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Testing for nonlinearity and low-dimensional dynamics in the slow solar wind

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Abstract

We focus on nonlinearity and possible deterministic behavior of the low-speed solar wind. We analyze time series of velocities of the streams including Alfvénic fluctuations measured by the Helios spacecraft in the inner heliosphere. Because of nonlinear behavior of that flow we use the average-mutual-information and false-nearest-neighbors methods. The fraction of false-nearest-neighbors drops to zero at dimension equal to four for both the radial velocity and Alfvénic velocity, independent of the distance from the Sun. The question of possible colored noise in the solar wind is also discussed. The obtained results clearly indicate that a low-dimensional attractor should exist. One can therefore hope to infer information about some complex nonlinear phenomena from the geometrical properties of the attractor or by solving a set of four nonlinear ordinary differential equations describing the system. One can expect that the attractor should contain information about the dynamic variations of the coronal streamers or it could represent a structure of the time sequence of near-Sun coronal fine-stream tubes.
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1. Introduction

The solar wind plasma flowing supersonically outward from the Sun has two forms: slow ($\approx$300 km s^{-1}) and fast ($\approx$900 km s^{-1}) (Schwenn, 1990). The fast wind is associated with coronal holes and is relatively uniform and stable, while the slow wind is quite variable in terms of velocities. By measuring quantities describing the slow plasma flow, we get irregular and aperiodic time series. Analysis of the time series with classical linear tools indicates stochastic origin of the dynamics of the flow. However, the solar wind plasma can be quite well modeled within the framework of the hydromagnetic theory and we should rather expect deterministic dynamics of the flow. Simultaneously aperiodic and deterministic flow is possible in case of chaotic low-dimensional dynamics. To detect this kind of plasma behavior we should use nonlinear time series analysis, which is a powerful tool for the reconstruction of attractors of dynamical systems.

The question of low-dimensional dynamics is of great importance for the solar wind community, because it allows us to investigate the nature of interplanetary hydromagnetic turbulence, e.g., Bruno et al. (2001). Therefore, following space physics applications, e.g., Kurths and Herzel (1987), Burlaga (1991), we consider the inner heliosphere and we use the average mutual information (AMI) suggested as a method to determine a reasonable delay for nonlinear systems (Fraser and Swinney, 1986);

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we also use the false-nearest-neighbor (FNN) method as a convenient procedure to determine the minimal sufficient embedding dimension (Kennel et al., 1992).

Both methods are reviewed in Section 2. Indication for a chaotic attractor in the slow solar wind has been given by Macek (1998), Macek and Redaelli (2000), and Redaelli and Macek (2001). This paper is aimed to extend the previous results by comparison with results obtained by the AMI and FNN methods and taking also into account Alfvénic fluctuations of the flow. We examine also spatial structure of the solar wind plasma dynamics. The AMI and FNN methods have been applied to time series analysis of velocities of the solar wind low-speed streams measured by the Helios 1 and the Helios 2 spacecrafts at 0.3 and 0.9 AU from the Sun. The obtained results clearly show that the plasma dynamics in the inner heliosphere is deterministic and the solar wind attractor can also be unfolded at the low embedding dimension. Results for time series taken at 0.3 and 0.9 AU from the Sun are similar, hence plasma dynamics in the case of the slow solar wind probably does not change substantially with distance from the Sun. On the contrary, as it has recently been shown by Bruno et al. (2003) plasma dynamics in the case of the fast solar wind is much more variable with changing distance from the Sun.

2. Methods

2.1. Average mutual information

The average mutual information is a generalization from the correlation function, which measures the linear correlations, to the case of nonlinear correlations between measurements. The method has been introduced by Fraser and Swinney (1986). A measure of a mutual information between elements of a time series $x(t_i)$, $i = 1,2,\ldots,N$ is

$$I(T) = \sum_{x(t_i),x(t_i+T)} P(x(t_i),x(t_i+T)) \times \log_2\left[\frac{P(x(t_i),x(t_i+T))}{P(x(t_i))P(x(t_i+T))}\right], \quad (1)$$

where $x(t_i)$ is the ith element of the time series, $T = k\Delta t$ ($k = 1,2,\ldots,k_{\max}$), $P(x(t_i))$ is the probability density at $x(t_i)$, $P(x(t_i),x(t_i+T))$ is the joint probability density at the pair $x(t_i)$, $x(t_i+T)$. The delay $T_{\rm m}$ of the first minimum of AMI is chosen as a delay time τ for time-delay reconstruction of a phase space (see Section 2.2).

2.2. False nearest neighbors

The false-nearest-neighbor method has been introduced first by Kennel et al. (1992), but we use the version from the paper by Rhodes and Morari (1997) for time series corrupted by noise. This method use the time-delay reconstruction (embedding) of a phase space, i.e., from a scalar time series $x(t_i)$, $i = 1,2,\ldots,N$, some m-dimensional vectors

$$\mathbf{X}(t_i) = [x(t_i), x(t_i+\tau), \ldots, x(t_i+(m-1)\tau)] \quad (2)$$

are constructed to trace out an orbit of a dynamical system. See Section 2.1 for description of choice of delay time τ. For deterministic autonomous systems any element of the time series can be predicted as a function involving time-delayed elements of the same series: $x(t_i+m\tau) = G[\mathbf{X}(t_i)]$. It can be used for determining the proper minimal dimension. If the dimension m is too small, there are many nearest neighbors $\mathbf{X}(t_i)$, $\mathbf{X}(t_j)$ that are close in the reconstructed space, but their images $x(t_i+m\tau)$, $x(t_j+m\tau)$ are very distant. Such pairs of points are false nearest neighbors and in order to determine large enough dimension for time-delay reconstruction of the phase space, one can check if the following condition is fulfilled (Rhodes and Morari, 1997):

$$\frac{|x(t_i+m\tau) - x(t_j+m\tau)|}{|\mathbf{X}(t_i) - \mathbf{X}(t_j)|} < R + \frac{2\epsilon R\sqrt{m} + 2\epsilon}{|\mathbf{X}(t_i) - \mathbf{X}(t_j)|}, \quad (3)$$

where R is a threshold (usually about 10–20) and ϵ is chosen appropriately to the maximal noise level in the time series. If inequality in Eq. (3) is not satisfied, the points are the false neighbors. Checking every point and its nearest neighbor in reconstructed space, one can determine a fraction of false nearest neighbors. If the system is deterministic, the false-nearest-neighbors fraction drops to zero at m equal to $m_{\rm p}$. Dimension $m_{\rm p}$ is the true dimension to unfold dynamics, but applying only criterion in Eq. (3) one can obtain the fraction of false-nearest-neighbors about zero at a certain dimension also for a time series from stochastic systems. Therefore, in the original method of Kennel et al. (1992) there is a second threshold A and the condition

$$\frac{\sqrt{[x(t_i+m\tau) - x(t_j+m\tau)]^2 + [\mathbf{X}(t_i) - \mathbf{X}(t_j)]^2}}{R_A} < A, \quad (4)$$

where

$$R_A^2 = \frac{1}{N}\sum_{i=1}^{N}[x(t_i) - \bar{x}]^2, \quad (5)$$

is a measure proportional to an attractor diameter and usually $A = 2$ is chosen. If expression in Eq. (4) is false, the nearest neighbors are also false. Rhodes and Morari (1997) argue that using conditions of Eqs. (3) and (4), one ensures reliable estimation of the correct dimension $m_{\rm p}$ in the case of limited length and time series corrupted by noise. We have confirmed this for time series for classical model systems (Lorenz system, Hénon map) with a small amount of dynamical or additive noise (less then 10%).

3. Experimental data

Having tested the methods considered here for a classical model system, we apply these methods for a real system in nature. Namely, we analyze the Helios 1 and 2 experimental data using plasma parameters measured in situ in the heliosphere near the Sun at 0.3 and 0.9 AU (Schwenn, 1990). The radial velocity component of the flow of the solar wind plasma, v, has been investigated by Macek (1998), Macek and Redaelli (2000), and Redaelli and Macek (2001). In this paper, we take also into account Alfvénic fluctuations of the flow. We also analyze $v_A = B/(\mu_o\rho)^{1/2}$, the Alfvénic velocity calculated from the experimental data: the radial component of the magnetic field of the plasma B and the mass density ρ (both B and ρ are taken as dependent on time and μ_o is the permeability of the free space). Namely, the following sets of data consisting of the radial velocity v and the Alfvénic velocity v_A are analyzed: the data set I (4282 points), measured by the Helios 1 spacecraft in 1975 from 67:08 to 69:11 (day:hour) at distances 0.33–0.35 AU from the Sun; this is nearly the same set as in the paper by Macek (1998), and the data for set II (15,894 points), measured by the Helios 2 spacecraft in 1977 from 348:00 to 358:00 (day:hour) at distances 0.82–0.89 AU from the Sun. The sampling time is $\Delta t = 40.5$ s.

Sometimes the raw data are nonequally sampled and linear interpolation has been applied. Admittedly, Matthaeus and Goldstein (1982) have shown that magnetohydrodynamic fluctuations in the solar wind are stationary. However, our analysis concerns much shorter time series and our raw data seems to be nonstationary (e.g., one can see the clear trend in the data shown in Fig. 1(a)). We focus rather on fast fluctuations in the slow solar wind and we are not interested in slowly varying trend. In the paper by Macek (1998), a quadratic trend was subtracted from the original data, but it led to the autocorrelation and the average-mutual-information functions decreasing very slowly with time. The characteristic decreasing delay time for these functions was $\sim 200\Delta t$ and so large delay time introduces some complications to that analysis. Slowly decreasing autocorrelation for a given time series means that this time series is oversampled or nonstationary. And the FNN method is based on looking for the nearest neighbors in embedding space. The two effects, i.e., oversampling and nonstationarity, introduce some problems in searching for nearest neighbors. Oversampling causes that two nearest neighbors that we find are rather two consequent points on a phase trajectory of the system than two points lying on the two different segments of the phase trajectory. Treating two consequent points on the trajectory as nearest neighbors in the FNN method we do not obtain actually any information about dynamics, because one point is a simple iteration of the second point. In the case of nonstationarity situation is even more complicated. If we are lucky then the FNN method applied to a nonstationary time series can give an optimal embedding dimension, which is higher than the proper dimension. The reason is that from the point of view of time series analysis, change of system parameters looks like additional degrees of freedom. In the worst case, for some types of nonstationarity, analyzing a nonstationary time series, we can get completely misleading results. In order to reduce these complications, in this work the moving average detrending (with window of 45 min) have been applied to the time series analyzed here. Fig. 1 shows an example of raw and detrended time series (radial velocity v, data set I). In Fig. 2 the logarithm of the spectral density as a function of the logarithm of the frequency is shown. As is usually characteristic for random and chaotic data, the obtained spectra are broad, revealing aperiodic behavior of the flow.

4. Results and discussion

Here, we present results of the analysis by means of the AMI and FNN methods for a real experimental situation. Namely, we show the results for solar wind velocities analysis. Finally, we present the graphs of the dependence $|x(t_i + m) - x(t_j + m)|$ on $|\mathbf{X}(t_i) - \mathbf{X}(t_j)|$ as introduced by Aittokallio et al. (1999), for the Lorenz

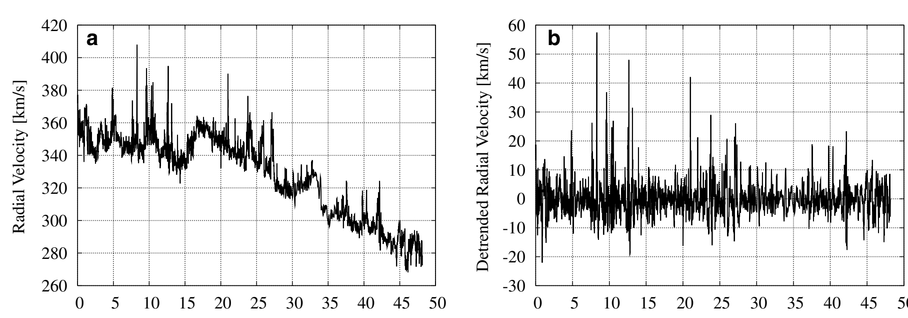

Fig. 1. (a) The radial velocity in the data set I, (b) the radial velocity detrended by subtracting moving average, data set I.

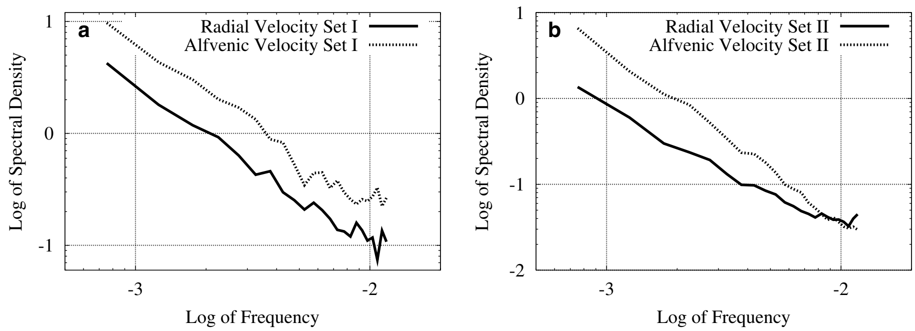

Fig. 2. The logarithm of the spectral density as a function of the logarithm of the frequency for the radial velocity v and the Alfvénic velocity v_A (a) in the data set I, (b) in the data set II.

model and the solar wind data, discussing also the case of colored noise.

We analyze time series of plasma parameters of the low-speed streams of the solar wind measured in situ by the Helios 1 and 2 spacecraft in the inner heliosphere at 0.3 and 0.9 AU from the Sun, as discussed in Section 3. We have chosen the values $R = 10$ and $A = 2$ in Eqs. (3) and (4). The appropriate values of ϵ for the solar wind have been chosen in such a way that the fraction of false nearest neighbors drops to zero at the same embedding dimension m for both the whole data set and for a shorter series, where only 20% of points have been taken into consideration. The dependence of the average mutual information on the delay time T for time series for the solar wind system is shown in Fig. 3, and the dependence of the calculated FNN fraction on the embedding dimension m is now presented in Fig. 4. As seen in Fig. 3, owing to the procedure of detrending by subtracting moving average, we have obtained the time series with the characteristic decreasing delay time of $\sim 15\Delta t$, which is much less than for the original data, $\sim 200\Delta t$. In addition, the slowly varying trends have been possibly removed to a high degree. The number of points at the averaging is chosen so as to obtain a sharply outlined minimum in the average mutual information dependence on the delay time. As one can see (Fig. 4) the obtained value of the embedding dimension m does not depend on the number of data in the time series, and it follows that ϵ parameter was chosen properly (Rhodes and Morari, 1997).

However, one can see in Fig. 2 that for the time series with solar wind velocities, at least in some range of frequencies, the dependence of the spectral density on the frequency is approximately linear on the logarithmic scale. Therefore, there is a possibility that we deal here with a colored noise, which can behave similarly to a deterministic system, providing also estimation of the low embedding dimension (Osborne and Provenzale, 1989). To exclude that possibility, in Fig. 5 we show graphs of dependencies of $|x(t_i + m) - x(t_j + m)|$ versus $|\mathbf{X}(t_i) - \mathbf{X}(t_j)|$, given in Eq. (3), as proposed by Aittokallio et al. (1999). One can see that the graph for the radial velocity of the solar wind shown in Fig. 5(b) is rather similar to the graph for the Lorenz system shown in Fig. 5(a) then that one for a colored noise time series, Fig. 5(c). Naturally, strongly asymmetric arrangement of points in a graph of $|x(t_i + m) - x(t_j + m)|$ versus $|\mathbf{X}(t_i) - \mathbf{X}(t_j)|$, which are concentrated in neighborhood of the origin of coordinates, is typical for deterministic systems and is directly connected with the deterministic origin of the dynamics of the system under study. The colored noise time series, where spectrum has a power-

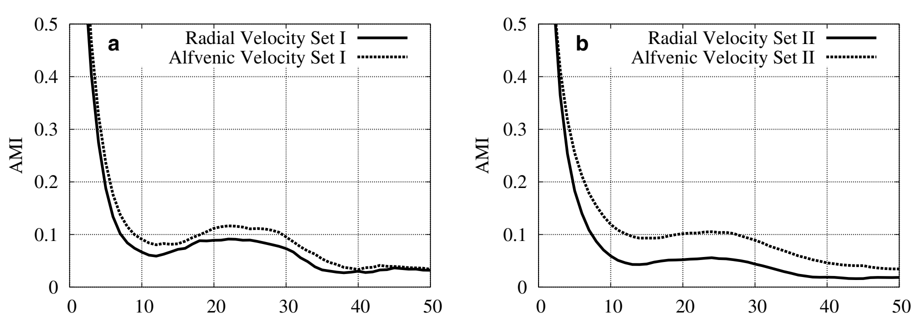

Fig. 3. The dependence of the AMI on the delay T for the solar wind radial velocity v and the Alfvénic velocity v_A (a) in the data set I, (b) in the data set II. Delay time is given in the sampling time units (i.e., 40.5 s).

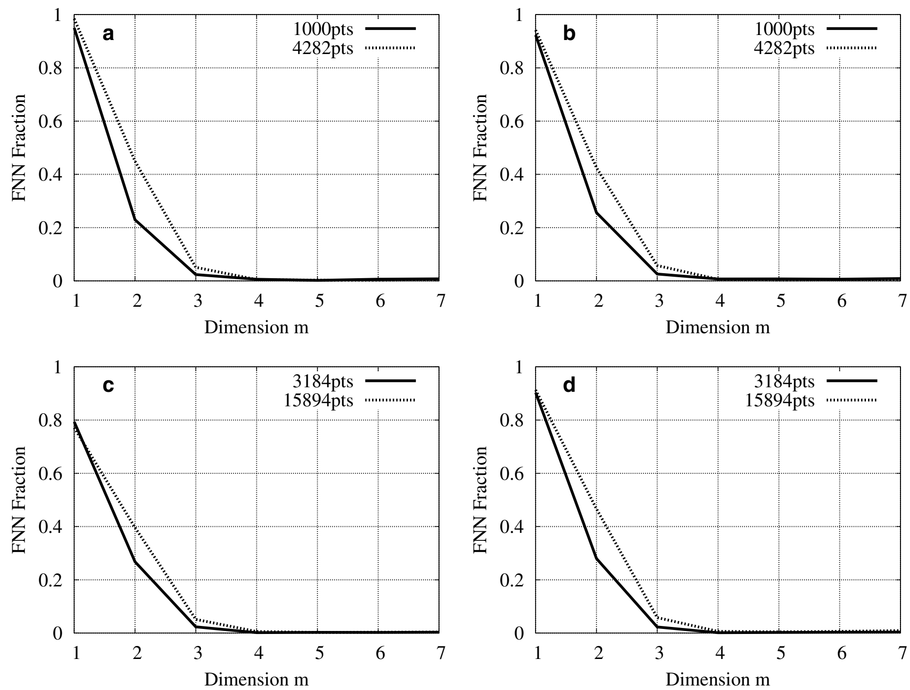

Fig. 4. The dependence of the FNN fraction on the dimension m for: (a) the radial velocity v in the data set I ($\tau = 12$, $\epsilon = 0.005$), (b) the Alfvénic velocity v_A in the data set I ($\tau = 12$, $\epsilon = 0.005$), (c) the radial velocity v in the data set II ($\tau = 14$, $\epsilon = 0.01$), and (d) the Alfvénic velocity v_A in the data set II ($\tau = 16$, $\epsilon = 0.005$).

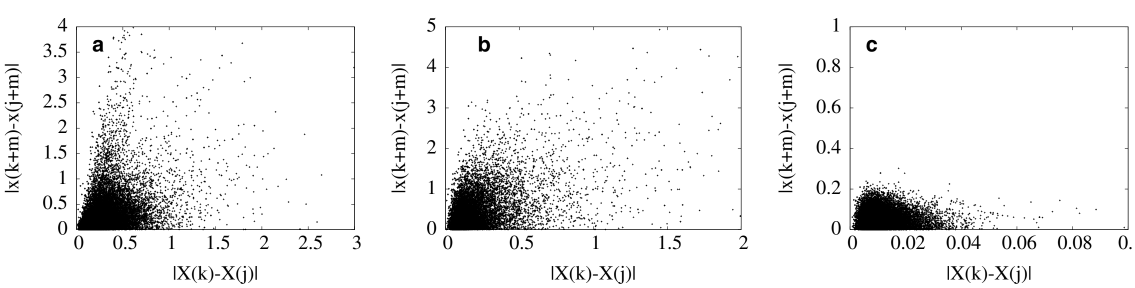

Fig. 5. The dependence of $|x(t_i + m) - x(t_j + m)|$ versus $|\mathbf{X}(t_i) - \mathbf{X}(t_j)|$ for: (a) a time series for the Lorenz system, (b) the radial velocity of the solar wind, and (c) colored noise with the spectral exponent $\alpha = 1.75$.

law dependence on frequency, $\propto f^{-\alpha}$ with a spectral exponent $\alpha = 1.75$, has been obtained by a program which is an implementation of the colored noise generation algorithm described by Osborne and Provenzale (1989).

The results clearly indicate that plasma dynamics in the inner heliosphere is deterministic and the minimal dimension ensuring the proper embedding in time-delay coordinates space is about four. It confirms the results obtained by Macek (1998) and Macek and Redaelli (2000), where it has also been argued that solar wind plasma dynamics could be chaotic. Therefore, there is some indication that complex plasma dynamics in the inner heliosphere can be modeled by an autonomous set of four nonlinear ordinary differential equations.

5. Conclusions

Results of our analysis of the solar wind data indicate that plasma dynamics in the solar wind is most likely deterministic and the proper embedding dimension is equal to four. Also results obtained for the radial velocity and the Alfvénic velocity are similar, and this fact shows that these two quantities are originating from the same dynamical system. The estimated proper embedding dimensions are the same for data obtained at 0.3 and 0.9 AU. Therefore, probably plasma dynamics for the slow solar wind does not change substantially with distance from the Sun. The question of possible colored noise in the solar wind has been also discussed and we have come to the conclusion that colored noise does not dominate dynamics of the solar wind flow.

The obtained results clearly indicate that a low-dimensional attractor should exist. One can therefore hope to infer information about some complex nonlinear phenomena from the geometrical properties of the attractor or by solving a set of four nonlinear ordinary differential equations describing the system. One can expect that the attractor should contain information about the dynamic variations of the coronal streamers or it could represent a structure of the time sequence of near-Sun coronal fine-stream tubes.

The common opinion is that fully developed turbulence in spatially extended systems cannot be explained in terms of pure low-dimensional chaotic dynamics. From the point of view of the dynamical systems theory the turbulent behavior is considered as a spatio-temporal chaos. In the case of chaotic dynamics in a spatially extended system, usually distant parts of the system become uncorrelated. Therefore, if the volume of the system increases, some additional degrees of freedom appear and low-dimensionality is not a good approximation anymore. However, we can distinguish another case besides the case of laminar flow and the case of fully developed turbulence, namely temporal chaos in spatially ordered structure. And our results suggest that this case can also be found in the interplanetary medium.

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