JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 115, A07104, doi:10.1029/2009JA015176, 2010



# Observation of the multifractal spectrum in solar wind turbulence by Ulysses at high latitudes

Anna Wawrzaszek[1] and Wieslaw M. Macek[2,3]

Received 7 December 2009; revised 27 January 2010; accepted 2 March 2010; published 13 July 2010.

[1] The aim of our study is to examine the question of multifractal scaling properties of turbulence in the solar wind at high latitudes. We analyze time series of the velocities of the solar wind during solar minimum (1994–1997, 2006–2007) at various heliographic latitudes measured in situ by Ulysses, which is the only mission that has investigated parameters of the solar wind out of the ecliptic plane also in the polar regions of the Sun. We consider the non-homogeneous energy transfer rate in the turbulent cascade leading to the phenomenon of intermittency. To quantify the degree of multifractality and the degree of asymmetric scaling of solar wind turbulence, we consider a generalized two-scale weighted Cantor set with two different scales describing nonuniform distribution of the kinetic energy flux between cascading eddies of various sizes. It is worth noting that both characteristics exhibit latitudinal dependence with some symmetry with respect to the ecliptic plane. Generally, at high latitudes during solar minimum in the fast solar wind streams we observe a somewhat smaller degree of multifractality and intermittency as compared with those at the ecliptic and a roughly symmetric multifractal singularity spectrum. The minimum of intermittency is observed at midlatitudes, possibly related to the transition from the region where the interaction of the fast and slow streams takes place to a more homogeneous region of the pure fast solar wind.

**Citation:** Wawrzaszek, A., and W. M. Macek (2010), Observation of the multifractal spectrum in solar wind turbulence by Ulysses at high latitudes, *J. Geophys. Res.*, *115*, A07104, doi:10.1029/2009JA015176.

## 1. Introduction

[2] As is known the solar wind is one of the many complex physical systems, which are characterized by intermittency [*Bruno and Carbone*, 2005]. For these systems energy at a given scale is not evenly distributed in space. Consequently, solar wind parameters, in particular magnetic field and velocity, exhibit strong intermittent behavior both during solar minimum and solar maximum [*Marsch and Tu*, 1994, 1997] at different heliocentric distance, in the inner [*Marsch and Liu*, 1993; *Sorriso-Valvo et al.*, 1999; *Bruno et al.*, 2001; *Hnat et al.*, 2003; *Szczepaniak and Macek*, 2008] and the outer heliosphere [*Burlaga*, 1991; *Burlaga et al.*, 1993; *Burlaga*, 2001].

[3] The launch of the Ulysses on 6th October 1990 provided for the first time the possibility to investigate the properties of the solar wind beyond the ecliptic plane. The mission has completed its third circumnavigation of the solar polar regions and finally came to an end on 30th June 2009. The magnetic data measured by Ulysses allowed to identify intermittency signatures also at high latitudes [*Horbury et al.*, 1995; *Horbury and Balogh*, 1997; *Pagel and Balogh*, 2002]. In particular, the evolution of intermittency in interplanetary magnetic fields with distance and latitude and nature of the turbulent cascade in fast solar wind has been studied [e.g., *Horbury and Balogh*, 2001; *Pagel and Balogh*, 2003; *Yordanova et al.*, 2009]. Naturally, there are essential differences between properties of ecliptic and polar solar wind turbulence [*Bruno and Carbone*, 2005]. For example, fluctuations of these parameters at high latitudes seem to be more homogeneous, evolving more slowly, but Alfvénic fluctuations becomes more important, as compared with those in the ecliptic [*Horbury and Balogh*, 2001].

[4] A detailed picture of energy transfer processes and intermittency are still not clear; there is no dominant model to quantify intermittency in a given system. One of the advanced and popular method used to look inside complex nature of intermittent turbulence is multifractal formalism [*Mandelbrot*, 1989]. In particular, the multifractal spectrum has been investigated using Voyager (magnetic field fluctuations) data in the outer heliosphere [e.g., *Burlaga*, 1991; *Burlaga et al.*, 1993; *Burlaga*, 2001] and using Helios (plasma) and Advanced Composition Explorer (ACE) data in the inner heliosphere [e.g., *Marsch et al.*, 1996; *Szczepaniak and Macek*, 2008; *Macek and Wawrzaszek*, 2010]. It has appeared that the multifractal singularity spectrum obtained for the solar wind data has an asymmetric shape and shows a substantial departure from the standard $p$ model [*Macek*, 2007; *Macek and Wawrzaszek*, 2009].

[1]Space Research Centre, Polish Academy of Sciences, Warsaw, Poland.
[2]Faculty of Mathematics and Natural Sciences, Cardinal Stefan Wyszyński University in Warsaw, Warsaw, Poland.
[3]Also at Space Research Centre, Polish Academy of Sciences, Warsaw, Poland.


0148-0227/10/2009JA015176

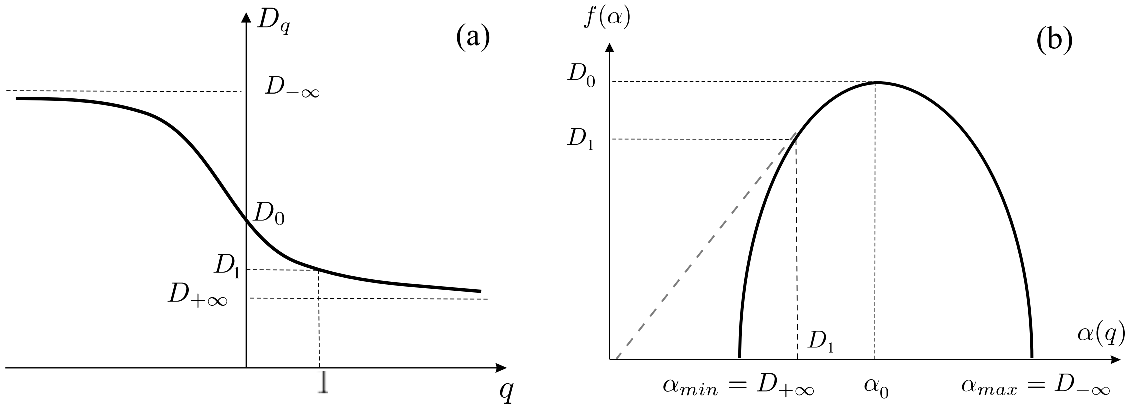


**Figure 1.** (a) The generalized dimensions $D_q$ as a function of any real $q$, $-\infty < q < +\infty$, and (b) the singularity multifractal spectrum $f(\alpha)$ versus the singularity strength $\alpha$ with some general properties: (1) the maximum value of $f(\alpha)$ is $D_0$; (2) $f(D_1) = D_1$; and (3) the line joining the origin to the point on the $f(\alpha)$ curve, where $\alpha = D_1$ is tangent to the curve [*Ott*, 1993].

[5] Therefore, to quantify scaling of solar wind turbulence, we have developed a generalized weighted two-scale Cantor set model using the partition technique [*Szczepaniak and Macek*, 2008]. We have already studied the inhomogeneous rate of the transfer of the energy flux indicating multifractal and intermittent behavior of solar wind turbulence in the inner (Helios and ACE) and outer heliosphere (Voyager) using fluctuations of the velocity of the flow of the solar wind at small scales. We have investigated the resulting spectrum of generalized dimensions and the corresponding multifractal singularity spectrum depending on the model parameters [*Macek and Szczepaniak*, 2008; *Macek and Wawrzaszek*, 2009; *Macek et al.*, 2009]. By using the cascade model with two different scaling parameters we have shown that the degree of multifractality of the velocity fluctuations of the solar wind in the inner and outer heliosphere in the ecliptic is different for slow and fast streams. As the solar activity increases the slow solar wind becomes more multifractal. Moreover, both the degree of multifractality and the degree of asymmetry of the singularity spectrum are correlated with the heliospheric distance, and we have observed the evolution of multifractal scaling in the heliosphere [*Macek and Wawrzaszek*, 2009].

[6] The aim of this study is to further examine the question of scaling properties of intermittent turbulence using velocities of the fast speed streams of the solar wind measured in situ by Ulysses during solar minimum out of the ecliptic plane. As far as we are aware of we have determined for the first time the full multifractal singularity spectrum for the plasma data in this region. By using our weighted two-scale Cantor set model in comparison with the simple one scale multifractal spectrum we show that the degree of multifractality for fluctuations at high latitudes during solar minimum for the fast solar wind is somewhat smaller as compared with those at the ecliptic and we observe roughly symmetric multifractal singularity spectra. It is worth noting that the minimum of intermittency is observed at midlatitudes (from ~50° to ~70°) possibly related to the transition from the region where the interaction of the fast and slow streams takes place to a more homogeneous region of the pure fast solar wind.

[7] This paper is organized as follows. In section 2 a multifractal formalism and the weighted two-scale Cantor set model are introduced, and the data are presented in section 3. The methods related the concept of the generalized dimensions and the singularity spectrum in the context of turbulence scaling are reviewed in section 4. The results of our analysis are presented and discussed in section 5. The importance of our multifractal cascade model also for solar wind turbulence at high latitudes is underlined in section 6.

## 2. Multifractal Cascade Model

### 2.1. Multifractal Formalism

[8] Theory of multifractals allows us an intuitive understanding of multiplicative processes and of the intermittent distributions of various characteristics of turbulence. As an extension of fractals, multifractals could be seen as objects that demonstrate various self-similarities at various scales. Consequently, the multifractals are described by an infinite number of the generalized dimensions, $D_q$, as depicted in Figure 1a and by the multifractal spectrum $f(\alpha)$ sketched in Figure 1b [*Halsey et al.*, 1986]. The generalized dimensions $D_q$ are calculated as a function of a continuous index $q$ [*Grassberger*, 1983; *Grassberger and Procaccia*, 1983; *Hentschel and Procaccia*, 1983; *Halsey et al.*, 1986]. This parameter $q$, where $-\infty < q < \infty$, can be compared to a microscope for exploring different regions of the singular measurements. In the case of turbulence cascade the generalized dimensions are related to inhomogeneity with which the energy is distributed between different eddies [*Meneveau and Sreenivasan*, 1991]. In this way they provide information about dynamics of multiplicative process of cascading eddies. Here high positive values of $q$ emphasize regions of intense energy transfer rate, while negative values of $q$ accentuate low-transfer rate regions. An alternative description can be formulated by using the singularity spectrum $f(\alpha)$ as a function of a singularity strength $\alpha$, which quantify multifractality of a given system [e.g., *Ott*, 1993]. This function describes singularities occurring in considered probability measure attributed to different

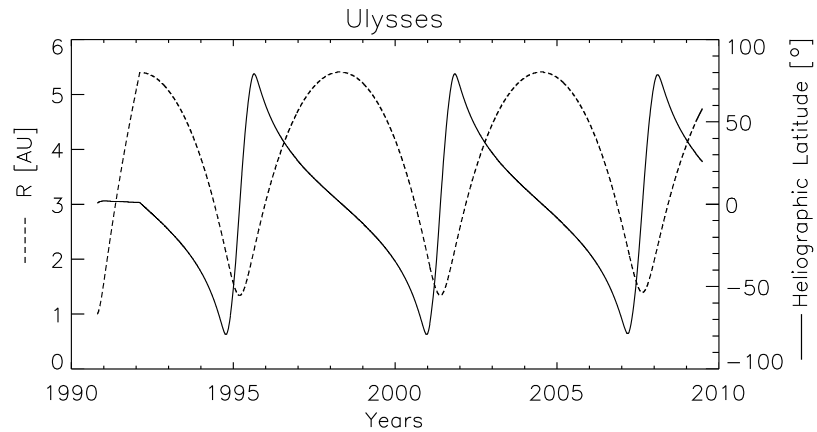


**Figure 2.** The heliographic latitude (continuous lines) and the heliocentric distance from the Sun (dashed lines) during each year of the Ulysses mission.

regions of the phase space of a given dynamical system. Admittedly, both functions $f(\alpha)$ and $D_q$ have the same information about multifractality. However, the singularity multifractal spectrum is easier to interpret theoretically by comparing the experimental results with the models under study.

### 2.2. Two-Scale Model

[9] In order to look into the multifractality and intermittency in the context of turbulence cascade, we consider generalized weighted Cantor set [*Macek and Szczepaniak*, 2008]. This model describe a standard scenario of cascading eddies, each breaking down into two new ones, but not necessarily equal and twice smaller. At each stage of construction of this generalized Cantor set we basically have two scaling parameters $l_1$ and $l_2$, where $l_1 + l_2 \leq L = 1$ (normalized) and two different probability measure $p_1 = p$ and $p_2 = 1 - p$. In particular, space filling turbulence is recovered for $l_1 + l_2 = 1$. Naturally, in the inertial region of the system of size $L = 1$, $\eta \ll l \ll 1$, the energy is not allowed to be dissipated directly, assuming $p_1 + p_2 = 1$, until the Kolmogorov scale $\eta$ is reached. However, in this range at each $n$th step of the binomial multiplicative process, the flux of kinetic energy density $\varepsilon$ transferred to smaller eddies (energy transfer rate) could be divided into nonequal fractions $p$ and $1 - p$ [cf. *Meneveau and Sreenivasan*, 1987]. Namely, in the first step of the two-scale model construction we have two eddies of sizes $l_1 = 1/r$ and $l_2 = 1/s$, satisfying $p/l_1 + (1 - p)/l_2 = 1$, or equivalently $rp + s\,(1 - p) = 1$. Therefore, the initial energy flux $\varepsilon_0$ is transferred to these eddies with the different proportions: $rp\varepsilon_0$ and $s(1 - p)\varepsilon_0$. In the next step the energy is divided between four eddies as follows: $(rp)^2\varepsilon_0$, $rsp(1 - p)\varepsilon_0$, $sr(1 - p)p\varepsilon_0$, and $s^2(1 - p)^2\varepsilon_0$. At $n$th step we have $N = 2^n$ eddies and partition of energy $\varepsilon$ can be described by the relation [*Burlaga et al.*, 1993]:

$$\varepsilon = \sum_{i=1}^{N} \varepsilon_i = \varepsilon_0 (rp + s(1-p))^n = \varepsilon_0 \sum_{k=0}^{n} \binom{n}{k} (rp)^{(n-k)} (s(1-p))^k. \tag{1}$$

This example of multifractal is characterized by two functions: the generalized dimensions and the singularity spectra. For any $q$ one obtains $D_q$ by solving numerically the following transcendental equation, e.g., [*Ott*, 1993]

$$\frac{p^q}{l_1^{(q-1)D_q}} + \frac{(1-p)^q}{l_2^{(q-1)D_q}} = 1. \tag{2}$$

## 3. Solar Wind Data

[10] It is worth noting that Ulysses' periodic (6.2 years) orbit with perihelion at 1.3 AU and aphelion at 5.4 AU and latitudinal excursion of ±82° gives us a new possibility enabling to study both latitudinal and radial dependence of the solar wind [*Smith et al.*, 1995; *Horbury et al.*, 1996]. The positions of the spacecraft as a function of heliographic distances and latitudes are depicted in Figure 2.

[11] In this paper we would like to determine multifractal characteristics of turbulence scaling such as the degree of multifractality and asymmetry of the multifractal singularity spectrum for the wealth of data provided by Ulysses space mission. We use plasma flow measurements as obtained from the SWOOPS instrument (Solar Wind Observations Over the Poles of the Sun). Namely, we analyze time series of velocities of the solar wind measured by Ulysses out of the ecliptic plane at different heliographic latitudes (+32° ÷ +40°, +47° ÷ +48°, +74° ÷ +78°, −40° ÷ −47°, −50° ÷ −56°, −69° ÷ −71°) and heliocentric distances of $R$ = 1.4–5.0 AU from the Sun. Here we have selected twelve-days time intervals of $v_x$ samples, each of 4096 data points, with sampling time of $\Delta t = 242$ s ≈ 4 min, for solar wind streams during solar minimum (1994–1997, 2006–2007). The heliographic latitudes and heliocentric distances of the analyzed data samples are summarized in Table 1. It has recently been suggested that longer samples in polar wind can be affected by non-stationarity [*Sorriso-Valvo et al.*, 1999; *Marino et al.*, 2008]. Therefore, it seems that this is an optimal choice of sample length, which provide us a possibility to divide the whole sample into $2^{12}$ segments. In our experience, this is sufficient to grasp the multifractal

**Table 1.** Heliographic Latitudes and Heliocentric Distances of the Analyzed Data Samples

| Year | Days | Heliographic Latitude | Heliocentric Distance |
|---|---|---|---|
| 1997[a] | 68–79 | +14° ÷ +15° | 4.9 AU |
| 1995 | 104–115 | +32° ÷ +40° | 1.4 AU |
| 1996 | 39–50 | +47° ÷ +48° | 3.3 AU |
| 1995 | 181–192 | +74° ÷ +78° | 1.8–1.9 AU |
| 1995 | 199–210 | +79° ÷ +80° | 1.9–2.0 AU |
| 2007 | 150–161 | −40° ÷ −47° | 1.6 AU |
| 1994 | 344–355 | −50° ÷ −56° | 1.6–1.7 AU |
| 2006 | 317–328 | −69° ÷ −71° | 2.8–2.9 AU |

[a]The case of the slow wind.

scaling of the solar wind fluctuations [*Macek*, 1998; *Macek and Redaelli*, 2000].

## 4. Methods of Data Analysis

[12] Description of the scaling properties and intermittency in the solar wind turbulence by determining the generalized dimensions and singularity spectra is the main problem considered in this article. For the direct estimation of the multifractal spectrum from the experimental data we use method proposed by *Chhabra and Jensen* [1989], as thoroughly discussed in our previous paper [*Macek and Wawrzaszek*, 2009]. In the first step of our analysis we construct multifractal measure [*Mandelbrot*, 1989] defining by using some approximation the transfer rate of the energy flux $\varepsilon$ in energy cascade. Namely, given a turbulent eddy of size $l$ with a velocity amplitude $u(x)$ at a point $x$ the transfer rate of this quantity $\varepsilon(x, l)$ is widely estimated by the third moment of increments of velocity fluctuations, e.g. [*Frisch*, 1995; *Frisch et al.*, 1978],

$$\varepsilon(x,l) \sim \frac{|u(x+l)-u(x)|^3}{l}, \tag{3}$$

where $u(x)$ and $u(x + l)$ are velocity components parallel to the longitudinal direction separated from a position $x$ by a distance $l$. Recently, limitations of this approximation are discussed and its hydromagnetic generalization for the Alfvénic fluctuations by using Ulysses data are considered [*Sorriso-Valvo et al.*, 2007; *Marino et al.*, 2008].

[13] Now, we decompose the signal in segments of size $l$ and then each segment is associated to an eddy. Therefore to each $i$th eddy of size $l$ in the turbulence cascade we associate a probability measure defined by

$$p(x_i,l) \equiv \frac{\varepsilon(x_i,l)}{\sum_{i=1}^{N}\varepsilon(x_i,l)} = p_i(l). \tag{4}$$

This quantity can be interpreted as a probability that the energy is transferred to an eddy of size $l$. As is usual, the temporal scales, measured in units of sampling time $\Delta t$, can be interpreted as the spatial scales, $x = v_{sw}t$, where $v_{sw}$ is the average solar wind speed (Taylor's hypothesis). At $n$-stage of this binomial multiplicative process of cascading eddies, as given by equation (1), we can obtain $2^n$ eddies of various sizes, ($i = 1,\ldots, N = 2^n$, in our case we have $n = 12$). In particular, we see that given a sampling time $\Delta t$, we have the smallest size $l = v_{sw}\Delta t$, and the quantity $p(t_i, \Delta t)$ may be thought of as the probability measure of the realization of the transfer rate at moment $t_i$ in the discrete time series [*Marsch et al.*, 1996, Equation (2)]. To give an illustration of intermittent nature of the energy transfer in the turbulence cascade, in Figures 3 and 4 we show the time trace $p(t)$ of the multifractal measure $p(t_i, \Delta t) = \varepsilon(t_i, \Delta t)/\Sigma\, \varepsilon(t_i, \Delta t)$ given by equations (3) and (4) and obtained using data of the velocity components $u = v_x$ (in time domain) as measured by Ulysses with a sampling time difference $\Delta t = 242\ s$ at different heliographic latitudes for the solar wind during solar minimum (1994–1997, 2006–2007). Admittedly, in the inertial range due to energy conservation the total energy transferred to all eddies at a given step of the energy cascade is constant. However, because of non-homogeneous distribution of energy from larger to smaller eddies the heights of intermittent pulses of $\varepsilon_i$ can vary with time difference $\Delta t$ and consequently with the size $l$ of the considered eddies [see, e.g., *Marsch et al.*, 1996, Figure 1; *Bruno and Carbone*, 2005, Figure 77]. This can adequately be modeled by one or two-scale weighted Cantor set model as given by equation (1). It is worth noting that intermittent pulses are somewhat stronger for the model with two different scaling parameters [cf. *Macek and Szczepaniak*, 2008, Figure 2].

[14] The multifractal scaling of this measure can be characterized by the generalized dimension, which is usually defined by [e.g., *Ott*, 1993]

$$D_q = \frac{1}{q-1}\lim_{l\to 0}\frac{\log\sum_{i=1}^{N}p_i^q(l)}{\log l}. \tag{5}$$

Hence, the slopes of the logarithm of a generalized average probability measure of cascading eddies $\overline{\mu}(q, l) \equiv \sqrt[q-1]{\langle (pi)^{q-1}\rangle_{av}}$, versus log $l$ (normalized) provides the generalized dimension as explained by *Macek and Wawrzaszek* [2009].

[15] More precisely, one may distinguish a probability measure from its geometrical support, which may or may not have fractal geometry. Then, if the measure has different fractal dimensions on different parts of the support, the measure is multifractal [*Mandelbrot*, 1989]. Therefore to each probability measure $p_i$ of $i$th eddy of size $l$ in the turbulence cascade we associate a singularity index $\alpha_i(l) \equiv \log p_i(l)/\log l$ [*Halsey et al.*, 1986]. Similarly, we define a one parameter $q$ family of (normalized) generalized pseudoprobability measures [*Chhabra and Jensen*, 1989; *Chhabra et al.*, 1989]

$$\mu_i(q,l) \equiv \frac{p_i^q(l)}{\sum_{i=1}^{N}p_i^q(l)}. \tag{6}$$

Now, with an associated fractal dimension index $f_i(q, l) \equiv \log \mu_i(q, l)/\log l$ for a given $q$ the multifractal singularity spectrum of dimensions is defined directly as the averages taken with respect to the measure $\mu(q, l)$ in equation (6) denoted from here on by $\langle\ldots\rangle$ (skipping a subscript av)

$$f(q) \equiv \lim_{l\to 0}\sum_{i=1}^{N}\mu_i(q,l)f_i(q,l) = \lim_{l\to 0}\frac{\langle\log\mu_i(q,l)\rangle}{\log(l)} \tag{7}$$

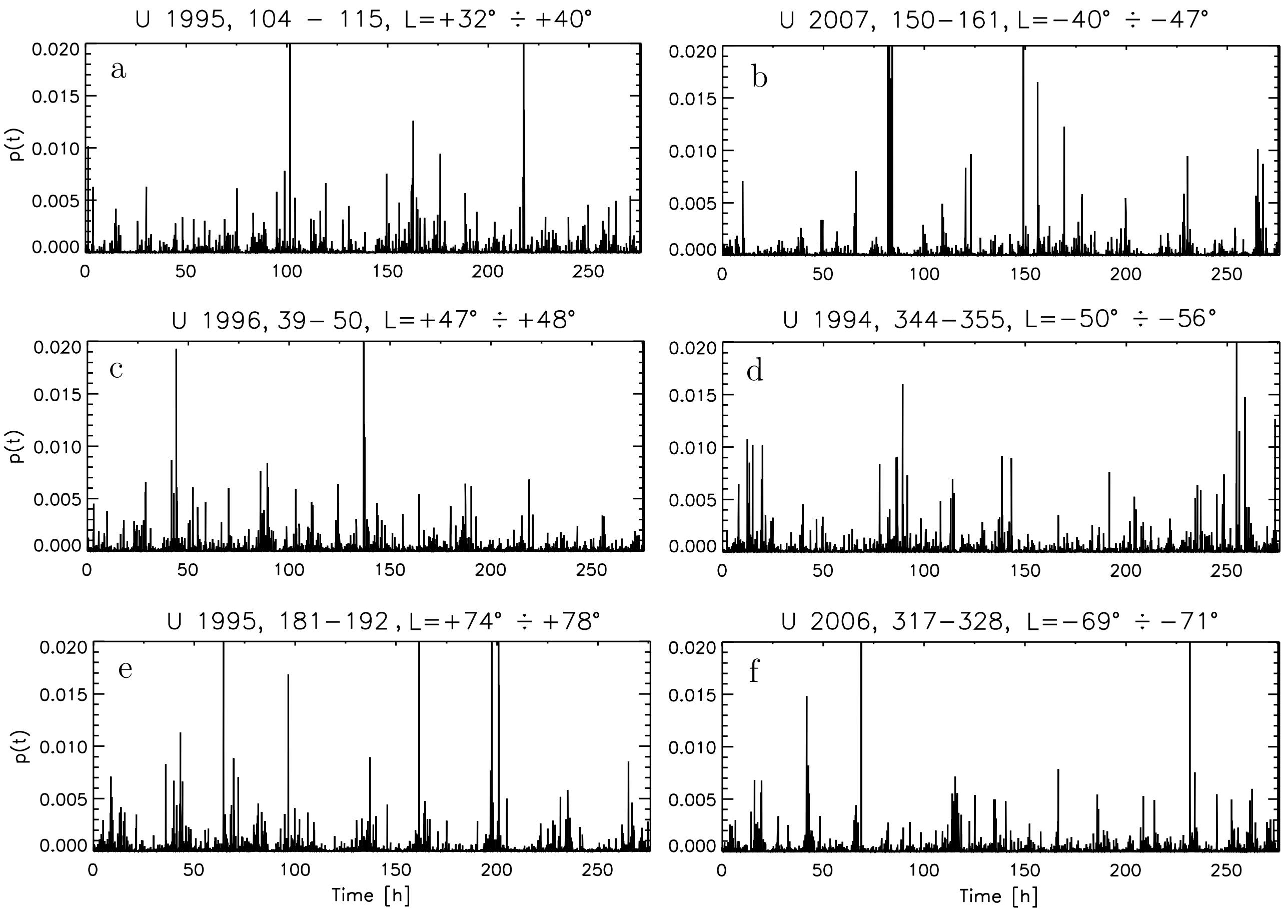


**Figure 3.** The time trace $p(t)$ of the normalized transfer rate of the energy flux $p(t_i, \Delta t) = \varepsilon\ (t_i, \Delta t)/\Sigma\ \varepsilon\ (t_i, \Delta t)$ ($i = 1,\ldots, N = 2^{12}$), obtained using data of the $v_x$ velocity components measured by Ulysses, with a sampling time $\Delta t = 242\ s$, during solar minimum (1994–1996, 2006–2007) (diamonds) at (a, c, e) $+32° \div +40°$, $+47° \div +48°$, $+74° \div +78°$ and (b, d, f) $-40° \div -47°$, $-50° \div -56°$, $-69° \div -71°$ for the fast solar wind.

and the corresponding average value of the singularity strength is given by [*Jensen et al.*, 1987]

$$\alpha(q) \equiv \lim_{l\to 0} \sum_{i=1}^{N} \mu_i(q,l)\alpha_i(l) = \lim_{l\to 0} \frac{\langle \log p_i(q,l)\rangle}{\log(l)}. \qquad (8)$$

We take $\Delta \equiv \alpha_{\max} - \alpha_{\min} = D_{-\infty} - D_{\infty}$ as the degree of multifractality [see, e.g., *Macek*, 2007]. Moreover, using $\alpha_0$, where $f(\alpha_0) = 1$, one can define a measure of asymmetry $A \equiv (\alpha_0 - \alpha_{\min})/(\alpha_{\max} - \alpha_0)$ [*Macek and Wawrzaszek*, 2009].

## 5. Results and Discussion

[16] In this section we discuss our main results on the generalized dimensions $D_q$ presented in Figure 5 and singularity spectra $f(\alpha)$ shown in Figure 6 obtained using

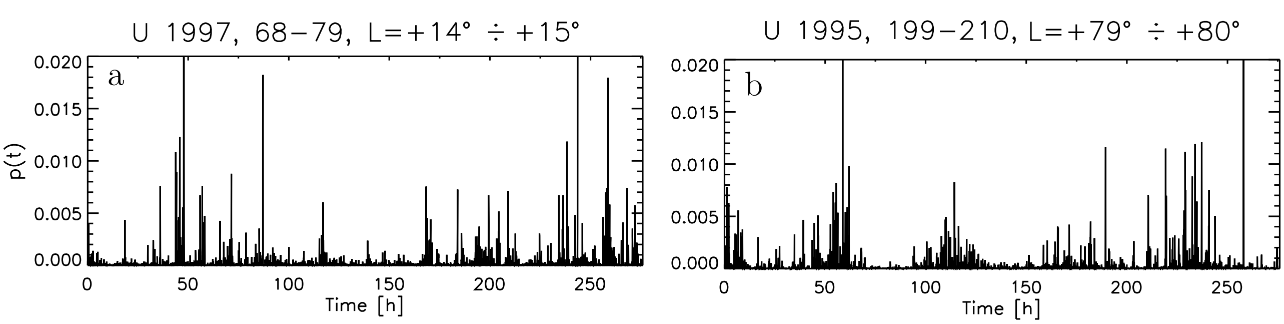


**Figure 4.** The time trace $p(t)$ of the normalized transfer rate of the energy flux $p(t_i, \Delta t) = \varepsilon\ (t_i, \Delta t)/\Sigma\ \varepsilon\ (t_i, \Delta t)$ ($i = 1,\ldots, N = 2^{12}$) obtained using data of the $v_x$ velocity components measured by Ulysses, with a sampling time $\Delta t = 242\ s$, during solar minimum (1995–1997) at $+14° \div +15°$ and $+79° \div +80°$ (diamonds) for the (a) slow and (b) fast solar wind, correspondingly.

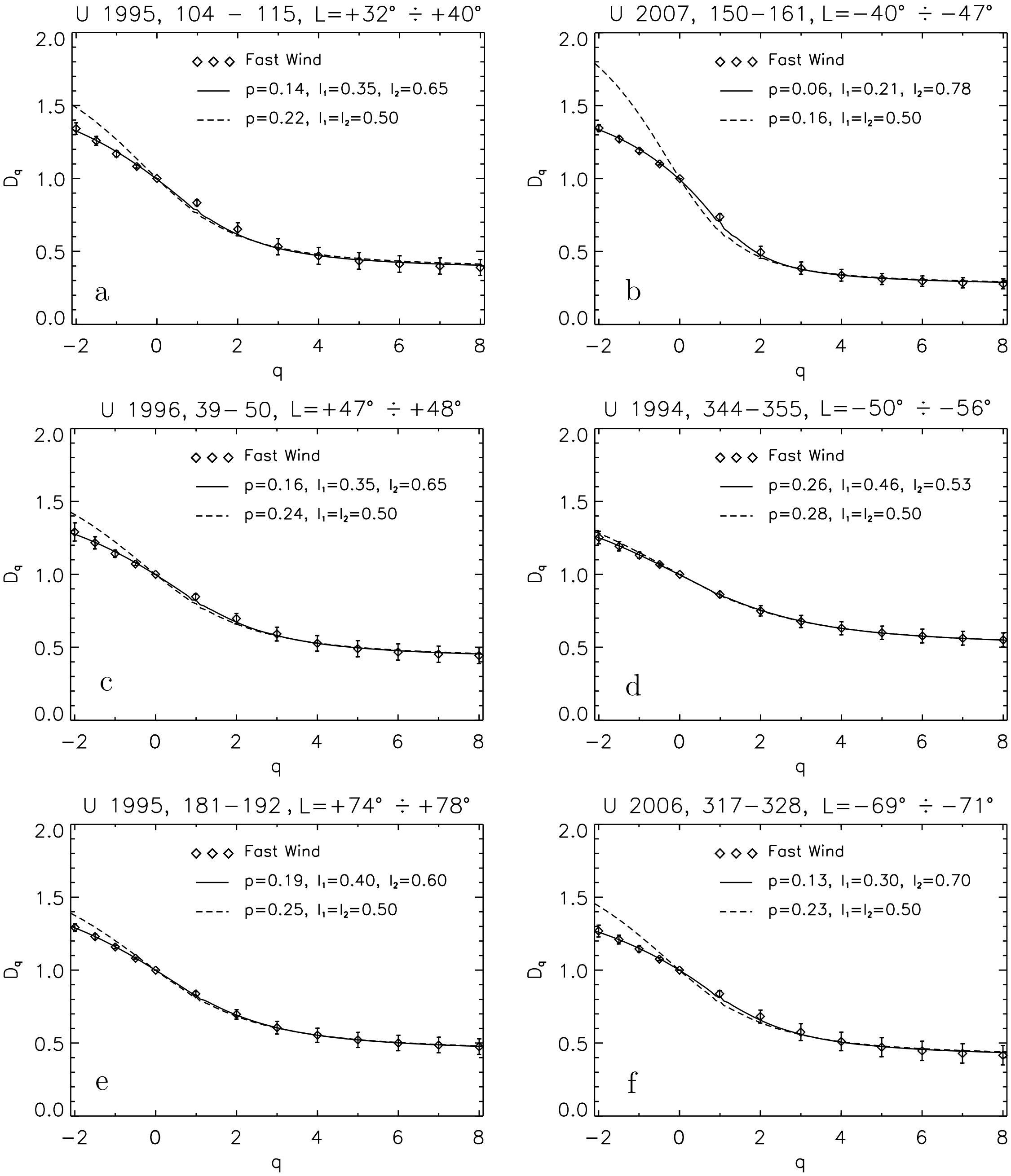


**Figure 5.** The generalized dimensions $D_q$ as a function of $q$. The values obtained for one-dimensional turbulence are calculated for the usual one-scale (dashed lines) $p$ model and the generalized two-scale (continuous lines) model with parameters fitted to the multifractal measure $\mu(q, l)$ obtained using data measured by Ulysses during solar minimum (1994–1996, 2006–2007) for the fast solar wind (diamonds) at (a, c, e) +32° ÷ +40°, +47° ÷ +48°, +74° ÷ +78° and (b, d, f) −40° ÷ −47°, −50° ÷ −56°, −69° ÷ −71°, correspondingly.

experimental values of the energy transfer rates for the fast solar wind measured by Ulysses during solar minimum (1994–1996, 2006–2007) at the following values of latitudes, namely above (Figure 5, left, and Figure 6, left) +32° ÷ +40°, +47° ÷ +48°, +74° ÷ +78° and below the ecliptic plane (Figure 5, right, and Figure 6, right) −40° ÷ −47°, −50° ÷ −56°, −69° ÷ −71°, correspondingly. For comparison, we have also considered slow solar wind near the ecliptic (+14° ÷ +15°) and fast solar wind from polar region of the Sun (+79° ÷ +80°), Figures 7 and 8.

[17] For $q \geq 0$ these results agree with the usual one-scale $p$ model fitted to the dimension spectra as obtained analytically using $l_1 = l_2 = 0.5$ in equation (2) and the corresponding value of the parameter $p$ from range $0.16 < p < 0.28$ for the solar wind streams during solar minimum (1994–1997, 2006–2007), as shown by dashed

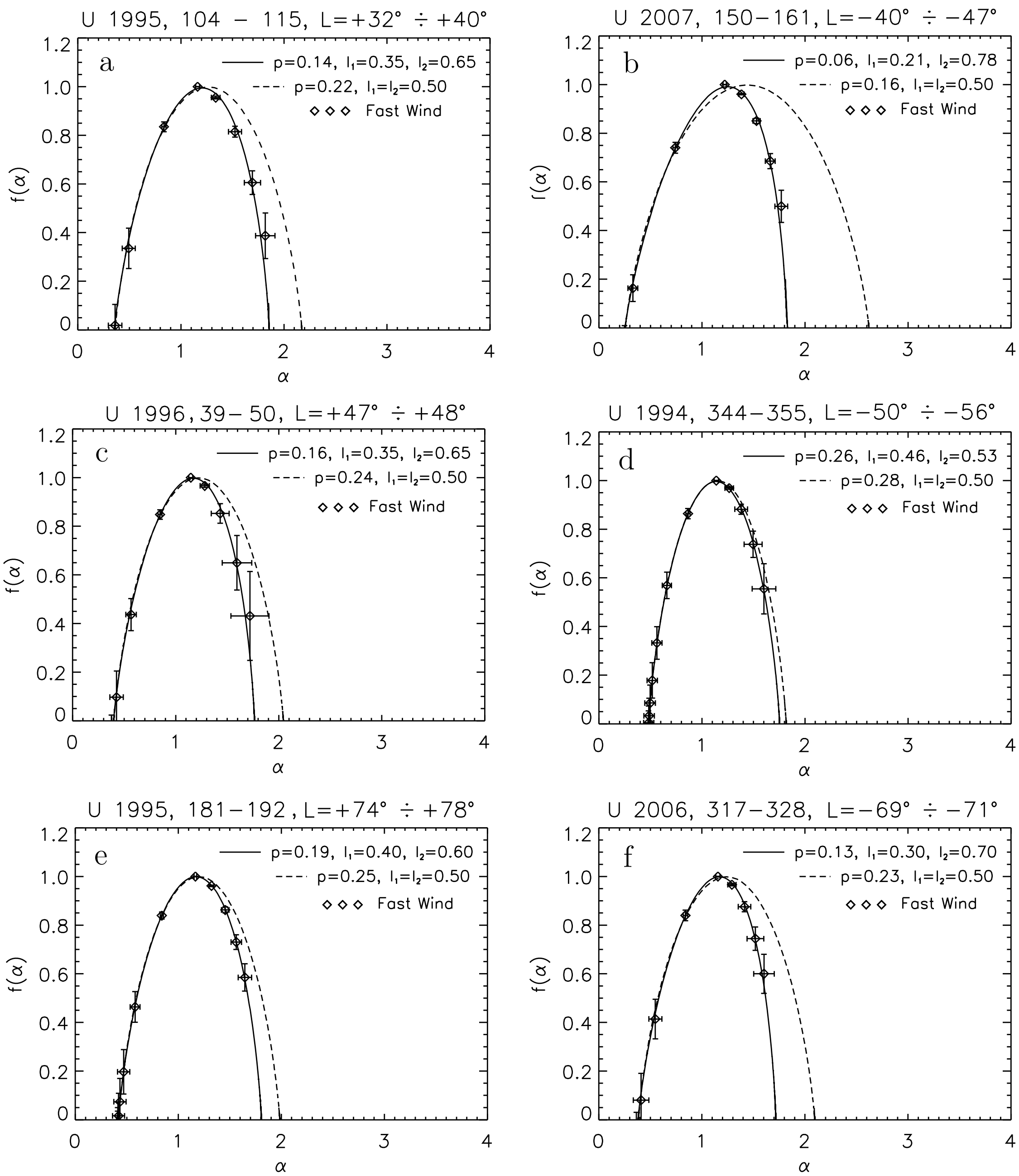


**Figure 6.** The singularity spectrum $f(\alpha)$ as a function of $\alpha$. The values obtained for one-dimensional turbulence are calculated for the usual one-scale (dashed lines) $p$ model and the generalized two-scale (continuous lines) model with parameters fitted to the multifractal measure $\mu(q, l)$ obtained using fast solar wind data measured by Ulysses during solar minimum (1994–1996, 2006–2007) (diamonds) at (a, c, e) +32° ÷ +40°, +47° ÷ +48°, +74° ÷ +78° and (b, d, f) −40° ÷ −47°, −50° ÷ −56°, −69° ÷ −71°, correspondingly.

lines. These values of $p$ obtained here are roughly consistent with the fitted value in the literature both for laboratory and the solar wind turbulence, which is in the range $0.1 < p < 0.3$ [e.g., *Carbone*, 1993; *Marsch et al.*, 1996; *Horbury et al.*, 1996; *Pagel and Balogh*, 2001]. In particular, taking only the positive integers for $q$, the scaling exponents of the $q$-order structure function can be related to the generalized dimensions $D_q$ by $\xi(q) = 1 + (q/m - 1)\, D_{q/m}$, for the Kolmogorov ($m$ = 3), and the Kraichan scaling ($m$ = 4), correspondingly [cf. *Carbone*, 1993; *Bruno and Carbone*, 2005]. We see that the results $D_q$ for positive $q$ obtained using the experimental values of velocities are relatively well fitted to the $p$ model, Figure 5. Therefore, the $p$ model is also consistent with the obtained values of scaling exponents of the structure functions. Now, since the degree of multifractality, which is a measure of deviation from the

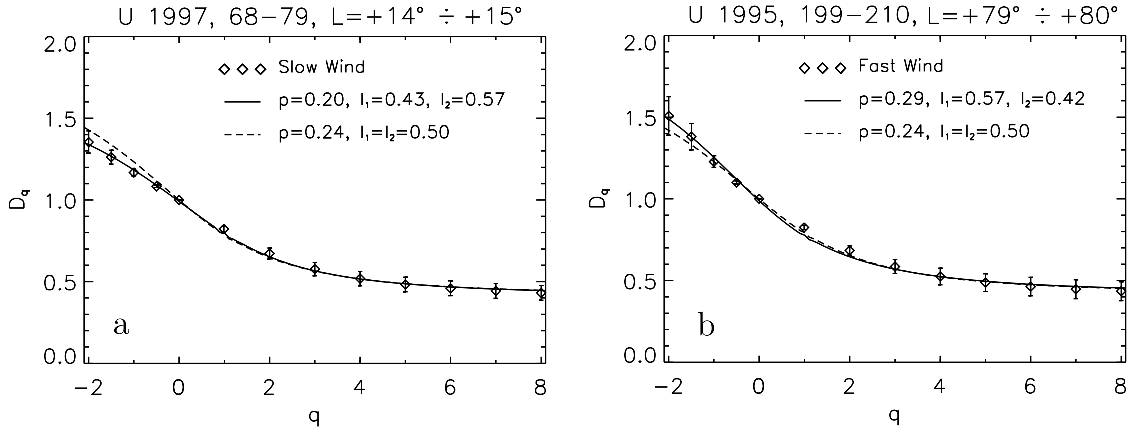


**Figure 7.** The generalized dimensions $D_q$ as a function of $q$. The values obtained for one-dimensional turbulence are calculated for the usual one-scale (dashed lines) $p$ model and the generalized two-scale (continuous lines) model with parameters fitted to the multifractal measure $\mu(q, l)$ obtained using data measured by Ulysses at +14° ÷ +15° and +79° ÷ +80° (diamonds) during solar minimum (1995–1997) for the (a) slow and (b) fast solar wind, correspondingly.

self-similarity, exhibits a minimum at midlatitudes, we have found that a curve for the power law indices of the increments of order $q$ is closer to the self-similar scaling in this region.

[18] On the contrary, for $q < 0$ the $p$ model in some cases cannot describe the observational results [*Marsch et al.*, 1996; *Szczepaniak and Macek*, 2008]. Admittedly for estimation of the degree of multifractality $\Delta = D_{-\infty} - D_{\infty}$ both positive and negative values of $q$ are needed. Here we show that in general the experimental values are consistent also with the spectrum of dimensions obtained numerically from equation (5) for the weighted two-scale Cantor set using an asymmetric scaling, i.e., using unequal scales $l_1 \neq l_2$, as is shown in Figures 5 and 7 by continuous lines. We also confirm the characteristic shape of the multifractal spectrum, as seen in Figures 6 and 8. In fact, one can see that the singularity spectrum $f(\alpha)$ is a concave function of $\alpha$, as schematically depicted in Figure 1. The width of this function, $\alpha_{\max} - \alpha_{\min}$, which is equal to $D_{-\infty} - D_{\infty}$, can be here identified as the degree of multifractality and intermittency, $\Delta$ [*Macek*, 2007; *Macek and Wawrzaszek*, 2009], which is somehow related to other measures of intermittency in the literature, e.g., flatness and kurtosis [*Frisch*, 1995; *Carbone*, 1994; *Szczepaniak and Macek*, 2008]. It is worth noting the universal character of the shape of the function $f(\alpha)$ in multifractal theory [see, e.g., *Ott*, 1993]. Moreover, the generalized two-scale Cantor set model is a convenient tool to investigate the asymmetry of this function; in a usual one-scale Cantor set model this function is necessarily symmetric.

[19] In general, we observe a latitudinal dependence of the multifractal characteristics of turbulence. The calculated degree of multifractality and asymmetry as a function on heliographic latitude for the fast solar wind are summarized in Figures 9 and 10a with some specific values listed in Table 2 (the case of the slow wind is denoted for 1997). We

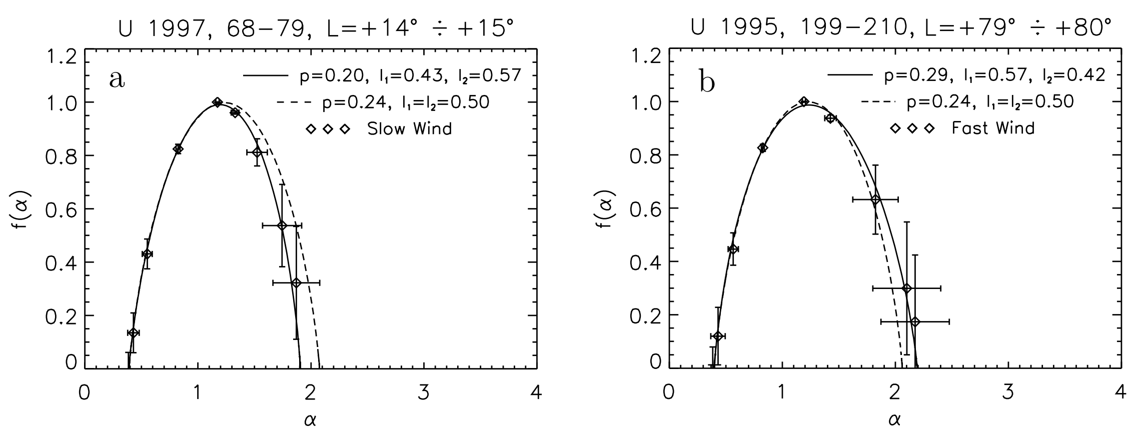


**Figure 8.** The singularity spectrum $f(\alpha)$ as a function of $\alpha$. The values obtained for one-dimensional turbulence are calculated for the usual one-scale (dashed lines) $p$ model and the generalized two-scale (continuous lines) model with parameters fitted to the multifractal measure $\mu(q, l)$ obtained using data measured by Ulysses during solar minimum (1995–1997) at +14° ÷ +15° and +79° ÷ +80° (diamonds) for the (a) slow and (b) fast solar wind, correspondingly.

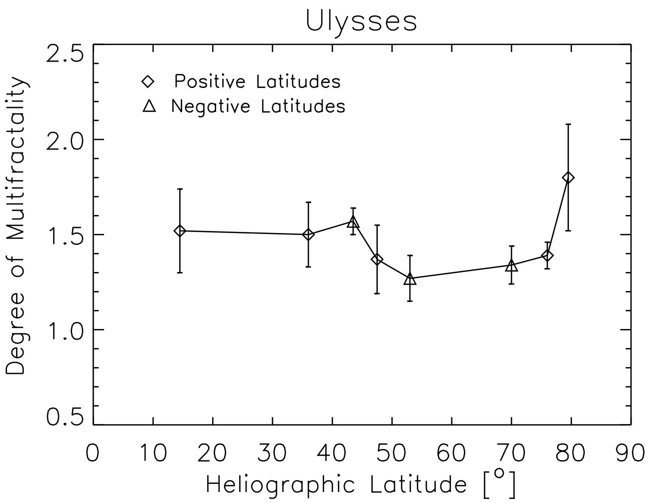


**Figure 9.** Degree of multifractality Δ (continuous line) for the slow (at 15°) and fast (above 15°) solar wind during solar minimum (1994–1996, 2006–2007) in dependence on heliographic latitude below (triangles) and above (diamonds) the ecliptic.

see from Table 2 that the degree of multifractality Δ and asymmetry *A* of the dimension spectra of the fast solar wind out of the ecliptic plane are similar for positive and the corresponding negative latitudes. Therefore, it seems that the values of these multifractal characteristics exhibits some symmetry with respect to the ecliptic plane. In particular, in a region from 50° to 70° we observe a minimum of the degree of multifractality (intermittency). This could be related to interactions between fast and slow streams, which usually can still take place at latitudes from 30° to 50°. Another possibility is appearance of first new solar spots for a subsequent solar cycle at some intermediate latitudes. At polar regions, where the pure fast streams are present, the degree of multifractality rises again. It is interesting that a similar behavior of flatness, which is another measure of intermittency, has been observed at high latitudes by using the magnetic data [*Yordanova et al.*, 2009].

**Table 2.** Degree of Multifractality Δ and Asymmetry *A* for the Energy Transfer Rate in the Out of Ecliptic Plane

| Heliographic Latitude | Heliocentric Distance | Multifractality Δ | Asymmetry *A* |
|---|---|---|---|
| +14° ÷ +15° (1997)[a] | 4.9 AU | 1.52 ± 0.22 | 1.14 ± 0.30 |
| +32° ÷ +40° (1995) | 1.4 AU | 1.50 ± 0.17 | 1.10 ± 0.20 |
| +47° ÷ +48° (1996) | 3.3 AU | 1.37 ± 0.18 | 1.21 ± 0.32 |
| +74° ÷ +78° (1995) | 1.8–1.9 AU | 1.39 ± 0.07 | 1.17 ± 0.12 |
| +79° ÷ +80° (1995) | 1.9–2.0 AU | 1.80 ± 0.28 | 0.80 ± 0.29 |
| −40° ÷ −47° (2007) | 1.6 AU | 1.57 ± 0.07 | 1.53 ± 0.20 |
| −50° ÷ −56° (1994) | 1.6–1.7 AU | 1.27 ± 0.12 | 1.07 ± 0.20 |
| −69° ÷ −71° (2006) | 2.8–2.9 AU | 1.34 ± 0.10 | 1.36 ± 0.25 |

[a]The case of the slow wind.

[20] Further, the degree of multifractality and asymmetry seem to be somewhat correlated. We see that when latitudes change from +32° ÷ +40° to −50° ÷ −56° then Δ decreases from 1.50 to 1.27, and the value of *A* changes only slightly from 1.10 to 1.07. Only at very high polar regions larger than 70° this correlation ceases. Moreover, the scaling of the fast streams from the polar region of the Sun exhibit more multifractal and asymmetric character, Δ = 1.80, *A* = 0.80, than that for the slow wind from the equatorial region, Δ = 1.52, *A* = 1.14 (in both cases we have relatively large errors of these parameters). Finally, in Figure 10b we show how the parameters of the two-scale Cantor set model *p* and $l_1$ (during solar minimum) depend on the heliographic latitudes, rising at ~50° and again above ~70°. It is clear that both parameters seem to be correlated.

[21] Let us now compare the results obtained in this work with those obtained previously at the ecliptic plane using the generalized two-scale cascade model. First, our analysis of the data obtained onboard ACE spacecraft at the Earth's orbit, especially in the fast solar wind, indicates multifractal structure with the degree of multifractality of Δ = 2.56 ± 0.16 and the degree of asymmetry *A* = 0.95 ± 0.11 during solar minimum [*Macek et al.*, 2009]. Similar values are obtained by Voyager spacecraft, e.g., at distance of 2.5 AU we have Δ = 2.12 ± 0.14 and *A* = 1.54 ± 0.24, and in the outer heliosphere at 25 AU we have also large values Δ = 2.93 ± 0.10 and rather asymmetric spectrum *A* = 0.66 ± 0.11 [*Macek and Wawrzaszek*, 2009]. Summarizing, we see that

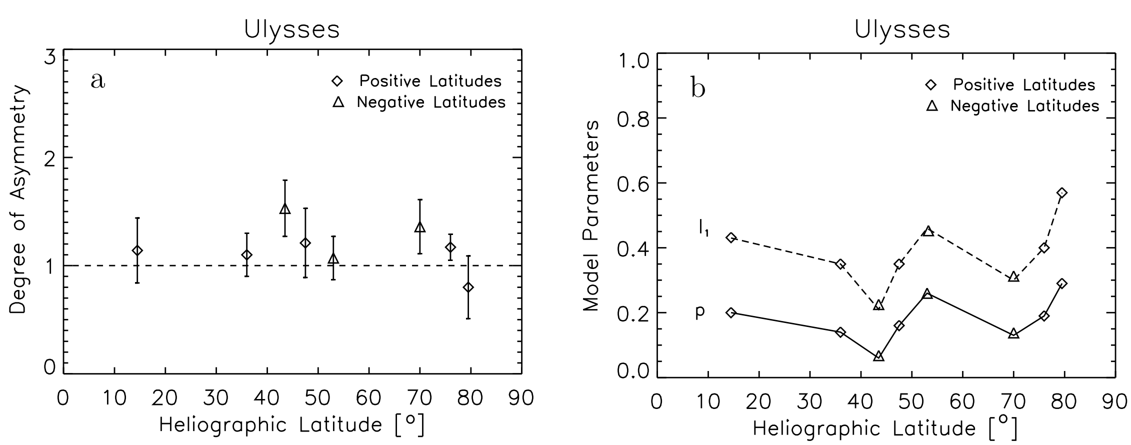


**Figure 10.** (a) Degree of asymmetry *A* and (b) change of two-scale model parameters *p* (continuous line) and $l_1$ (dashed line) in dependence on heliographic latitude during solar minimum (1994–1996, 2006–2007).

at high latitudes during solar minimum in the fast solar wind we observe somewhat smaller degree of multifractality and intermittency as compared with those at the ecliptic. These results are consistent with previous results confirming that slow wind intermittency is higher than that for the fast wind [e.g., *Sorriso-Valvo et al.*, 1999], and that intermittency in the fast wind increases with the heliocentric distance, including high latitudes [e.g., *Bruno et al.*, 2001, 2003]. In addition, symmetric multifractal singularity spectra are observed at high latitudes, in contrast to often significant asymmetry of the multifractal singularity spectrum at the ecliptic wind. This demonstrate that solar wind turbulence may exhibit somewhat different scaling at various latitudes resulting from different dynamics of the ecliptic and polar winds. Notwithstanding of the complexity of solar wind fluctuations it appears that the standard one-scale $p$ model can roughly describe these nonlinear fluctuations out of the ecliptic, hopefully also in the polar regions. However, the generalized two-scale Cantor set model is necessary for describing scaling of solar wind intermittent turbulence near the ecliptic.

[22] A generalization of this fluid approach as given by equation (3) by including magnetic field effects for full isotropic, homogeneous, and incompressible fluid has already been considered by *Politano and Pouquet* [1998]. In particular, the magnetic fluctuations have also been taken into consideration and the average transfer rate, assumed to be constant on all scales, has also been estimated by *Sorriso-Valvo et al.* [2007] and *Marino et al.* [2008]. Certainly, it would be interesting to extend our calculations using the generalized two-scale weighted Cantor set model to investigate possible intermittent and non-homogeneous behavior of the Alfvénic fluctuations in space plasmas, also in the polar wind, which will be a subject of our studies in the near future.

## 6. Conclusions

[23] We have studied the inhomogeneous rate of the transfer of the energy flux indicating multifractal and intermittent behavior of solar wind turbulence out of the ecliptic plane. In particular, we have identified somewhat smaller degree of multifractality at high latitudes during solar minimum for the fast solar wind, where turbulence evolves more slowly as compared with that at the ecliptic.

[24] By investigating Ulysses data we have shown that the degree of multifractality and asymmetry of the fast solar wind exhibit latitudinal dependence with some symmetry with respect to the ecliptic plane. Both quantities seem to be correlated during solar minimum for latitudes below 70°. The multifractal singularity spectra become roughly symmetric. The minimum intermittency is observed at midlatitudes and is possibly related to the transition from the region where the interaction of the fast and slow streams takes place to a more homogeneous region of the pure fast solar wind.

[25] Basically, the multifractal spectra for solar wind are consistent with the generalized $p$ model for both positive and negative $q$, but rather with different scaling parameters for sizes of eddies in the ecliptic, while in most cases the usual $p$ model is sufficient to reproduce the spectrum of the fast solar wind out of the ecliptic plane.

[26] However, in general, the generalized two-scale weighted Cantor set model can properly grasp possible asymmetry of the universal multifractal singularity spectrum. Hence this model appears to be an universal tool for describing scaling of solar wind turbulence allowing for a unifying description of its multifractal characteristics; this model should then be valid for turbulence at various scales in the whole heliosphere. Therefore, we propose this cascade model describing intermittent energy transfer for analysis of turbulence in various environments.

[27] **Acknowledgments.** We would like to thank the SWOOPS instrument team of Ulysses for providing velocity data. This work has been supported by the Polish Ministry of Science and Higher Education (MNiSW) through grant NN 202 4127 33.

[28] Philippa Browning thanks Luca Sorriso-Valvo and another reviewer for their assistance in evaluating this paper.

## References

Bruno, R., and V. Carbone (2005), The solar wind as a turbulence laboratory, *Living Rev. Sol. Phys.*, *2*, lrsp-2005-4.

Bruno, R., V. Carbone, P. Veltri, E. Pietropaolo, and B. Bavassano (2001), Identifying intermittency events in the solar wind, *Planet. Space Sci.*, *49*, 1201–1210.

Bruno, R., V. Carbone, L. Sorriso-Valvo, and B. Bavassano (2003), Radial evolution of solar wind intermittency in the inner heliosphere, *J. Geophys. Res.*, *108*(A3), 1130, doi:10.1029/2002JA009615.

Burlaga, L. (1991), Intermittent turbulence in the solar wind, *J. Geophys. Res.*, *96*(15), 5847–5851.

Burlaga, L. F. (2001), Lognormal and multifractal distributions of the heliospheric magnetic field, *J. Geophys. Res.*, *106*, 15,917–15,927, doi:10.1029/2000JA000107.

Burlaga, L. F., J. Perko, and J. Pirraglia (1993), Cosmic-ray modulation, merged interaction regions, and multifractals, *Astrophys. J.*, *407*, 347–358, doi:10.1086/172517.

Carbone, V. (1993), Cascade model for intermittency in fully developed magnetohydrodynamic turbulence, *Phys. Rev. Lett.*, *71*, 1546–1548, doi:10.1103/PhysRevLett.71.1546.

Carbone, V. (1994), Scaling exponents of the velocity structure functions in the interplanetary medium, *Ann. Geophys.*, *12*, 585–590.

Chhabra, A., and R. V. Jensen (1989), Direct determination of the f($\alpha$) singularity spectrum, *Phys. Rev. Lett.*, *62*(12), 1327–1330, doi:10.1103/PhysRevLett.62.1327.

Chhabra, A. B., C. Meneveau, R. V. Jensen, and K. R. Sreenivasan (1989), Direct determination of the f($\alpha$) singularity spectrum and its application to fully developed turbulence, *Phys. Rev. A*, *40*(9), 5284–5294, doi:10.1103/PhysRevA.40.5284.

Frisch, U. (1995), *Turbulence: The legacy of A. N. Kolmogorov*, Cambridge Univ. Press, Cambridge, U. K.

Frisch, U., P.-L. Sulem, and M. Nelkin (1978), A simple dynamical model of intermittent fully developed turbulence, *J. Fluid Mech.*, *87*, 719–736.

Grassberger, P. (1983), Generalized dimensions of strange attractors, *Phys. Lett. A*, *97*, 227–230, doi:10.1016/0375-9601(83)90753-3.

Grassberger, P., and I. Procaccia (1983), Measuring the strangeness of strange attractors, *Physica D*, *9*, 189–208, doi:10.1016/0167-2789(83)90298-1.

Halsey, T. C., M. H. Jensen, L. P. Kadanoff, I. Procaccia, and B. I. Shraiman (1986), Fractal measures and their singularities: The characterization of strange sets, *Phys. Rev. A*, *33*(2), 1141–1151, doi:10.1103/PhysRevA.33.1141.

Hentschel, H. G. E., and I. Procaccia (1983), The infinite number of generalized dimensions of fractals and strange attractors, *Physica D*, *8*, 435–444, doi:10.1016/0167-2789(83)90235-X.

Hnat, B., S. C. Chapman, and G. Rowlands (2003), Intermittency, scaling, and the Fokker-Planck approach to fluctuations of the solar wind bulk plasma parameters as seen by the WIND spacecraft, *Phys. Rev. E*, *67*(5), 056404, doi:10.1103/PhysRevE.67.056404.

Horbury, T., and A. Balogh (1997), Structure function measurements of the intermittent MHD turbulent cascade, *Nonlinear Processes Geophys.*, *4*, 185–199.

Horbury, T. S., and A. Balogh (2001), Evolution of magnetic field fluctuations in high-speed solar wind streams: Ulysses and Helios observations, *J. Geophys. Res.*, *106*, 15,929–15,940, doi:10.1029/2000JA000108.

Horbury, T., A. Balogh, R. Forsyth, and E. Smith (1995), Observations of evolving turbulence in the polar solar wind, *Geophys. Res. Lett.*, *22*, 3401–3404.

Horbury, T. S., A. Balogh, R. J. Forsyth, and E. J. Smith (1996), Magnetic field signatures of unevolved turbulence in solar polar flows, *J. Geophys. Res.*, *101*, 405–413, doi:10.1029/95JA01343.

Jensen, M. H., L. P. Kadanoff, and I. Procaccia (1987), Scaling structure and thermodynamics of strange sets, *Phys. Rev. A*, *36*, 1409–1420, doi:10.1103/PhysRevA.36.1409.

Macek, W. M. (1998), Testing for an attractor in the solar wind flow, *Physica D*, *122*, 254–264, doi:10.1016/S0167-2789(98)00098-0.

Macek, W. M. (2007), Multifractality and intermittency in the solar wind, *Nonlinear Processes Geophys.*, *14*, 695–700.

Macek, W. M., and S. Redaelli (2000), Estimation of the entropy of the solar wind flow, *Phys. Rev. E*, *62*, 6496–6504, doi:10.1103/PhysRevE.62.6496.

Macek, W. M., and A. Szczepaniak (2008), Generalized two-scale weighted Cantor set model for solar wind turbulence, *Geophys. Res. Lett.*, *35*, L02108, doi:10.1029/2007GL032263.

Macek, W. M., and A. Wawrzaszek (2009), Evolution of asymmetric multifractal scaling of solar wind turbulence in the outer heliosphere, *J. Geophys. Res.*, *114*, A03108, doi:10.1029/2008JA013795.

Macek, W. M., and A. Wawrzaszek (2010), Multifractal structure of small and large scales fluctuations of interplanetary magnetic fields, *Planet. Space Sci.*, doi:10.1016/j.pss.2010.02.006, in press.

Macek, W. M., A. Wawrzaszek, and T. Hada (2009), Multiscale multifractal intermittent turbulence in space plasmas, *J. Plasma Fusion Res. Ser.*, *8*, 142–147.

Mandelbrot, B. B. (1989), Multifractal measures, especially for the geophysicist, *Pure Appl. Geophys.*, *131*, 5–42, doi:10.1007/BF00874478.

Marino, R., L. Sorriso-Valvo, V. Carbone, A. Noullez, R. Bruno, and B. Bavassano (2008), Heating the solar wind by a magnetohydrodynamic turbulent energy cascade, *Astrophys. J.*, *677*, L71–L74, doi:10.1086/587957.

Marsch, E., and S. Liu (1993), Structure functions and intermittency of velocity fluctuations in the inner solar wind, *Ann. Geophys.*, *11*, 227–238.

Marsch, E., and C.-Y. Tu (1994), Non-Gaussian probability distributions of solar wind fluctuations, *Ann. Geophys.*, *12*(12), 1127–1138.

Marsch, E., and C.-Y. Tu (1997), Intermittency, non-Gaussian statistics and fractal scaling of MHD fluctuations in the solar wind, *Nonlinear Processes Geophys.*, *4*, 101–124.

Marsch, E., C.-Y. Tu, and H. Rosenbauer (1996), Multifractal scaling of the kinetic energy flux in solar wind turbulence, *Ann. Geophys.*, *14*, 259–269.

Meneveau, C., and K. R. Sreenivasan (1987), Simple multifractal cascade model for fully developed turbulence, *Phys. Rev. Lett.*, *59*, 1424–1427, doi:10.1103/PhysRevLett.59.1424.

Meneveau, C., and K. R. Sreenivasan (1991), The multifractal nature of turbulent energy dissipation, *J. Fluid Mech.*, *224*, 429–484, doi:10.1017/S0022112091001830.

Ott, E. (1993), *Chaos in Dynamical Systems*, Cambridge Univ. Press, Cambridge, U. K.

Pagel, C., and A. Balogh (2001), A study of magnetic fluctuations and their anomalous scaling in the solar wind: The Ulysses fast-latitude scan, *Nonlinear Processes Geophys.*, *8*(4/5), 313–330.

Pagel, C., and A. Balogh (2002), Intermittency in the solar wind: A comparison between solar minimum and maximum using Ulysses data, *J. Geophys. Res.*, *107*(A8), 1178, doi:10.1029/2002JA009331.

Pagel, C., and A. Balogh (2003), Radial dependence of intermittency in the fast polar solar wind magnetic field using Ulysses, *J. Geophys. Res.*, *108*(A1), 1012, doi:10.1029/2002JA009498.

Politano, H., and A. Pouquet (1998), Dynamical length scales for turbulent magnetized flows, *Geophys. Res. Lett.*, *25*, 273–276, doi:10.1029/97GL03642.

Smith, E. J., A. Balogh, R. P. Lepping, M. Neugebauer, J. Phillips, and B. T. Tsurutani (1995), Ulysses observations of latitude gradients in the heliospheric magnetic field, *Adv. Space Res.*, *16*, 165–170, doi:10.1016/0273-1177(95)00328-C.

Sorriso-Valvo, L., V. Carbone, P. Veltri, G. Consolini, and R. Bruno (1999), Intermittency in the solar wind turbulence through probability distribution functions of fluctuations, *Geophys. Res. Lett.*, *26*, 1801–1804.

Sorriso-Valvo, L., R. Marino, V. Carbone, A. Noullez, F. Lepreti, P. Veltri, R. Bruno, B. Bavassano, and E. Pietropaolo (2007), Observation of inertial energy cascade in interplanetary space plasma, *Phys. Rev. Lett.*, *99*(11), 115001, doi:10.1103/PhysRevLett.99.115001.

Szczepaniak, A., and W. M. Macek (2008), Asymmetric multifractal model for solar wind intermittent turbulence, *Nonlinear Processes Geophys.*, *15*, 615–620.

Yordanova, E., A. Balogh, A. Noullez, and R. von Steiger (2009), Turbulence and intermittency in the heliospheric magnetic field in fast and slow solar wind, *J. Geophys. Res.*, *114*, A08101, doi:10.1029/2009JA014067.

W. M. Macek and A. Wawrzaszek, Space Research Centre, Polish Academy of Sciences, Bartycka 18 A, 00-716 Warszawa, Poland. (macek@cbk.waw.pl)