Multifractal Turbulence in the Heliosphere and the Heliosheath

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Plan of Presentation

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Abstract

We present first results of the multifractal scaling of the fluctuations of the interplanetary magnetic field strength as measured onboard Voyager 2 in the very distant heliosphere and even in the heliosheath. More specifically, we analyze the spectra observed by Voyager 2 in a wide range of heliospheric distances from 6 to 90 astronomical units (AU) that are compared with those of Voyager 1 already analyzed between 7 and 107 AU, which is now approaching the heliopause.

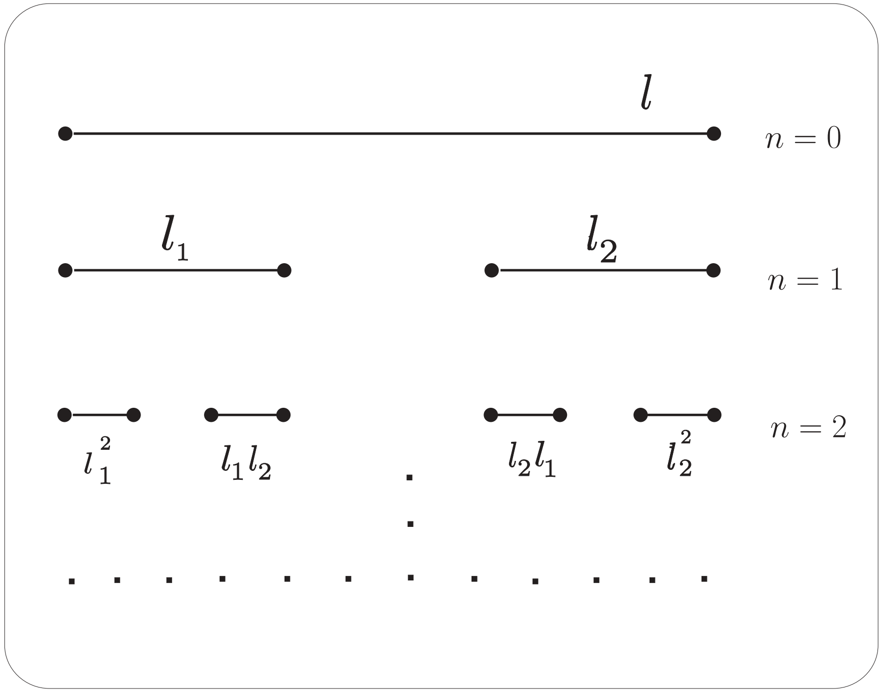
We focus on the singularity multifractal spectrum before and after crossing the termination heliospheric shock by Voyager 1 at 94 AU and Voyager 2 at 84 AU from the Sun. It is worth noting that the spectrum is prevalently right-skewed inside the whole heliosphere. Moreover, we have observed a change of the asymmetry of the spectrum at the termination shock.

We show that the degree of multifractality falls steadily with the distance from the Sun. In addition, the multifractal structure is apparently modulated by the solar activity, with a time shift of several years, corresponding to a distance of about 10 AU, resulting from the evolution of the whole heliosphere. Hence this basic result also brings significant additional support to some earlier claims suggesting that the solar wind termination shock is asymmetric.

Prologue

A **fractal** is a rough or fragmented geometrical object that can be subdivided in parts, each of which is (at least approximately) a reduced-size copy of the whole. Fractals are generally *self-similar* and independent of scale (fractal dimension).

A **multifractal** is a set of intertwined fractals. Self-similarity of multifractals is scale dependent (spectrum of dimensions). A deviation from a strict self-similarity is also called **intermittency**.



Two-scale **Cantor** set.

Importance of Multifractality

The general aim of the present paper is to report on the new developments in systems exhibiting turbulent behavior by using multifractals, with application to phenomenological approach to this complex issue. Following the idea of Kolmogorov (1941) and Kraichnan (1965) various multifractal models of turbulence have been developed (Meneveau and Sreenivasan, 1987; Carbone, 1993; Frisch, 1995).

In particular, multifractal scaling of this flux in solar wind turbulence using Helios (plasma) data in the inner heliosphere has been analyzed by March et al. (1996). It is known that fluctuations of the solar magnetic fields may also exhibit multifractal scaling laws. The multifractal spectrum has been investigated using magnetic field data measured *in situ* by Voyager in the outer heliosphere up to large distances from the Sun (Burlaga, 1991, 1995, 2004) and even in the heliosheath (Burlaga and Ness, 2010; Burlaga et al., 2006).

To quantify scaling of solar wind turbulence we have developed a generalized two-scale weighted Cantor set model using the partition technique (Macek 2007; Macek and Szczepaniak, 2008), which leads to complementary information about the multifractal nature of the fluctuations as the rank-ordered multifractal analysis (cf. Lamy et al., 2010).

We have investigated the spectrum of generalized dimensions and the corresponding multifractal singularity spectrum depending on one probability measure parameter and two rescaling parameters. In this way we have looked at the inhomogeneous rate of the transfer of the energy flux indicating multifractal and intermittent behavior of solar wind turbulence. In particular, we have studied in detail fluctuations of the velocity of the flow of the solar wind, as measured in the inner heliosphere by Helios (Macek and Szczepaniak, 2008), Advanced Composition Explorer (ACE) (Szczepaniak and Macek, 2008), and Voyager in the outer heliosphere (Macek and Wawrzaszek, 2009), including Ulysses observations at high heliospheric latitudes (Wawrzaszek and Macek, 2010).

Voyager 1 crossed the termination heliospheric shock, which separates the Solar System plasma from the surrounding heliosheath, with the subsonic solar wind, on 16 December 2004 at heliocentric distances of 94 AU (at present its distance to the Sun is about 122 AU approaching the heliopause). Please note that (using the pressure balance) the distance to the nose of the heliopause has been estimated to be $\sim$120 AU (Macek, 1998). Later, in 2007 also Voyager 2 crossed the termination shock at least five times at distances of 84 AU (now is at 100 AU).

Admittedly, variations of the magnetic field strength observed by Voyager 2 have also been analyzed including those prior and after crossing the termination shock up to distances of $\sim$90 AU in 2009 (from 2007.7 to 2009.4), see (Burlaga et al., 2008, 2009, 2010). However, the multifractal spectrum using Voyager 2 data has only been analyzed at 25 AU by Burlaga (1991). but in a more distant heliosphere and especially near the termination shock this multifractal analysis is still missing.

Therefore, the aim of our present study, which is a substantial extension of the previous GRL letter (2011) is to investigate the multifractal scaling for both Voyager 1 and 2 data that will allow us to infer new information about the multifractal structure of the heliospheric magnetic fields in the northern and southern hemisphere, including the correlation with the solar cycle.

In particular, we show that multifractal structure is modulated by the solar activity with some time delay and confirm that the degree of multifractality is also decreasing with distance: before shock crossing is greater than that in the heliosheath. Moreover, we demonstrate that the multifractal spectrum is asymmetric before shock crossing, in contrast to the nearly symmetric spectrum observed in the heliosheath; the solar wind in the outer heliosphere may exhibit strong asymmetric scaling, where the spectrum is prevalently right-skewed. The obtained delay between Voyager 1 and 2 can certainly be correlated with the evolution of the heliosphere, providing an additional support to some earlier independent claims that the solar wind termination shock itself is possibly asymmetric (Stone et al., 2008).

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Fractal

A measure (volume) V of a set as a function of size l

$$V(l) \sim l^{D_\mathrm{F}}$$

The number of elements of size l needed to cover the set

$$N(l) \sim l^{-D_\mathrm{F}}$$

The fractal dimension

$$D_\mathrm{F} = \lim_{l \to 0} \frac{\ln N(l)}{\ln 1/l}$$

Multifractal

A (probability) measure versus singularity strength, α

$$p_i(l) \propto l^{\alpha_i}$$

The number of elements in a small range from α to $\alpha + \mathrm{d}\alpha$

$$N_l(\alpha) \sim l^{-f(\alpha)}$$

The multifractal singularity spectrum

$$f(\alpha) = \lim_{\varepsilon \to 0} \lim_{l \to 0} \frac{\ln[N_l(\alpha+\varepsilon) - N_l(\alpha-\varepsilon)]}{\ln 1/l}$$

The generalized dimension

$$D_q = \frac{1}{q-1} \lim_{l \to 0} \frac{\ln \sum_{k=1}^{N} (p_k)^q}{\ln l}$$

Multifractal Characteristics

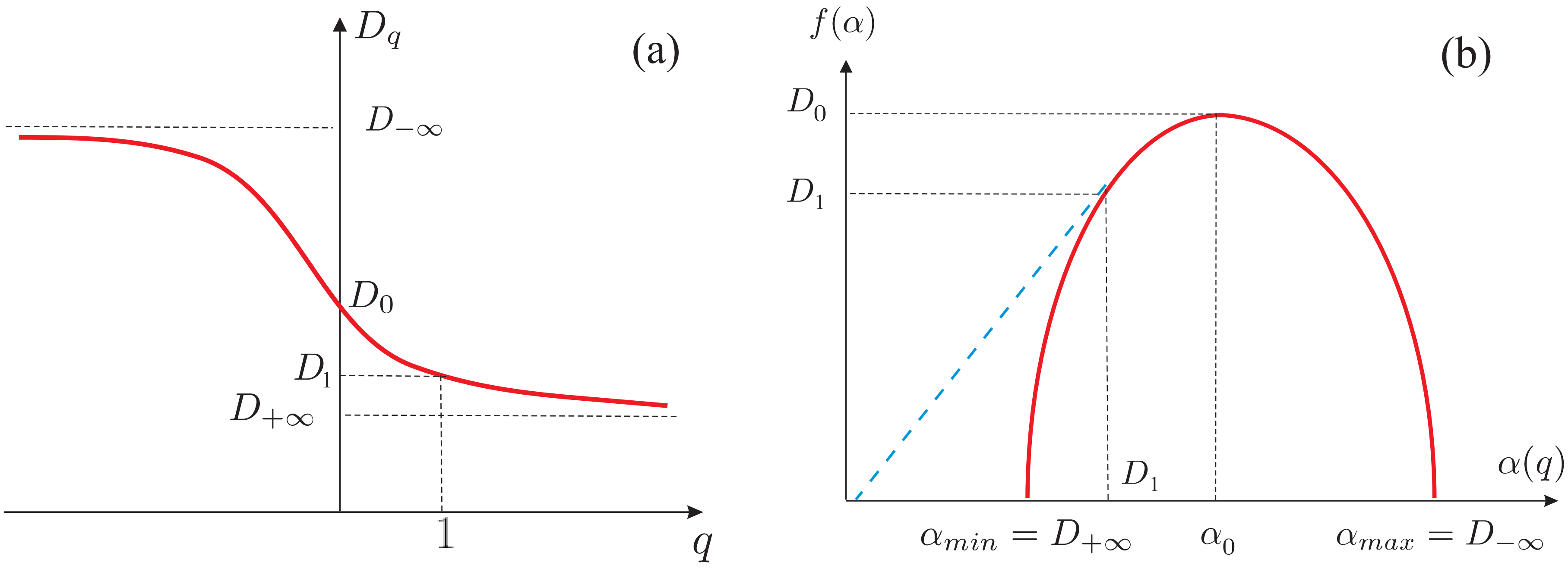


Fig. 1. (a) The generalized dimensions D_q as a function of any real q, $-\infty < q < \infty$, and (b) the singularity multifractal spectrum $f(\alpha)$ versus the singularity strength α with some general properties: (1) the maximum value of $f(\alpha)$ is D_0; (2) $f(D_1) = D_1$; and (3) the line joining the origin to the point on the $f(\alpha)$ curve where $\alpha = D_1$ is tangent to the curve (Ott *et al.*, 1994).

Generalized Scaling Property

The generalized dimensions are important characteristics of *complex* dynamical systems; they quantify multifractality of a given system (Ott, 1993). In the case of turbulence cascade these generalized measures are related to inhomogeneity with which the energy is distributed between different eddies (Meneveau and Sreenivasan, 1991). In this way they provide information about dynamics of multiplicative process of cascading eddies. Here high positive values of $q > 1$ emphasize regions of intense fluctuations larger than the average, while negative values of q accentuate fluctuations lower than the average (cf. Burlaga 1995).

Using ($\sum {p_i}^q \equiv \langle {p_i}^{q-1} \rangle_{\mathrm{av}}$) a generalized average probability measure

$$\bar{\mu}(q,l) \equiv \sqrt[q-1]{\langle (p_i)^{q-1} \rangle_{\mathrm{av}}} \tag{1}$$

we can identify D_q as scaling of the measure with size l

$$\bar{\mu}(q,l) \propto l^{D_q} \tag{2}$$

Hence, the slopes of the logarithm of $\bar{\mu}(q,l)$ of Eq. (2) versus $\log l$ (normalized) provides

$$D_q = \lim_{l \to 0} \frac{\log \bar{\mu}(q,l)}{\log l} \tag{3}$$

Measures and Multifractality

Similarly, we define a one-parameter q family of (normalized) generalized pseudoprobability measures (Chhabra and Jensen, 1989; Chhabra *et al.*, 1989)

$$\mu_i(q,l) \equiv \frac{p_i^q(l)}{\sum_{i=1}^{N} p_i^q(l)} \tag{4}$$

Now, with an associated fractal dimension index $f_i(q,l) \equiv \log \mu_i(q,l)/\log l$ for a given q the multifractal singularity spectrum of dimensions is defined directly as the average taken with respect to the measure $\mu(q,l)$ in Eq. (4) denoted by $\langle \ldots \rangle$

$$f(q) \ \equiv \ \lim_{l \to 0} \sum_{i=1}^{N} \mu_i(q,l) \ f_i(q,l) = \lim_{l \to 0} \ \frac{\langle \log \mu_i(q,l) \rangle}{\log(l)} \tag{5}$$

and the corresponding average value of the singularity strength is given by (Chhabra and Jensen, 1987)

$$\alpha(q) \ \equiv \ \lim_{l \to 0} \sum_{i=1}^{N} \mu_i(q,l) \ \alpha_i(l) = \lim_{l \to 0} \ \frac{\langle \log p_i(l) \rangle}{\log(l)}. \tag{6}$$

Methods of Data Analysis

Magnetic Field Strength Fluctuations and Generalized Measures

Given the normalized time series $B(t_i)$, where $i = 1, \ldots, N = 2^n$ (we take n = 8), to each interval of temporal scale Δt (using $\Delta t = 2^k$, with $k = 0, 1, \ldots, n$) we associate some probability measure

$$p(x_j, l) \equiv \frac{1}{N} \sum_{i=1+(j-1)\Delta t}^{j\Delta t} B(t_i) = p_j(l), \tag{7}$$

where $j = 2^{n-k}$, i.e., calculated by using the successive (daily) average values $\langle B(t_i, \Delta t) \rangle$ of $B(t_i)$ between t_i and $t_i + \Delta t$. At a position $x = v_{\mathrm{sw}} t$, at time t, where v_{sw} is the average solar wind speed, this quantity can be interpreted as a probability that the magnetic flux is transferred to a segment of a spatial scale $l = v_{\mathrm{sw}} \Delta t$ (Taylor's hypothesis).

The average value of the qth moment of the magnetic field strength B should scale as

$$\langle B^q(l) \rangle \sim l^{\gamma(q)}, \tag{8}$$

with the exponent $\gamma(q) = (q-1)(D_q - 1)$ as shown by Burlaga et al. (1995).

Mutifractal Models for Turbulence

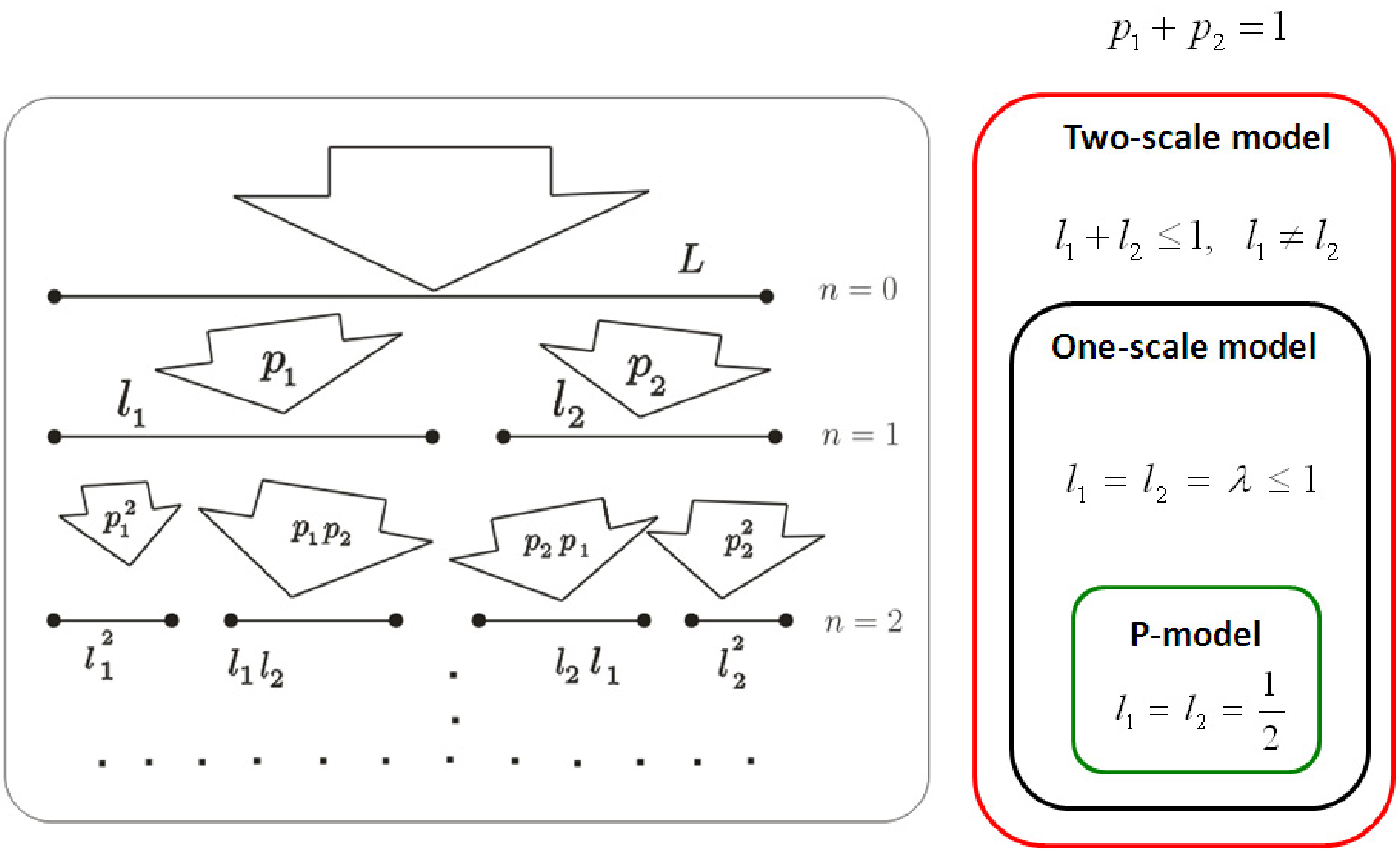


Fig. 1. Generalized two-scale Cantor set model for turbulence (Macek, 2007).

Solutions

Transcendental equation (for $n \to \infty$)

$$\frac{p^q}{l_1^{\tau(q)}} + \frac{(1-p)^q}{l_2^{\tau(q)}} = 1 \tag{9}$$

For $l_1 = l_2 = \lambda$ and any q in Eq. (9) one has for the generalized dimensions

$$\tau(q) \equiv (q-1)D_q = \frac{\ln[p^q + (1-p)^q]}{\ln \lambda}. \tag{10}$$

Space filling turbulence ($\lambda = 1/2$):
the multifractal cascade $p-$model for fully developed turbulence,
the generalized weighted Cantor set (Meneveau and Sreenivasan, 1987).
The usual middle one-third Cantor set (without any multifractality):
$p = 1/2$ and $\lambda = 1/3$.

Degree of Multifractality and Asymmetry

The difference of the maximum and minimum dimension (the least dense and most dense points in the phase space) is given, e.g., by Macek (2006, 2007)

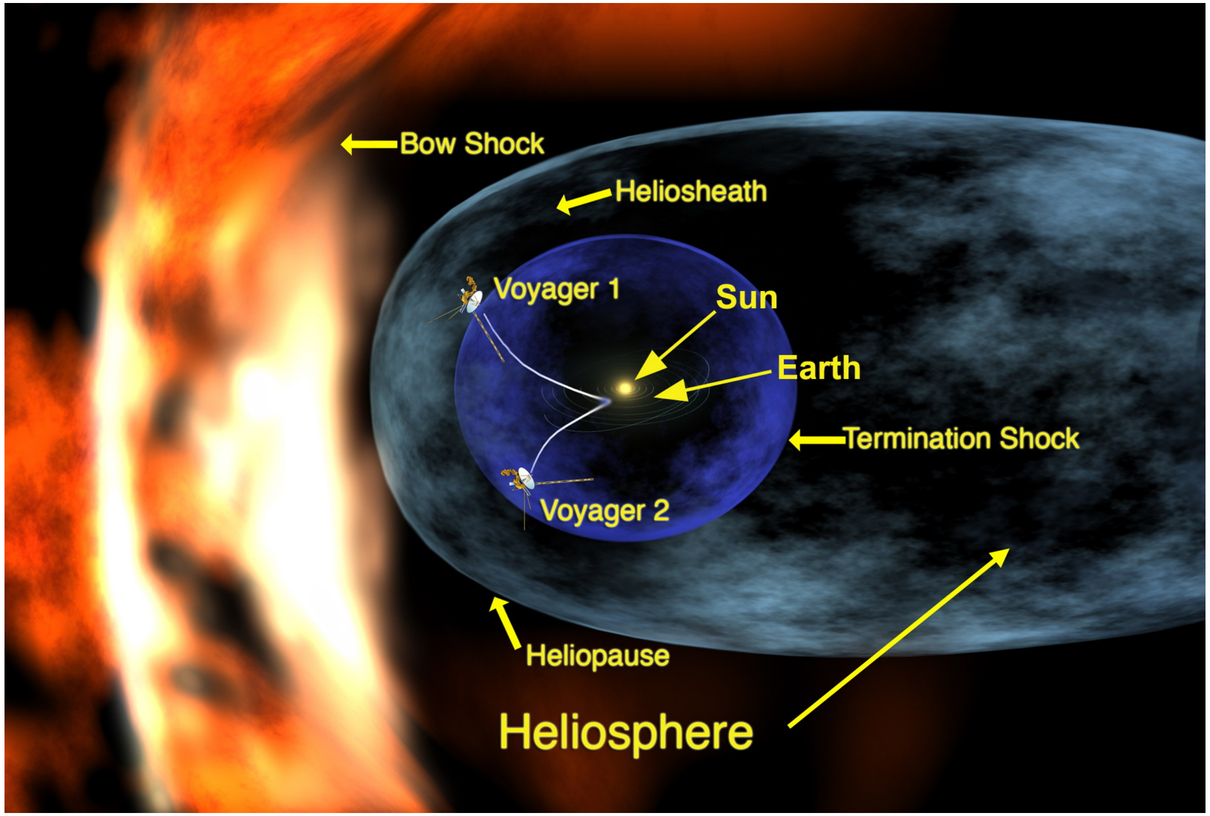
$$\Delta \equiv \alpha_{\max} - \alpha_{\min} = D_{-\infty} - D_{\infty} = \left| \frac{\log(1-p)}{\log l_2} - \frac{\log(p)}{\log l_1} \right|. \tag{11}$$

In the limit $p \to 0$ this difference rises to infinity (degree of multifractality).

The degree of multifractality Δ is simply related to the deviation from a simple self-similarity. That is why Δ is also a measure of intermittency, which is in contrast to self-similarity (Frisch, 1995, chapter 8).

Using the value of the strength of singularity α_0 at which the singularity spectrum has its maximum $f(\alpha_0) = 1$ we define a measure of asymmetry by

$$A \equiv \frac{\alpha_0 - \alpha_{\min}}{\alpha_{\max} - \alpha_0}. \tag{12}$$



Schematic of the Heliospheric Boundaries.

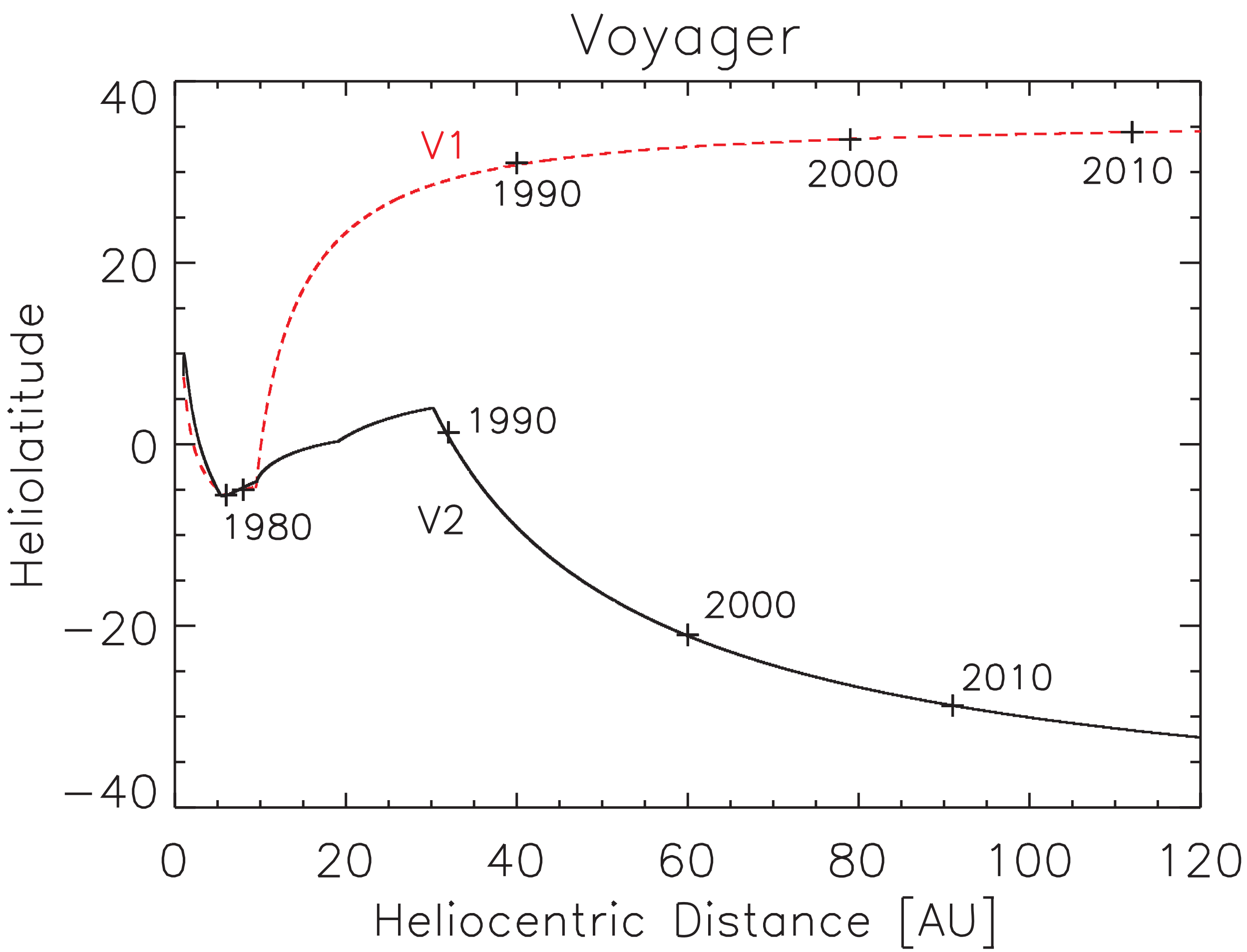


Fig. 2. The heliospheric distances from the Sun and the heliographic latitudes during each year of the Voyager mission. Voyager 1 and 2 spacecraft are located above and below the solar equatorial plane, respectively.

Voyager Spacecraft

V1

7 – 60 AU (1980 – 1995)
70 – 90 AU (1999 – 2003)
95 – 107 AU (2005 – 2008)

V2

7 – 40 AU (1980 – 1990)
60 – 80 AU (2002 – 2006)
85 – 90 AU (2008 – 2009)

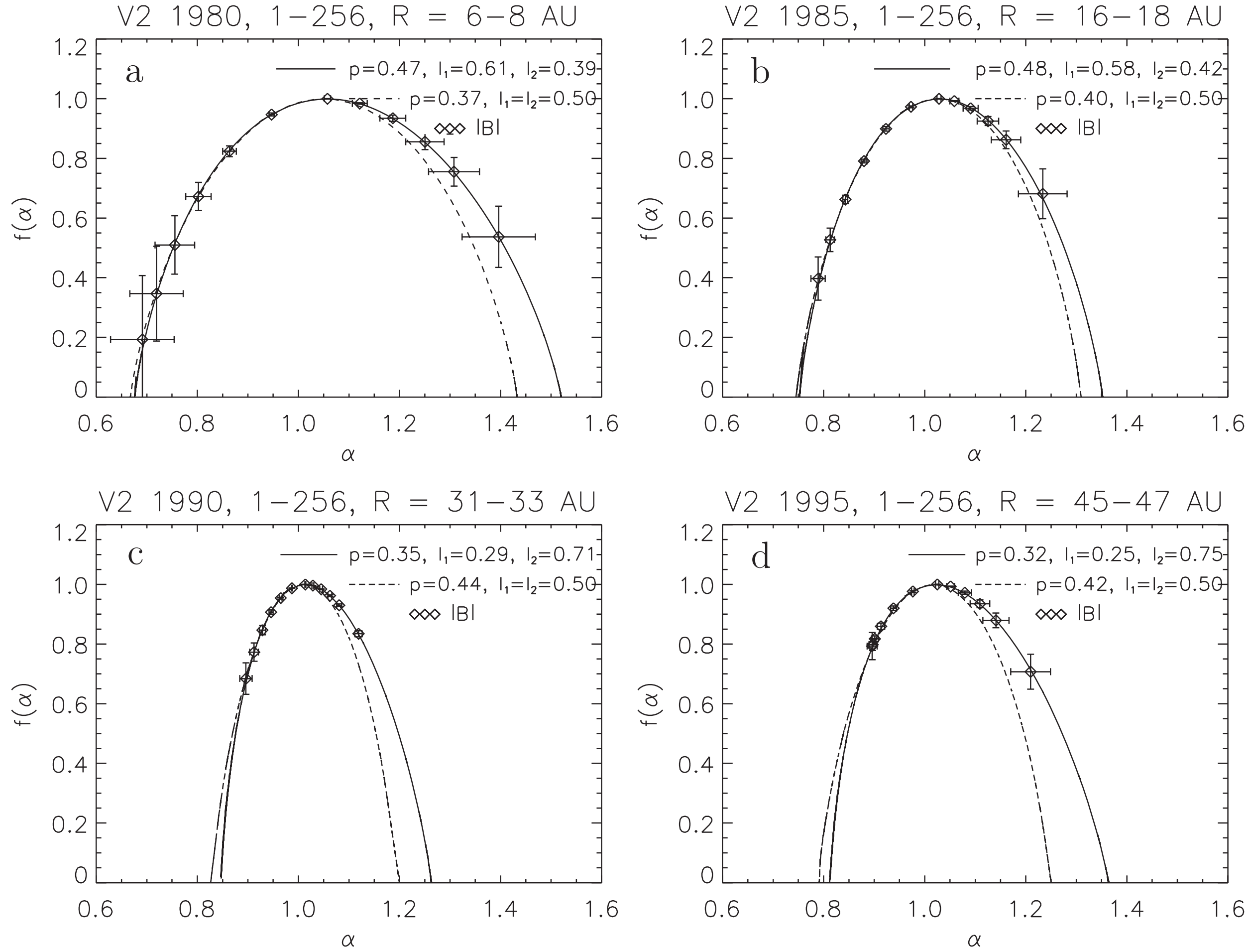


Fig. 3. The singularity spectrum $f(\alpha)$ as a function of a singularity strength α. The values are calculated for the weighted two-scale (continuous lines) model and the usual one-scale (dashed lines) p-model with the parameters fitted using the magnetic fields (diamonds) measured by Voyager 2 in the near heliosphere at (a) 6–8 AU (b) 16–18 AU, (c) 31–33 AU, and (d) 45–47 AU, correspondingly (Macek et al. 2012).

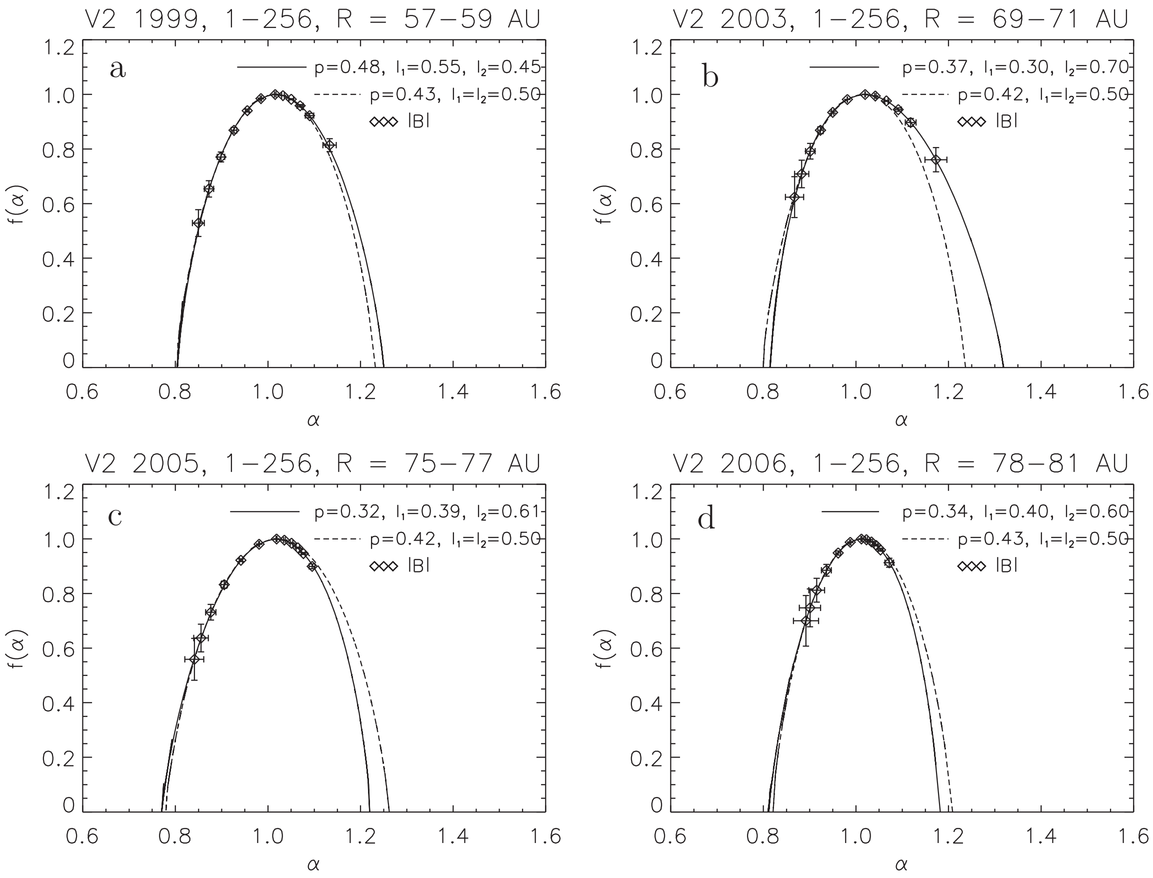


Fig. 4. The singularity spectrum $f(\alpha)$ as a function of a singularity strength α. The values are calculated for the weighted two-scale (continuous lines) model and the usual one-scale (dashed lines) p-model with the parameters fitted using the magnetic fields (diamonds) measured by Voyager 2 in the distant heliosphere at various distances before crossing the termination shock, (a) 57–59 AU (b) 69–71 AU, (c) 75–77 AU, and (d) 78–81 AU, correspondingly (Macek et al. 2012).

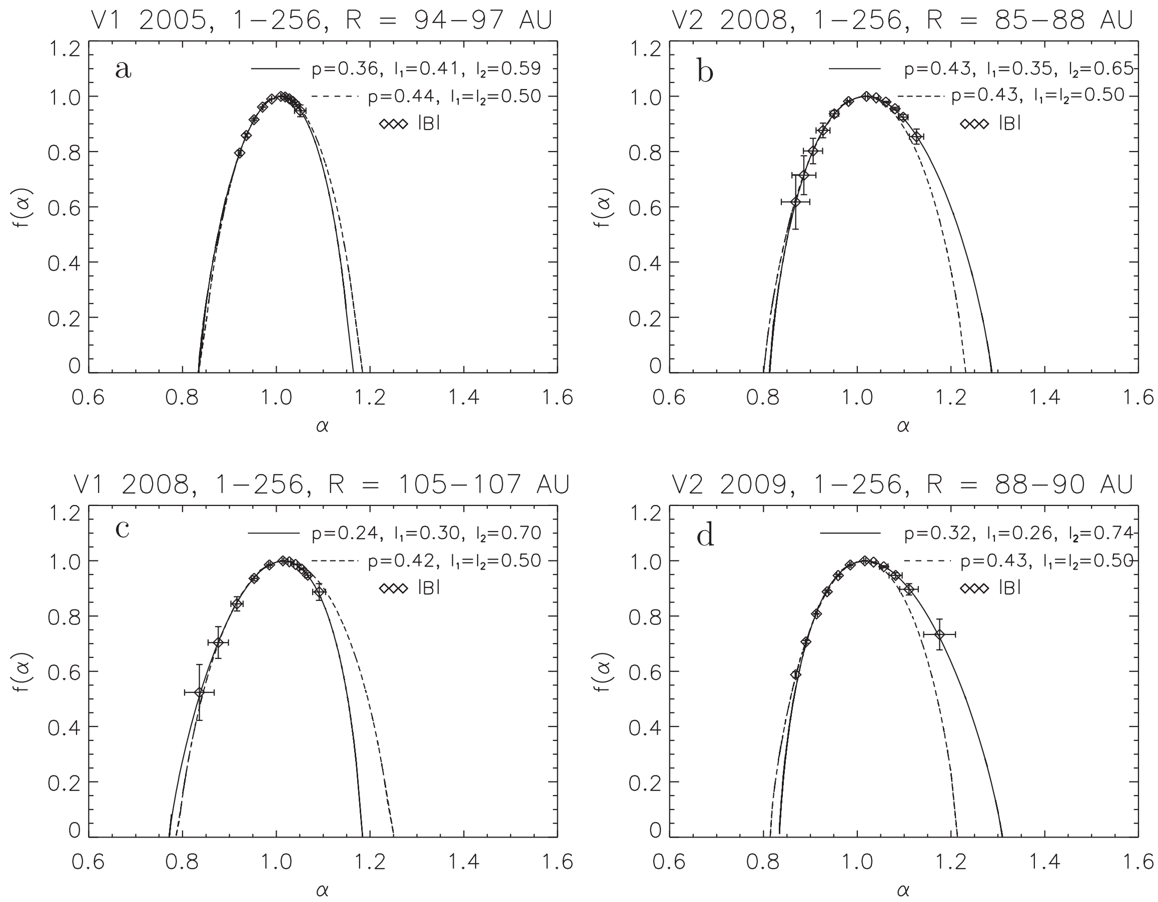


Fig. 5. The singularity spectrum $f(\alpha)$ as a function of a singularity strength α. The values are calculated for the weighted two-scale (continuous lines) model and the usual one-scale (dashed lines) p-model with the parameters fitted using the magnetic fields (diamonds) measured by Voyager 1 in the heliosheath at various heliocentric distances of (a) 94–97 AU and (c) 105–107 AU AU, and Voyager 2 at (b) 85–88 AU and (d) 88–90 AU, correspondingly (Macek et al. 2012).

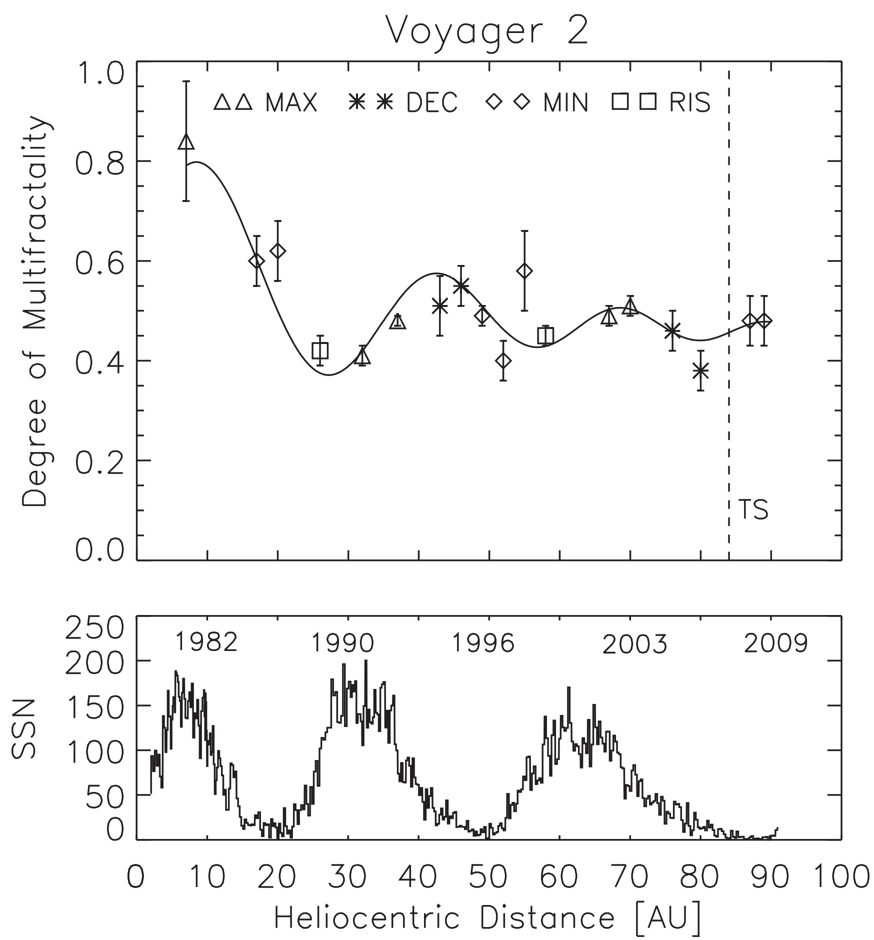


Fig. 6. The degree of multifractality Δ in the heliosphere as a function of the distances from the Sun fitted to a periodically decreasing function shown by a continuous line during solar minimum (MIN) and solar maximum (MAX), declining (DEC) and rising (RIS) phases of solar cycles. The crossing of the termination shock by Voyager 2 is marked by a vertical dashed line (TS). Below are shown the Sunspot Numbers (SSN) during the period of 1980–2009 (Macek et al. 2012).

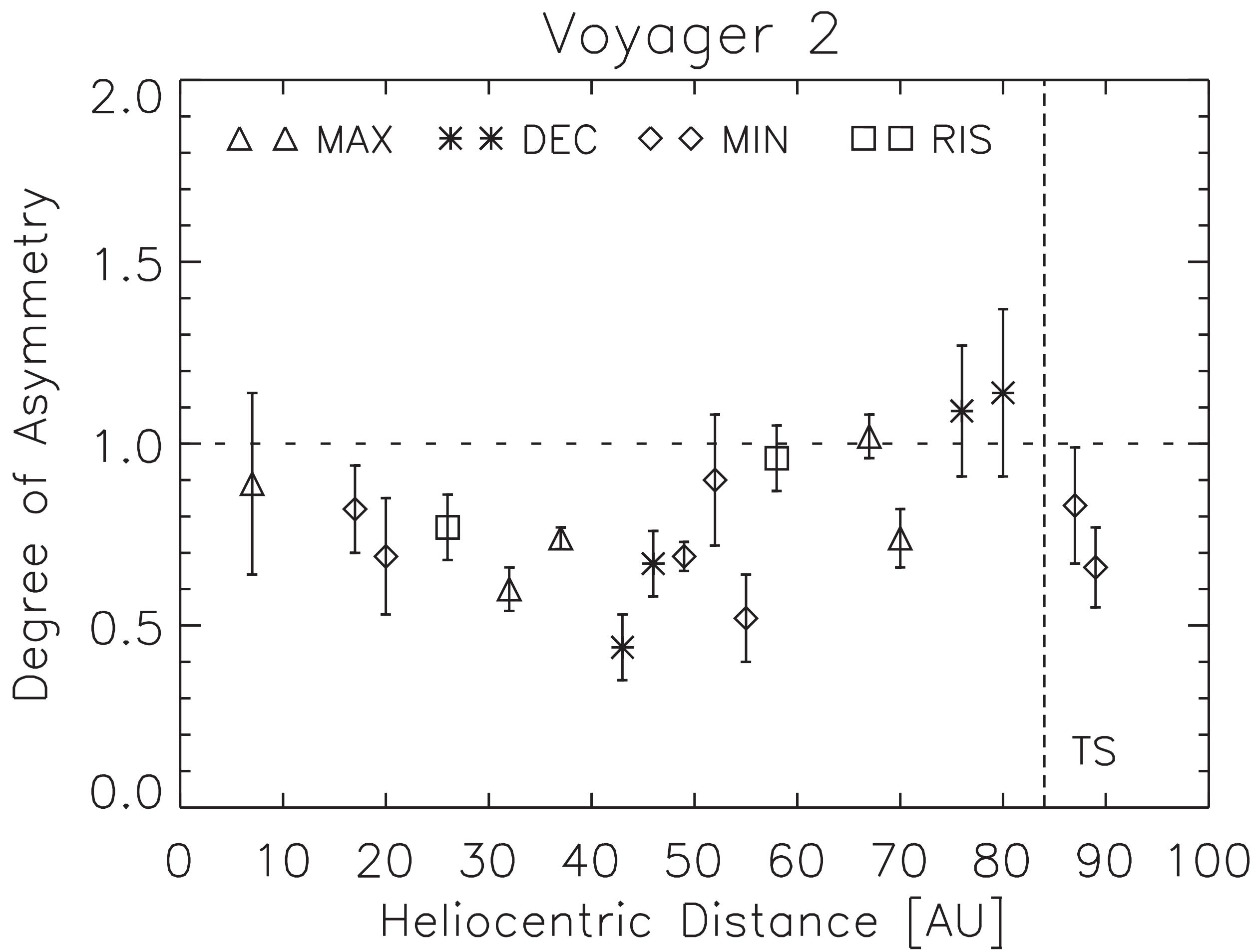


Fig. 7. The degree of asymmetry A of the multifractal spectrum in the heliosphere as a function of the heliospheric distances during solar minimum (MIN) and solar maximum (MAX), declining (DEC) and rising (RIS) phases of solar cycles. The value $A = 1$ corresponds to the one-scale symmetric model (dotted). The crossing of the termination shock by Voyager 2 is marked by a vertical (dashed) line, TS (Macek et al. 2012).

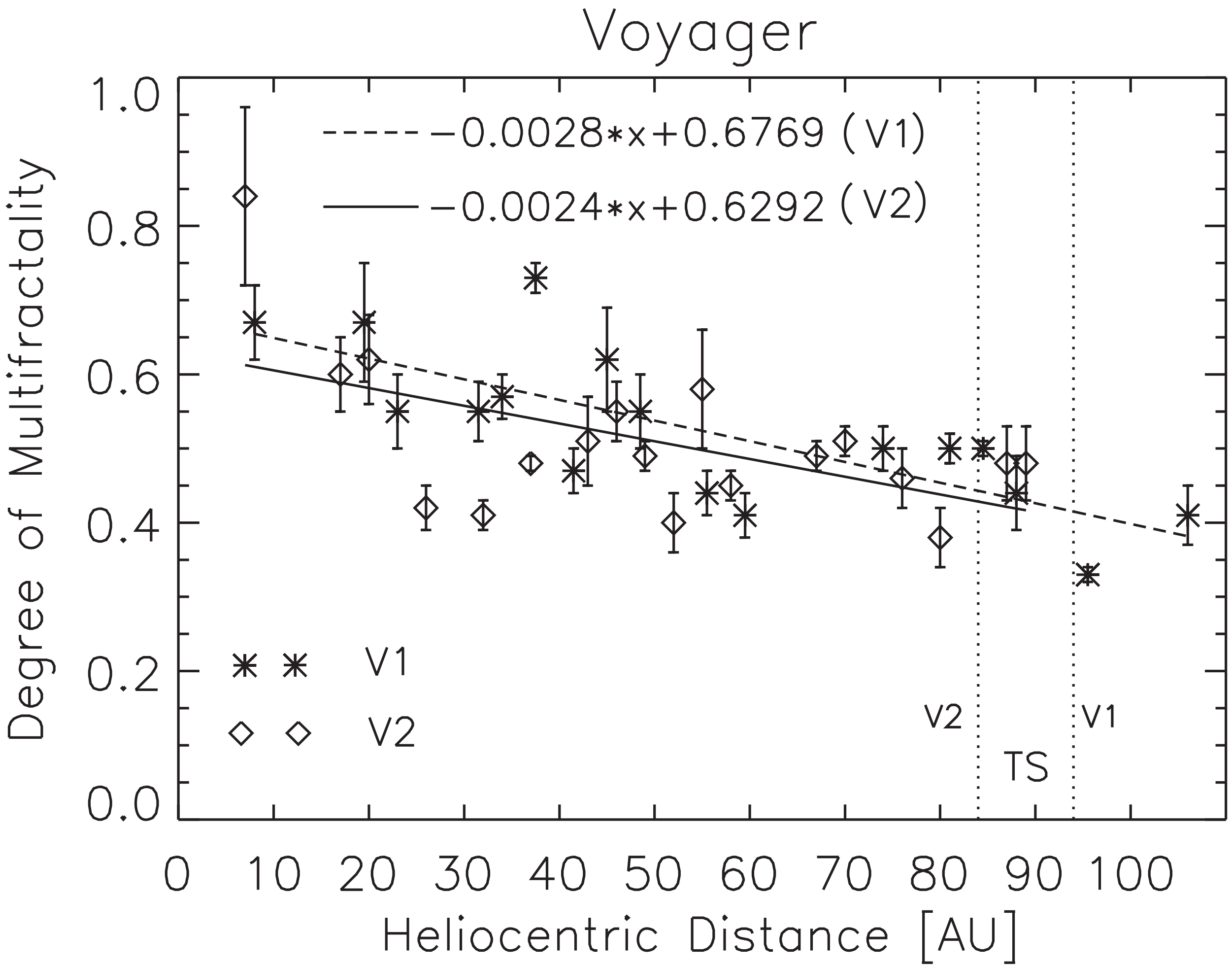


Fig. 8. The degree of multifractality Δ in the heliosphere versus the heliospheric distances for Voyager 1 and 2 with fits to straight lines. The crossings of the termination shock (TS) by both spacecraft are marked by vertical dashed lines. (Macek et al. 2012).

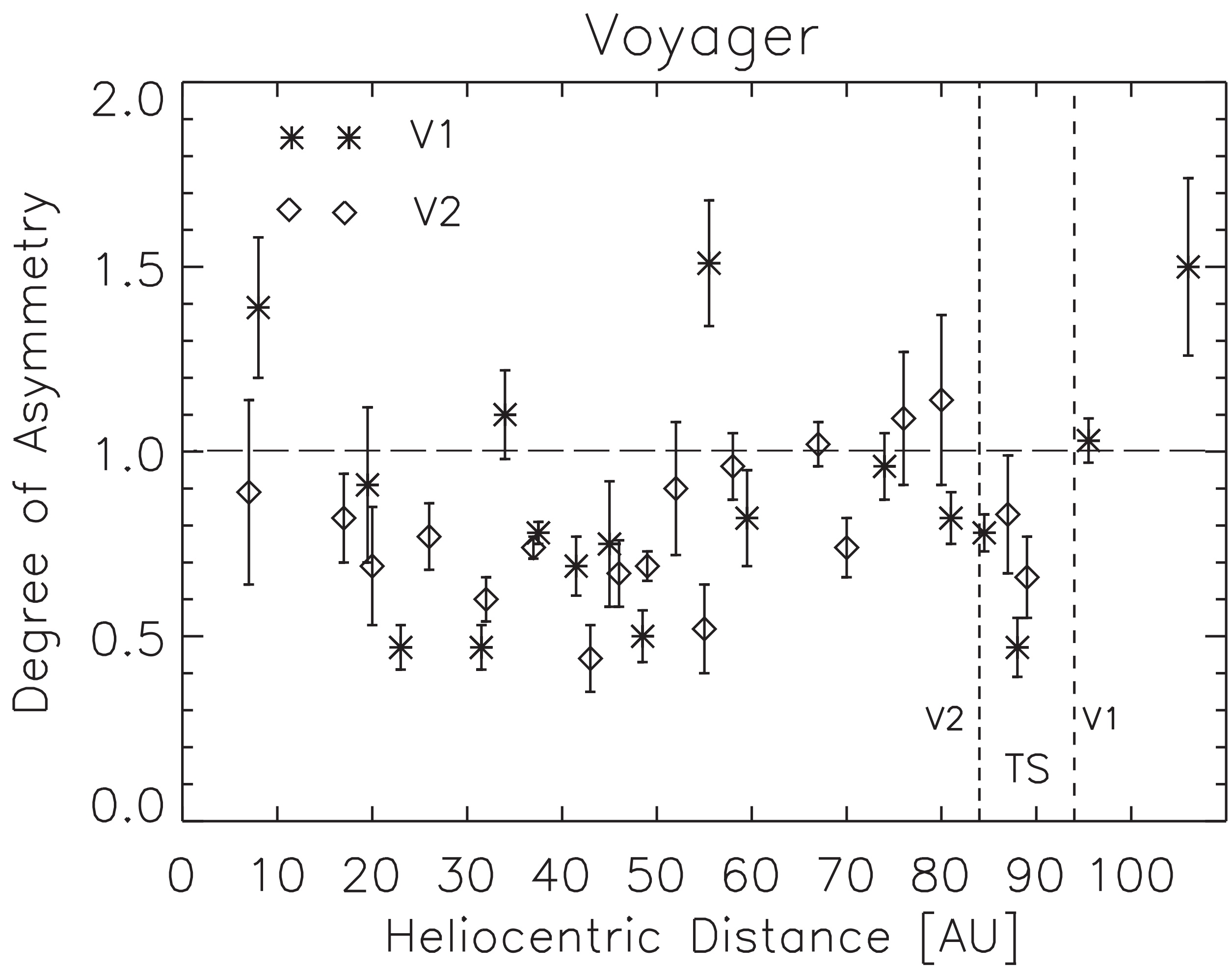


Fig. 9. The degree of asymmetry A of the multifractal spectrum in the heliosphere as a function of the heliospheric distance; the value $A = 1$ (dotted) corresponds to the one-scale symmetric model. The crossings of the termination shock (TS) by Voyager 1 and 2 are marked by vertical dashed lines (Macek et al. 2012).

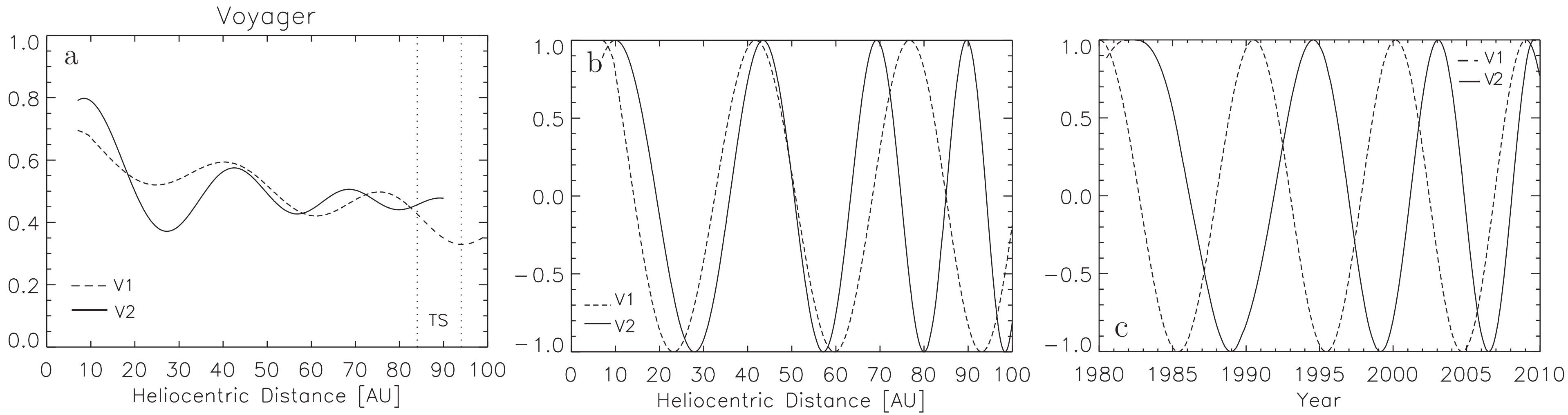


Fig. 10. Comparison of the periodic parts of functions shown in case (a), (corresponding to Figure 6) fitted to the obtained degree of multifractality as functions of heliocentric distances (b) and time (c) for Voyager 1 (dashed lines) and 2 (continuous lines) (Macek et al. 2012).

Termination shock $\Rightarrow$

Heliosheath

	7 – 40 **AU**	40 – 60 **AU**	70 – 90 **AU**	95 – 107 **AU**
Burlaga	$\mathbf{\Delta} = 0.64$ $\mathbf{A} = 0.69$	$\mathbf{\Delta} = 0.69$ $\mathbf{A} = 0.63$	$\mathbf{\Delta} = 0.69$ $\mathbf{A} = 0.63$	$\mathbf{\Delta} = 0.34$ $\mathbf{A} = 0.89$
Two-Scale Model	$\mathbf{\Delta} = 0.55 - 0.73$ $\mathbf{A} = 0.47 - 1.39$	$\mathbf{\Delta} = 0.41 - 0.62$ $\mathbf{A} = 0.51 - 1.51$	$\mathbf{\Delta} = 0.44 - 0.50$ $\mathbf{A} = 0.47 - 0.96$	$\mathbf{\Delta} = 0.33 - 0.41$ $\mathbf{A} = 1.03 - 1.51$

Heliopause $\Rightarrow$

Figure 1: Degree of multifractality Δ and asymmetry A for the magnetic field strengths in the outer heliosphere and beyond the termination shock (Macek et al. 2011).

Conclusions

- Using our weighted two-scale Cantor set model, which is a convenient tool to investigate the asymmetry of the multifractal spectrum, we confirm the characteristic shape of the universal multifractal singularity spectrum. $f(\alpha)$ is a downward concave function of scaling indices α.
- For the first time we show that the degree of multifractality for magnetic field fluctuations of the solar wind falls steadily with the distance from the Sun and seems to be modulated by the solar activity with a time delay of several years, corresponding to a difference of distances of about 10 AU in the very distant heliosphere.
- Moreover, we have again observed a change of the asymmetry of the spectrum when crossing the termination shock by Voyager 1 and 2. Consequently, a concentration of magnetic fields shrinks (stretches) resulting in thinner (fatter) flux tubes or stronger (weaker) current concentration in the heliosheath. Admittedly, we can still have approximately symmetric spectrum in the heliosheath, where the plasma is expected to be roughly in equilibrium in the transition to the interstellar medium.
- We also confirm our earlier results that before the shock crossing, especially during solar maximum, turbulence is more multifractal than that in the heliosheath (Macek et al., 2011).

- In addition, in the whole heliosphere during solar minimum the spectrum is dominated by values of $\alpha > 1$, which could change when crossing the termination heliospheric shock. This may represents a first direct *in situ* information of interest in the astrophysical context. In fact, a density of measure dominated by $\alpha < 1$ ($\alpha > 1$) would imply that the magnetic field in the very local interstellar medium is roughly confined in thin (thick) filaments of high (low) magnetic density.
- That means that it would be interesting to investigate whether the heliosheath is dominated by 'voids' of magnetic fields in the very local interstellar medium related to the presence of very localized intense magnetic field structures (cf. Frisch, 1995).
- Such informations, obtained *in situ* and not only remotely (through scintillations observations), are important in the context of question of interstellar turbulence and stellar formation modeling (e.g., Spangler et al. 2009). Finally, it is worth mentioning that our analysis brings significant additional support to earlier results suggesting that the solar wind termination shock is asymmetric (Stone et al., 2008).

Epilogue

Within the complex dynamics of the solar wind's and interstellar medium fluctuating intermittent plasma parameters, there is a detectable, hidden order described by a Cantor set that exhibits a multifractal structure.

This means that the observed **intermittent** behavior of magnetic fluctuations in the heliosphere and the very local interstellar medium may result from intrinsic *nonlinear* dynamics rather than from random external forces.

Thank you!

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