Multifractal Turbulence at the Termination Shock

Wiesław M. Macek [1,2], Anna Wawrzaszek[2]

[1]Faculty of Mathematics and Natural Sciences, Cardinal Stefan Wyszyński University, Wóycickiego 1/3, 01-938 Warsaw, Poland
[2]Space Research Centre, Polish Academy of Sciences, Bartycka 18 A, 00-716 Warsaw, Poland

ABSTRACT

We analyze time series of the solar wind magnetic field measured in situ by Voyager 1 and 2 spacecraft in the outer heliosphere [1-4] The aim of this study is to examine the question of scaling properties of intermittent solar wind turbulence at the termination shock at 84 and 94 AU from the Sun. To quantify scaling of solar wind turbulence, we consider a generalized two-scale weighted Cantor set with possibly two different scales [5, 6]. We investigate the resulting spectrum of generalized dimensions and the corresponding multifractal singularity spectrum depending on one probability measure parameter and two rescaling parameters. We observe the evolution of multifractal scaling of the solar wind in the outer heliosphere [7]. In particular, we demonstrate that this scaling is asymmetric before shock crossing, in contrast to the symmetric spectrum observed in the heliosheath. Moreover, we show that the degree of multifractality for magnetic filed fluctuations of the solar wind before shock crossing is greater than that for the heliosheath; the solar wind in the outer heliosphere may exhibit strong asymmetric scaling. It is worth noting that for the multifractal two-scale phenomenological model a good agreement with the data is obtained. Hence we propose this model as a useful tool for analysis of intermittent turbulence also at the heliospheric boundaries.

1 Theoretical Model

1.1 Multifractal Formalism

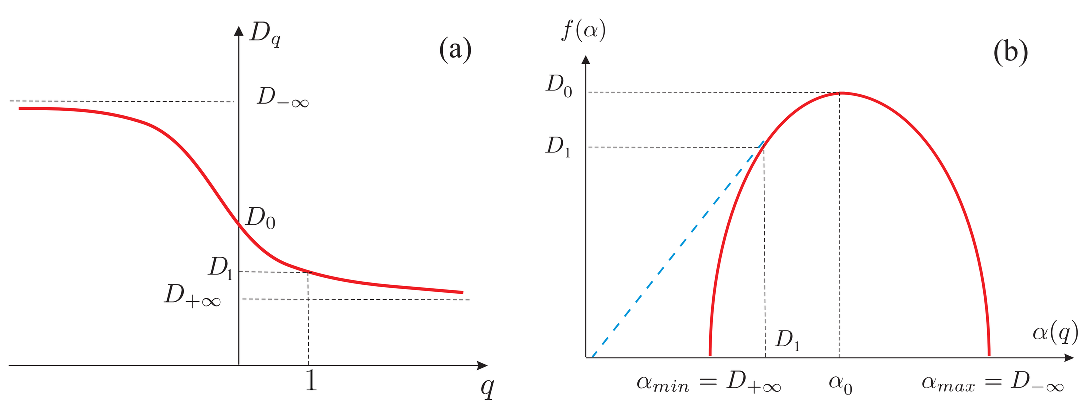
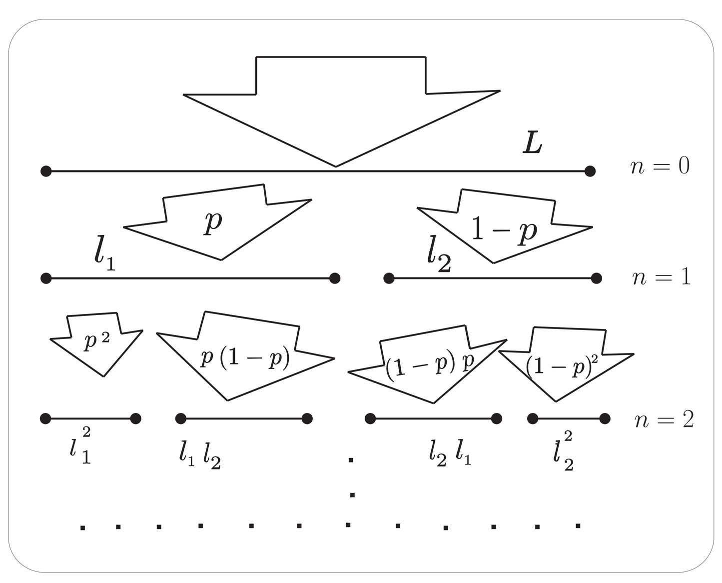


Fig. 1: A multifractal is an object that demonstrate various self-similarities, described by (a) a multifractal spectrum of dimensions D_q and (b) a singularity spectrum $f(\alpha)$.

1.2 Generalized P model

At each stage of construction of the weighted two-scale Cantor set we have two scaling parameters (l_1, l_2) and two different weights p and $1-p$. To obtain generalized dimensions and singularity spectra for this multifractal set we use the partition function at the n-th level of construction

$$\Gamma_n^q(l_1, l_2, p) = \left(\frac{p^q}{l_1^{\tau(q)}} + \frac{(1-p)^q}{l_2^{\tau(q)}} \right)^n = 1 \quad (1)$$



$$D_q = \frac{\tau(q)}{(q-1)}$$

$$\alpha(q) = \frac{d\,\tau(q)}{dq}$$

$$f(\alpha) = q\,\alpha(q) - \tau(q)$$

Fig. 1: Two-scale weighted Cantor set model for asymmetric solar wind turbulence [5].

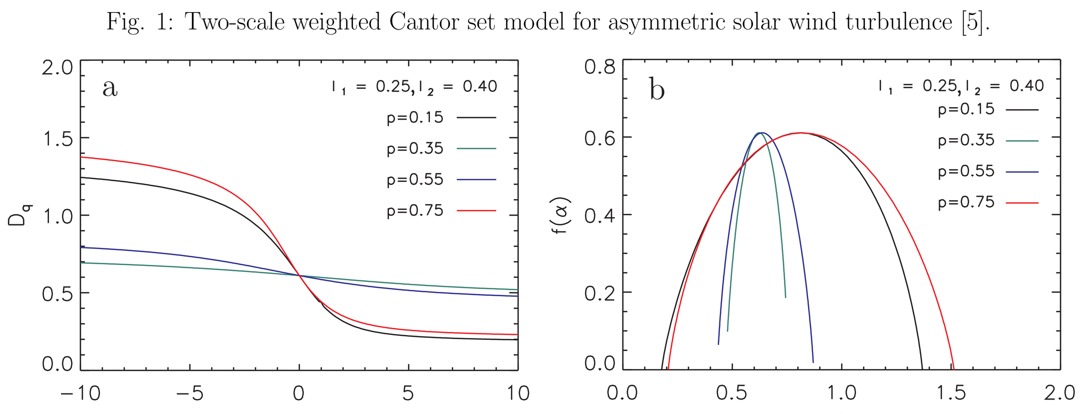


Fig. 2: The generalized dimensions D_q (a) and multifractality spectra $f(\alpha)$ (b). The values of D_q and $f(\alpha)$ are calculated numerically for the two-scale weighted Cantor set using different values of p.

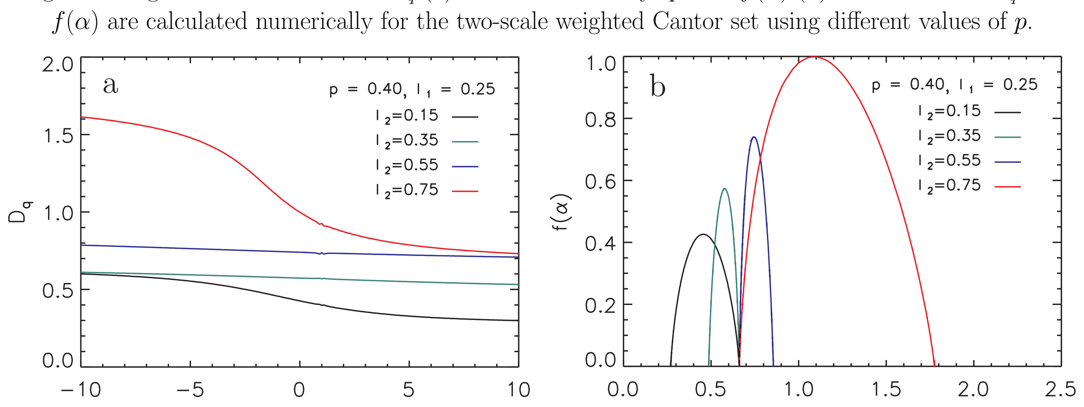


Fig. 3: The values of D_q and $f(\alpha)$ are calculated numerically for the two-scale weighted Cantor set using different values of l_2.

1.2 Comparison With the P model

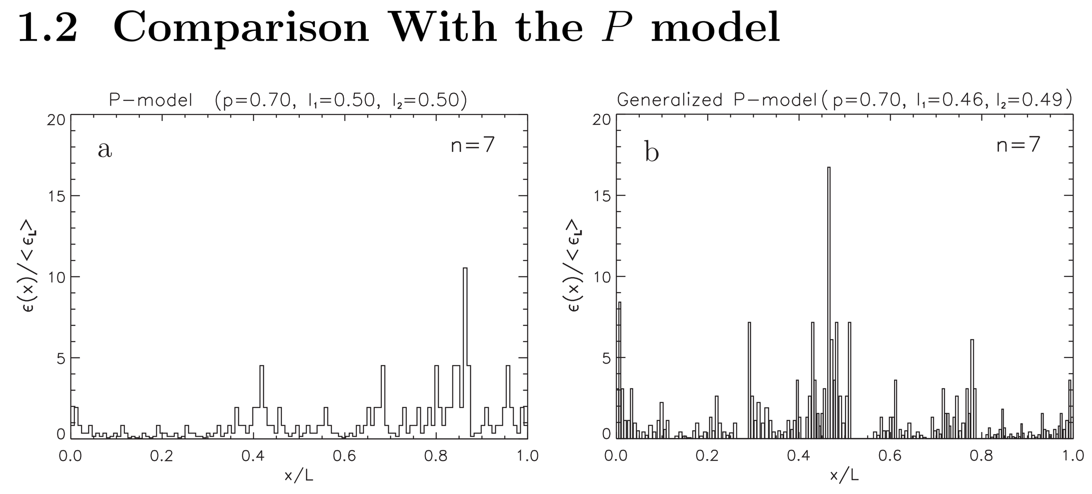


Fig. 4: The multifractal measure $\mu = \varepsilon / \langle \varepsilon_L \rangle$ on the unit interval for (a) the usual one-scale p-model and (b) the generalized two-scale cascade model. Intermittent pulses are stronger for the model with two different scaling parameters [5].

3 Solar Wind Data

25 AU (1987-1988)
40 AU (1989)
85 AU (2002)
95 AU (2005)

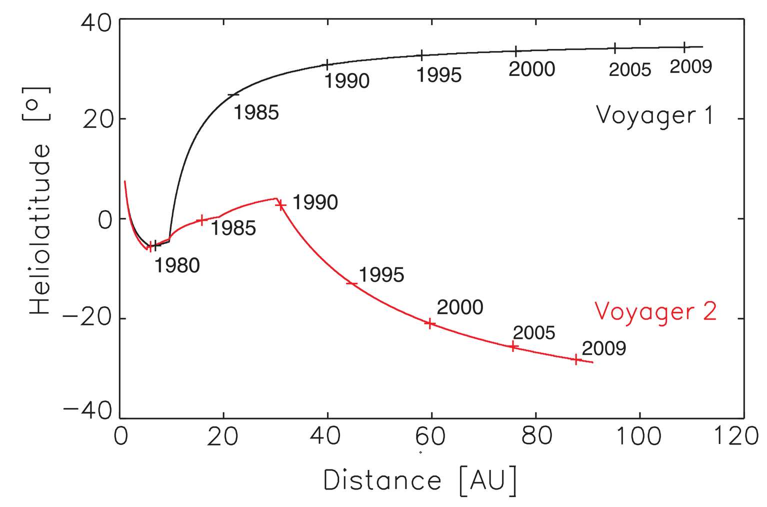


Fig. 5: Voyager 1 (black line) and 2 (red line) heliolatitude versus distance from the Sun.

4 Results - The Singularity Spectrum

4.1 25 AU

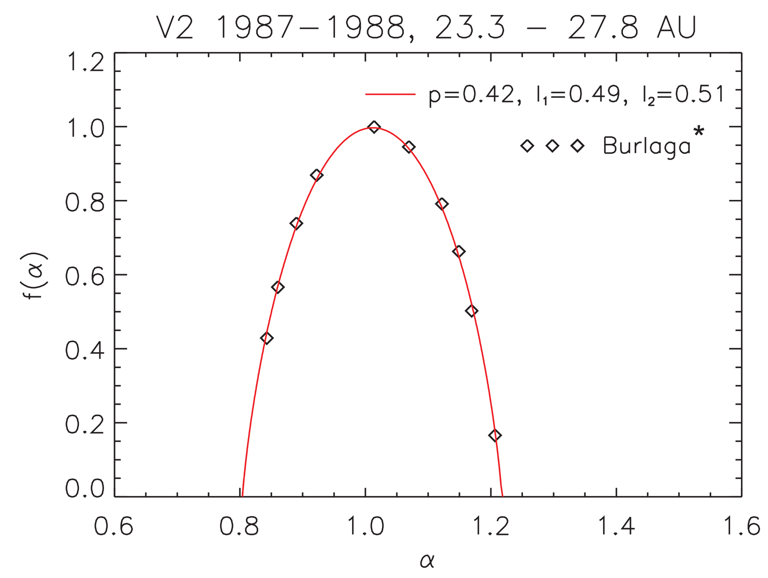


Fig. 6.1: The multifractal spectrum derived from the Voyager 2 observations of the magnetic field strength at 25 AU (diamonds). Burlaga has proposed a fourth-order polynomial fit [1]. The solid curve is a fit using of the two-scale model, which is in a good agreement with these points and exhibits symmetric character of the spectrum.

4.2 40 AU

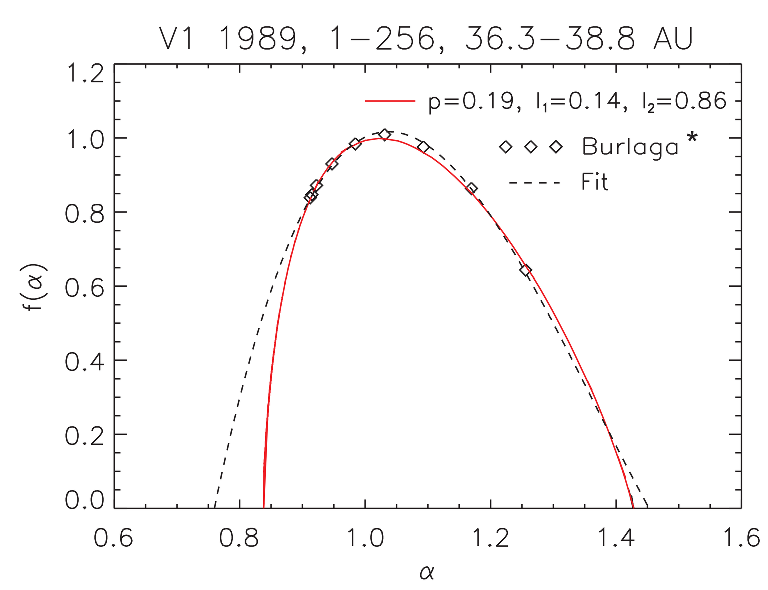


Fig. 6.2: The multifractal spectrum derived from the Voyager 1 observations of magnetic fields at 40 AU (diamonds). The dashed line is a fit using a cubic polynomial [2]. The fit of the generalized two-scale model (continuous line) shows a good agreement with the data and exhibits asymmetric character of the spectrum.

4.3 85 AU

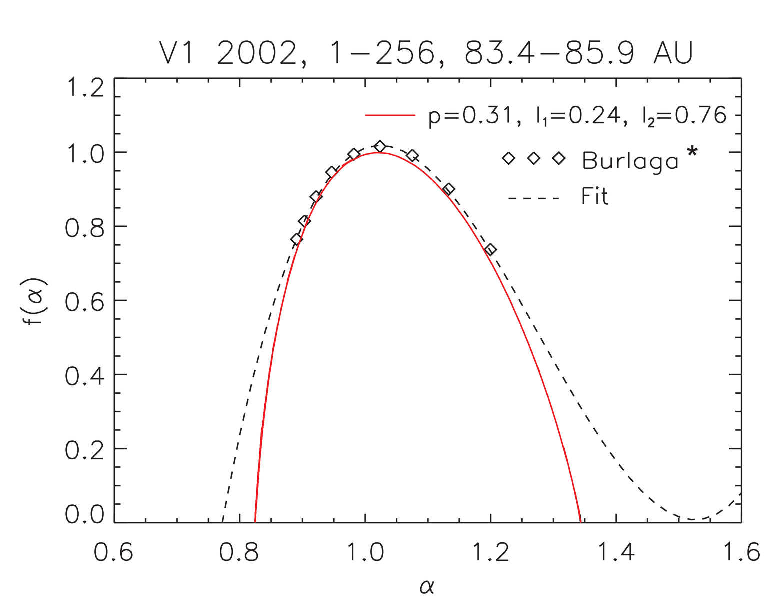


Fig. 6.3: The multifractal spectrum derived from the Voyager 1 observations of magnetic fields near 85 AU (diamonds). The dashed line is a fit using a cubic polynomial [3]. The fit of the generalized two-scale model (continuous line) shows a good agreement with the data exhibiting somewhat smaller degree of multifractality.

4.4 95 AU

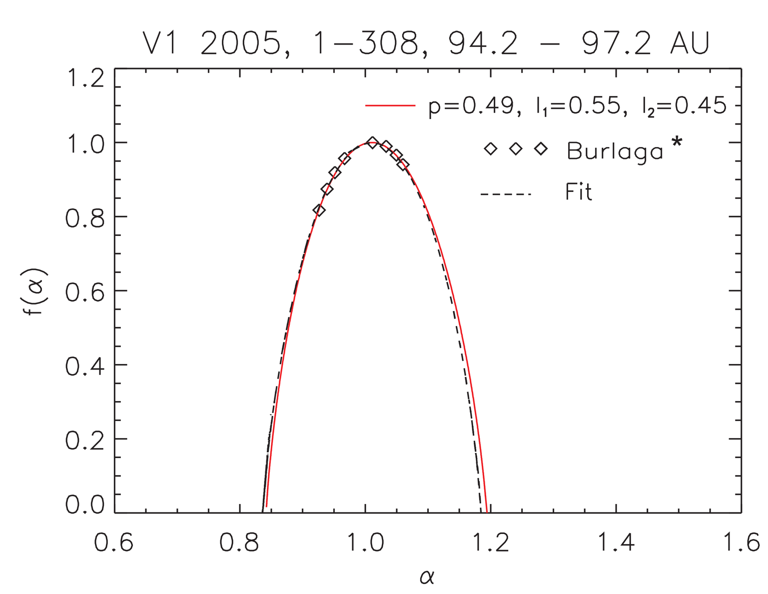


Fig. 6.4: The multifractal spectrum of the magnetic field strength observed by the Voyager 1 in the heliosheath at 95 AU (diamonds) together with a quadratic fit to the p model with $p = 0.56$ [4]. The fit of the generalized two-scale model (continuous line) exhibits symmetric character of the multifractal spectrum in a good agreement with the p model.

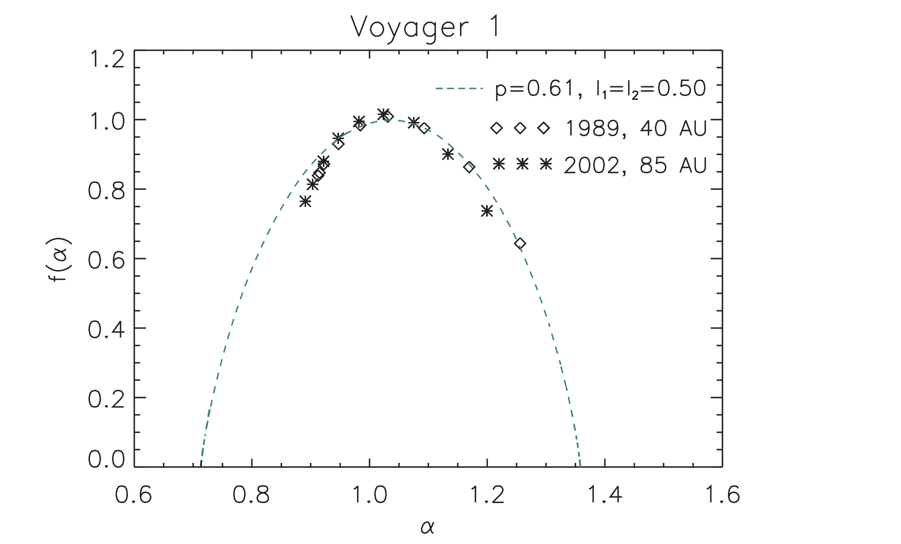


Fig. 6.5: The multifractal spectrum derived from the Voyager 1 observations of the magnetic field strength at 40 AU (diamonds) and 85 AU (stars) together with a fit of the p model with $p = 0.61$ [4].

4.5 Degree of Multifractality and Asymmetry

$$\Delta \equiv \alpha_{\max} - \alpha_{\min} = D_{-\infty} - D_{\infty} = \left| \frac{\log(1-p)}{\log l_2} - \frac{\log(p)}{\log l_1} \right|$$

$$A \equiv \frac{\alpha_0 - \alpha_{\min}}{\alpha_{\max} - \alpha_0}\; f(\alpha_0) = 1$$

Table 1. Degree of Multifractality Δ and Asymmetry A for the magnetic field strength observed in the Outer Heliosphere.

	$\sim$ 25 AU	$\sim$ 40 AU	$\sim$ 85 AU	$\sim$ 95 AU
Burlaga	–	$\Delta = 0.65$	$\Delta = 0.68$	$\Delta = 0.34$
Two-scale model	$\Delta = 0.41$	$\Delta = 0.59$	$\Delta = 0.52$	$\Delta = 0.35$
Asymmetry	$A = 1.01$	$A = 0.46$	$A = 0.60$	$A = 0.91$

5 Conclusions

- **We have studied the inhomogeneous rate of the transfer of the energy flux indicating multifractal and intermittent behavior of solar wind turbulence in the inner and outer heliosphere [6-7].**
- **Basically, the generalized dimensions for solar wind are consistent with the generalized p model for both positive and negative q, but rather with different scaling parameters for sizes of eddies, while the usual p model can only reproduce the spectrum for $q \geq 0$. We have demonstrated that a much better agreement of the two-scale model with the real data is obtained, especially for $q < 0$.**
- **The degree of multifractality for the solar wind during solar minimum not very far from the Sun is greater for fast streams than that for the slow streams.**
- **Both the degree of multifractality and degree of asymmetry are correlated with the heliospheric distance and we observe the evolution of multifractal scaling in the outer heliosphere.**
- **We also show that the degree of multifractality for magnetic field fluctuations of the solar wind before shock crossing is greater than that for the heliosheath.**
- **It is worth noting that the multifractal scaling is often rather asymmetric, especially for the fast wind during solar minimum. In particular, we demonstrate that this scaling is strongly asymmetric before shock crossing.**
- **In contrast to the asymmetric spectrum observed in the outer heliosphere the spectrum becomes symmetric after the shock crossing, i. e. in the heliosheath, where the plasma is roughly in equilibrium.**
- **Our results provide supporting evidence for multifractal structure of the solar wind in the outer heliosphere. One can expect that the fluctuations in the solar wind magnetic field should contain information about the dynamic variations of the solar wind plasma also at the heliospheric boundaries.**
- **In general, the proposed generalized two-scale weighted Cantor set model should also be valid for non space filling turbulence. Therefore we propose this new cascade model describing intermittent energy transfer for analysis of turbulence in various environments.**

References

[1] Burlaga, L. (1991), Multifractal structure of the interplanetary magnetic field–Voyager 2 observations near 25 AU, 1987–1988, *Geophys. Res. Lett.*, *18*, 69–72.

[2] Burlaga, L. F., C. Wang, N. F. Ness (2003), A model and observations of the multifractal spectrum of the heliospheric magnetic field strength fluctuations near 40 AU, *Geophys. Res. Lett.*, *30*, 501-504, doi:10.1029/2003GL016903.

[3] Burlaga, L. F. (2004), Multifractal structure of the large-scale heliospheric magnetic field strength fluctuations near 85 AU, *Nonlin. Proc. Geophys.*, *11*, 441–445.

[4] Burlaga, L. F., N. F. Ness, M. H. Acuna (2006), Multiscale structure of magnetic fields in the heliosheath, *J. Geophys. Res.*, *111*, A09112, doi:10.1029/2006JA011850.

[5] W. M. Macek and A. Szczepaniak (2008), Generalized two-scale weighted Cantor set model for solar wind turbulence, Geophys. Res. Lett., 35, L02108, doi:10.1029/2007GL032263.

[6] A. Szczepaniak and W. M. Macek (2008), Asymmetric multifractal model for solar wind intermittent turbulence, *Nonlin. Processes Geophys.*, *15*, 615–620 , http://www.nonlin-processes-geophys.net/15/615/2008/.

[7] W. M. Macek and A. Wawrzaszek (2009), Evolution of asymmetric multifractal scaling of solar wind turbulence in the outer heliosphere, *J. Geophys. Res.*, *114*, A 03108, doi:10.1029/2008JA013795.

Email: macek@cbk.waw.pl
http://www.cbk.waw.pl/~macek