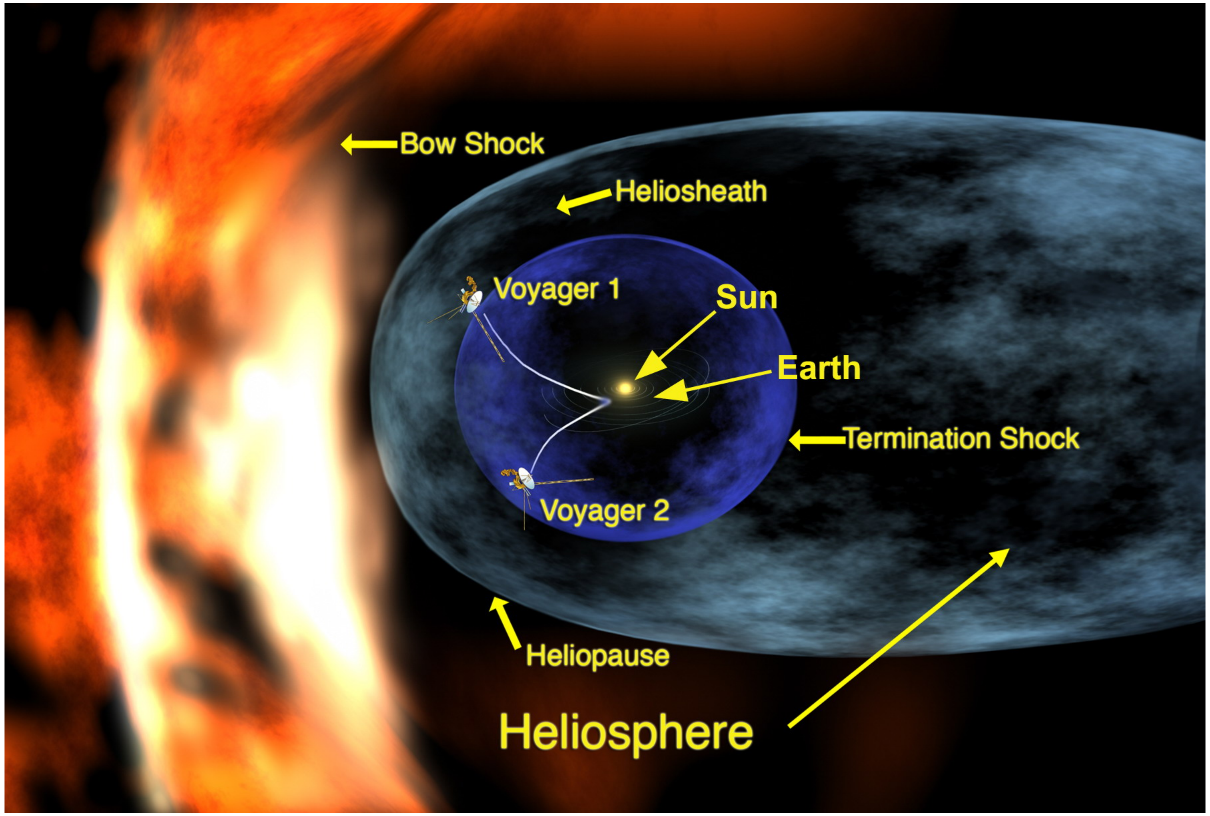
$$\Delta \equiv \alpha_{\max} - \alpha_{\min} = D_{-\infty} - D_{\infty} = \left| \frac{\log(1-p)}{\log l_2} - \frac{\log(p)}{\log l_1} \right|. \tag{8}$$

In the limit $p \to 0$ this difference rises to infinity (degree of multifractality).

The degree of multifractality Δ is simply related to the deviation from a simple self-similarity. That is why Δ is also a measure of intermittency, which is in contrast to self-similarity (Frisch, 1995, chapter 8).

Using the value of the strength of singularity α_0 at which the singularity spectrum has its maximum $f(\alpha_0) = 1$ we define a measure of asymmetry by

$$A \equiv \frac{\alpha_0 - \alpha_{\min}}{\alpha_{\max} - \alpha_0}. \tag{9}$$



Schematic of the Heliospheric Boundaries.

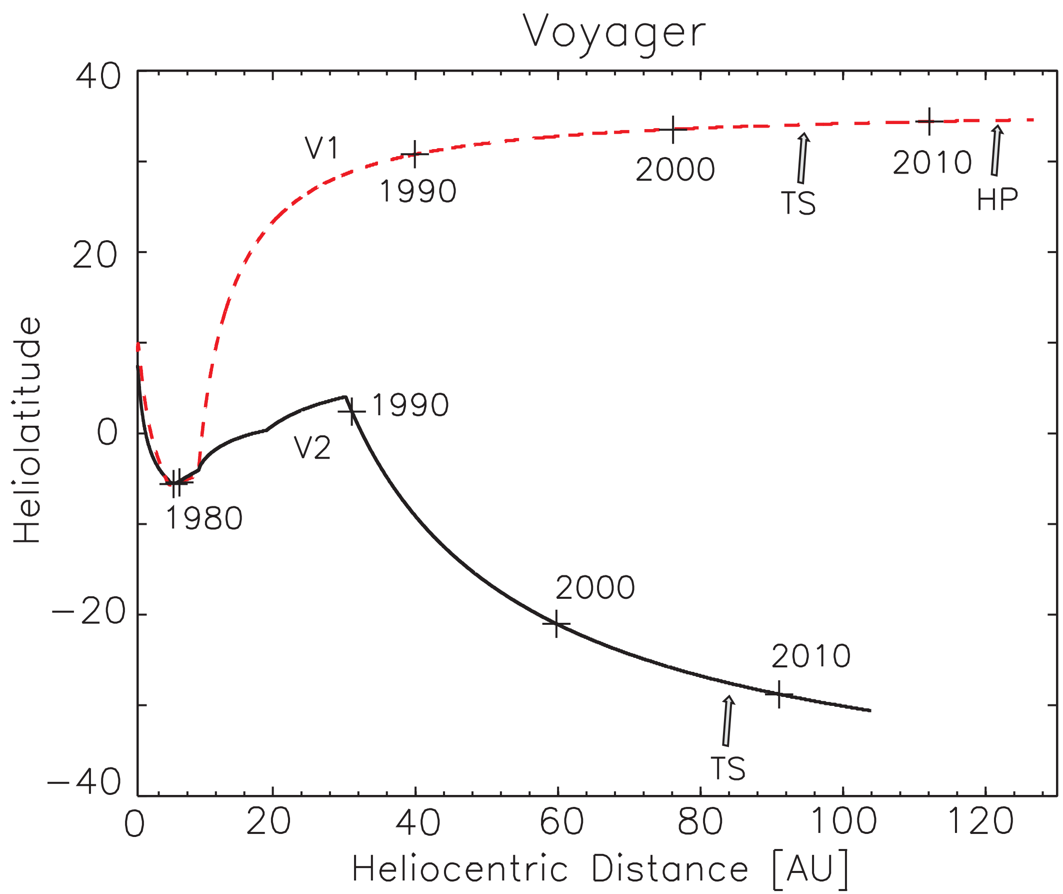
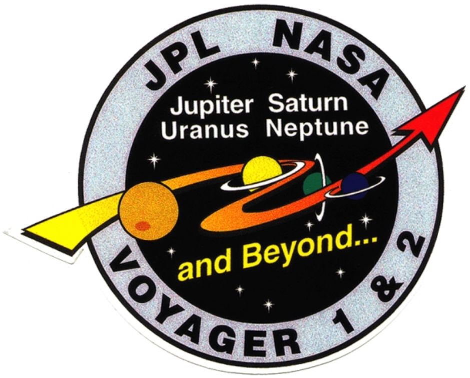


Fig. 2. The heliospheric distances from the Sun and the heliographic latitudes during each year of the Voyager mission. Voyager 1 and 2 spacecraft are located above and below the solar equatorial plane, respectively.

Voyager 1 Spacecraft



7 – 60 AU (1980 – 1995)
70 – 90 AU (1999 – 2003)
95 – 107 AU (2005 – 2008)
108 – 115 AU (2009 – 2010)

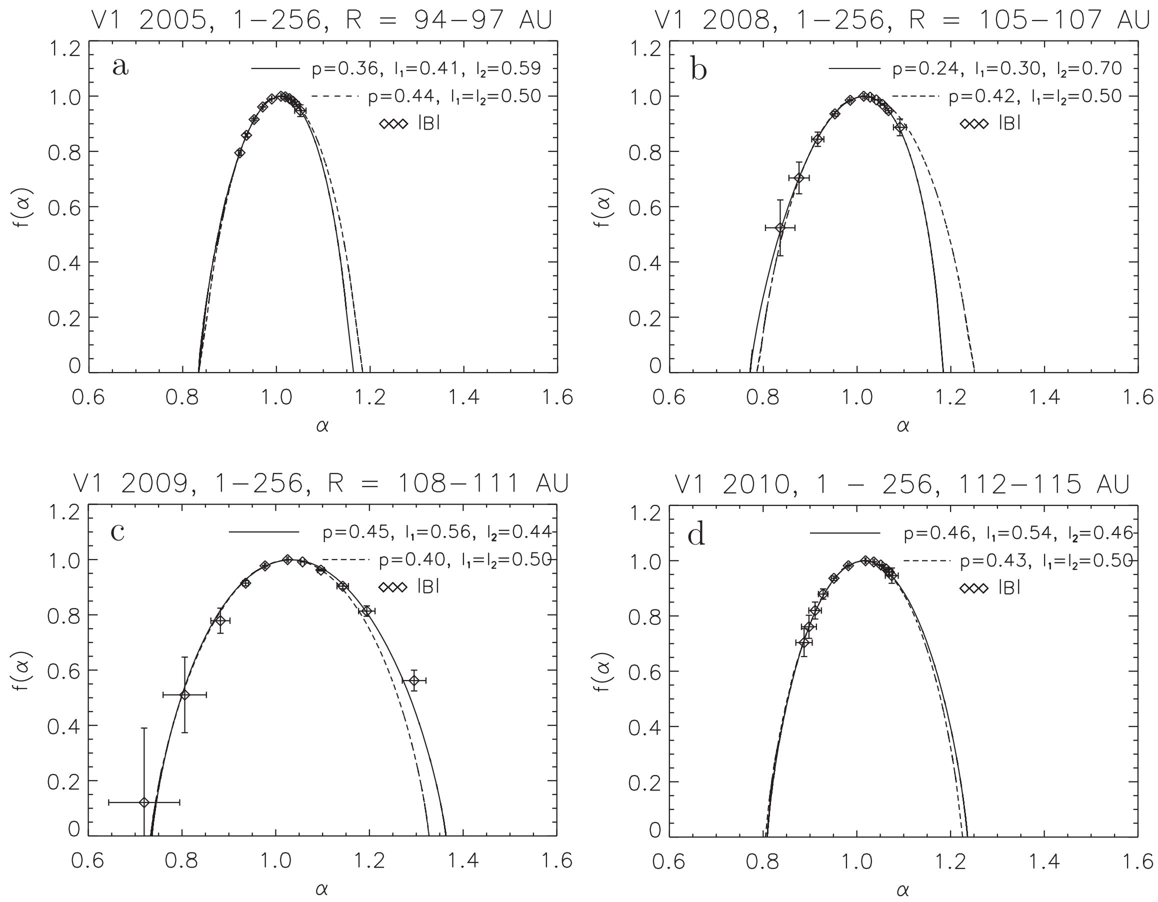


Fig. 1. The singularity spectrum $f(\alpha)$ as a function of a singularity strength α. The values are calculated for the weighted two-scale (continuous lines) model and the usual one-scale (dashed lines) p-model with the parameters fitted using the magnetic fields (diamonds) measured by Voyager 1 in the heliosheath at various distances before crossing the heliopause, (a) 94–97 AU (b) 105–107 AU, (c) 108–111 AU, and (d) 112–115 AU, correspondingly (cf. Macek et al., 2011, 2012).

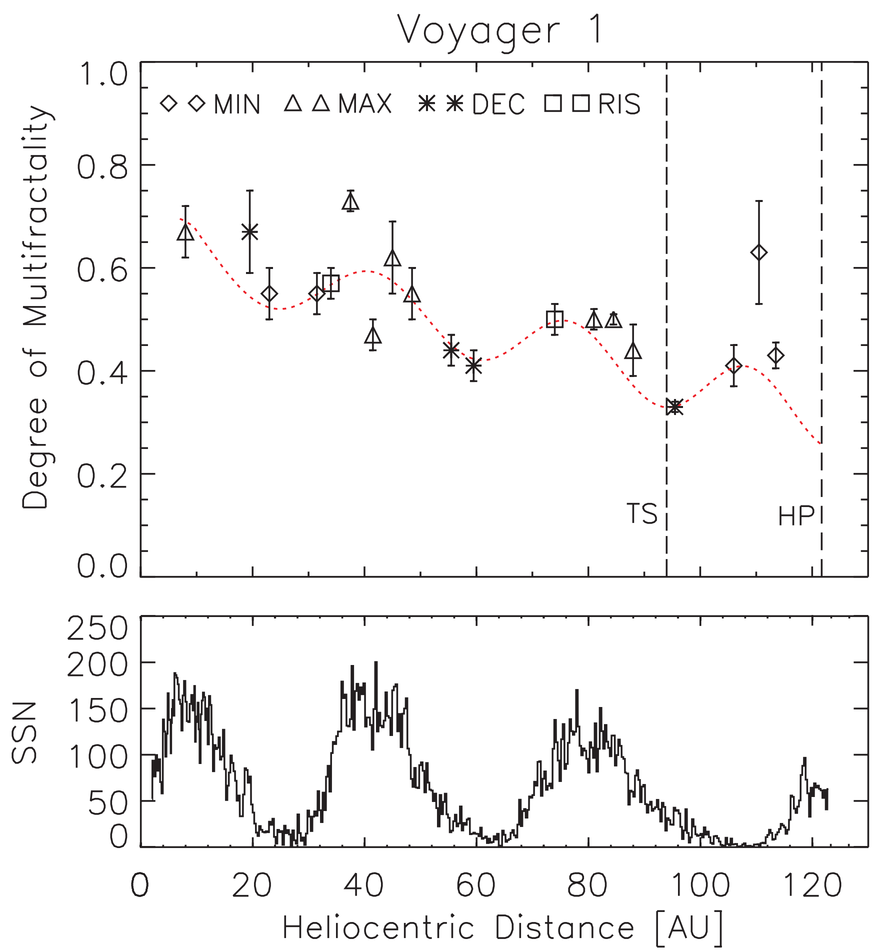


Fig. 2. The degree of multifractality Δ in the heliosphere versus the heliospheric distances compared to a periodically decreasing function (dotted) during solar minimum (MIN) and solar maximum (MAX), declining (DEC) and rising (RIS) phases of solar cycles, with the corresponding averages shown by continuous lines. The crossing of the termination shock (TS) and the heliopause (HP) by Voyager 1 are marked by vertical dashed lines. Below is shown the Sunspot Number (SSN) during years 1980–2010 (cf. Macek et al. 2011).

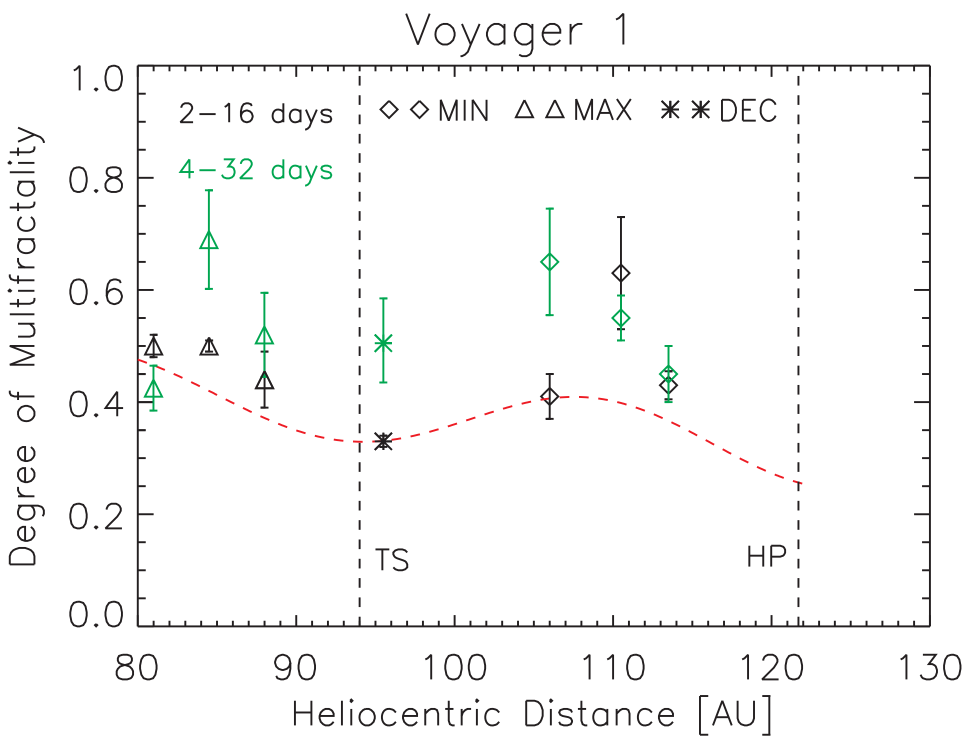


Fig. 3. The degree of multifractality Δ at the heliosphere boundaries between crossing the termination heliospheric shock (TS) and the heliopause (HP) by Voyager 1 (marked by vertical dashed lines) for different scaling range.

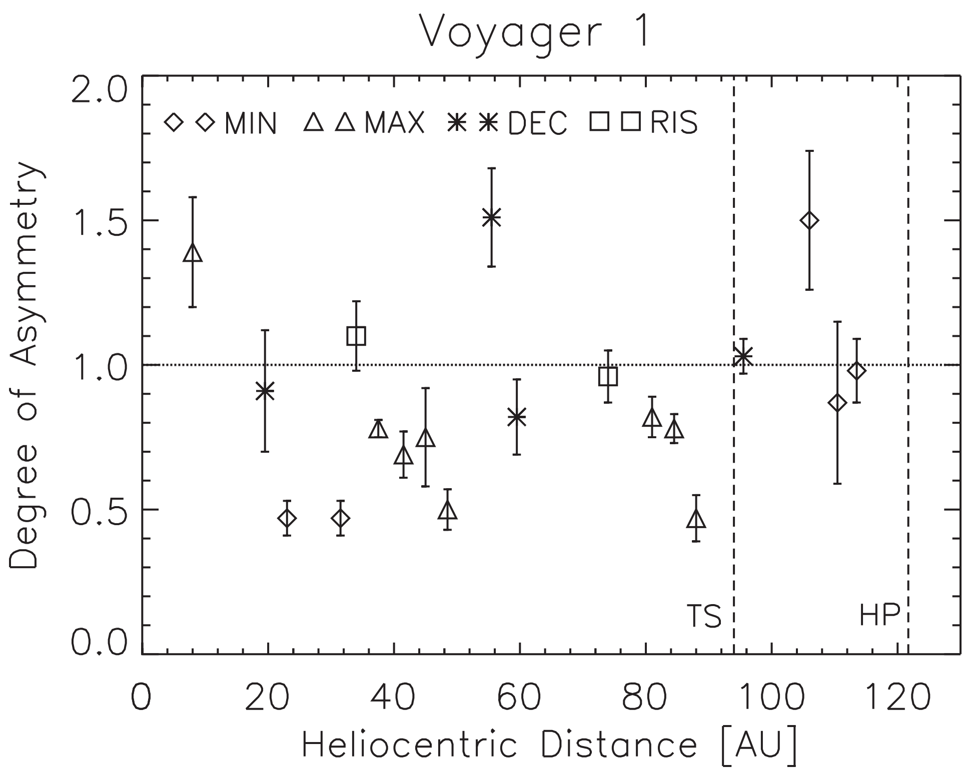


Fig. 3. The degree of asymmetry A of the multifractal spectrum in the heliosphere as a function of the heliospheric distance during solar minimum (MIN) and solar maximum (MAX), declining (DEC) and rising (RIS) phases of solar cycles with the corresponding averages denoted by continuous lines; the value $A = 1$ (dotted) corresponds to the one-scale symmetric model. The crossing of the termination shock (TS) and the heliopause (HP) by Voyager 1 are marked by vertical dashed lines (cf. Macek et al. 2011).

			Heliosheath	
	40 – 60 **AU**	70 – 90 **AU**	95 – 107 **AU**	108 – 115 **AU**
Burlaga	$\boldsymbol{\Delta} = 0.69$ $\mathbf{A} = 0.63$	$\boldsymbol{\Delta} = 0.69$ $\mathbf{A} = 0.63$	$\boldsymbol{\Delta} = 0.34$ $\mathbf{A} = 0.89$	$\boldsymbol{\Delta} = 0.34$ $\mathbf{A} = 1.0$
Two-scale Model	$\boldsymbol{\Delta} = 0.41 - 0.62$ $\mathbf{A} = 0.51 - 1.51$	$\boldsymbol{\Delta} = 0.44 - 0.50$ $\mathbf{A} = 0.47 - 0.96$	$\boldsymbol{\Delta} = 0.33 - 0.41$ $\mathbf{A} = 1.03 - 1.51$	$\boldsymbol{\Delta} = 0.44 - 0.63$ $\mathbf{A} = 0.87 - 0.98$

Termination shock $\Rightarrow$ Heliopause $\Rightarrow$

Figure 1: Degree of multifractality Δ and asymmetry A for the magnetic field strengths in the distant heliosphere and beyond the termination shock ahead of the heliopause (cf. Macek et al. 2011, 2012).

Conclusions

- Using our weighted two-scale Cantor set model, which is a convenient tool to investigate the asymmetry of the multifractal spectrum, we confirm the characteristic shape of the universal multifractal singularity spectrum. $f(\alpha)$ is a downward concave function of scaling indices α.
- We show that the degree of multifractality for magnetic field fluctuations of the solar wind falls steadily with the distance from the Sun and seems to be modulated by the solar activity.
- Moreover, we have considered the multifractal spectra of fluctuations of the interplanetary magnetic field strength before and after shock crossing by Voyager 1. In contrast to the right-skewed asymmetric spectrum with singularity strength $\alpha > 1$ inside the heliosphere, the spectrum becomes more left-skewed, $\alpha < 1$, or approximately symmetric after the shock crossing in the heliosheath, where the plasma is expected to be roughly in equilibrium in the transition to the interstellar medium.
- We also confirm the results obtained by Burlaga et al. (2006) that before the shock crossing, especially during solar maximum, turbulence is more multifractal than that in the heliosheath.

- In addition, outside the heliosphere during solar minimum the spectrum seems to be dominated by values of $\alpha < 1$. This is very interesting because it represents a first direct *in situ* information of interest in the astrophysical context beyond the heliosphere. In fact, a density of measure dominated by $\alpha < 1$ implies that the magnetic field in the very local interstellar medium is roughly confined in thin filaments of high magnetic density.
- That is the heliosheath is possibly more dominated by 'voids' of magnetic fields, thus implying that magnetic turbulence tends to be more 'passive' (cf. Frisch, 1995) in the very local interstellar medium.
- These informations, obtained *in situ* rather than through scintillations observations, are relevant in the context of interstellar turbulence confirming stellar formation modeling (e.g., Spangler et al. 2009), related to the presence of very localized intense magnetic field structures.

Epilogue

Within the complex dynamics of the solar wind's and interstellar medium fluctuating intermittent plasma parameters, there is a detectable, hidden order described by a Cantor set that exhibits a multifractal structure.

This means that the observed **intermittent** behavior of magnetic fluctuations in the heliosphere and the very local interstellar medium may result from intrinsic *nonlinear* dynamics rather than from random external forces.

Thank you!

Image courtesy of ESO

Generalized Scaling Property

The generalized dimensions are important characteristics of *complex* dynamical systems; they quantify multifractality of a given system (Ott, 1993). In the case of turbulence cascade these generalized measures are related to inhomogeneity with which the energy is distributed between different eddies (Meneveau and Sreenivasan, 1991). In this way they provide information about dynamics of multiplicative process of cascading eddies. Here high positive values of $q > 1$ emphasize regions of intense fluctuations larger than the average, while negative values of q accentuate fluctuations lower than the average (cf. Burlaga 1995).

0Using ($\sum p_i{}^q \equiv \langle p_i{}^{q-1} \rangle_{\mathrm{av}}$) a generalized average probability measure

$$\bar{\mu}(q,l) \equiv \sqrt[q-1]{\langle (p_i)^{q-1} \rangle_{\mathrm{av}}} \tag{10}$$

we can identify D_q as scaling of the measure with size l

$$\bar{\mu}(q,l) \propto l^{D_q} \tag{11}$$

Hence, the slopes of the logarithm of $\bar{\mu}(q,l)$ of Eq. (11) versus $\log l$ (normalized) provides

$$D_q = \lim_{l \to 0} \frac{\log \bar{\mu}(q,l)}{\log l} \tag{12}$$

Table 1: The degree of multifractality Δ and asymmetry A of the multifractal spectrum for the magnetic field strength observed by Voyager 1 in the heliosheath depending on scale range

		Year 2009	
Scale Range, Days	$2-16$	$2-32$	$4-32$
P-model	$\Delta = 0.59$	$\Delta = 0.53$	$\Delta = 0.55$
TS-model	$\Delta = 0.63 \pm 0.10$	$\Delta = 0.60 \pm 0.08$	$\Delta = 0.55 \pm 0.04$
	$A = 0.87 \pm 0.28$	$A = 0.85 \pm 0.22$	$A = 0.69 \pm 0.10$
		Year 2010	
Scale Range, Days	$2-16$	$2-32$	$4-32$
P-model	$\Delta = 0.41$	$\Delta = 0.41$	$\Delta = 0.41$
TS-model	$\Delta = 0.44 \pm 0.03$	$\Delta = 0.44 \pm 0.03$	$\Delta = 0.45 \pm 0.05$
	$A = 0.98 \pm 0.11$	$A = 0.76 \pm 0.11$	$A = 0.82 \pm 0.19$

Table 2: The degree of multifractality Δ and asymmetry A of the multifractal spectrum for the magnetic field strength observed by Voyager 1 at various heliospheric distances, before and after crossing the termination shock, as calculated by L. Burlaga (LB) and by the authors of the paper by Macek et al. (2011) (WM).

Heliocentric Distance	**Year**	**Multifractality** Δ (WM)	Δ (LB)	**Asymmetry** A(WM)	A (LB)
$7-40$ AU	1980-1989	$0.55-0.73$	0.64	$0.47-1.39$	0.69
$40-60$ AU	1990-1995	$0.41-0.62$	0.69	$0.51-1.51$	0.63
$70-90$ AU	1999-2003	$0.44-0.50$	0.69	$0.47-0.96$	0.63
$95-107$ AU	2005-2008	$0.33-0.41$	0.34	$1.03-1.51$	0.89
$108-115$ AU	2009-2010	$0.44-0.63$	0.34	$0.87-0.98$	1.0

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