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NEW METHODS AND RESULTS OF RESEARCH ON THE PROBLEM OF THE EARTH'S FIGURE

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NOWE METODY I WYNIKI BADANIA FIGURY ZIEMI

S t r e s z c z e n i e

Artykuł przypomina w zarysach zasady wyznaczania figury Ziemi metodami: astronomiczno-geodezyjną i grawimetryczną ze zwróceniem uwagi na słabe punkty teorii tych metod. Dalej omówione są rezultaty poszukiwań parametrów figury Ziemi metodą obserwacji sztucznych satelitów i przytoczone są wyniki liczbowe. Przedstawione są próby łącznego wyrównania danych otrzymanych trzema metodami oraz podane są związane z tym problemem koncepcje własne autora: wprowadzenie do metody geometrycznej i grawimetrycznej wyników otrzymanych metodą satelitarną oraz odmienny od przyjętego przez W.Kaulę sposób łącznego wyrównania. Wyprowadzone są wzory i wykonany rachunek dla zbadania wpływu obecności trzeciej harmoniki kulistej P_3 na wynik wyznaczenia spłaszczenia z pomiarów stopnia. Obliczono, że przy uwzględnieniu tej poprawki wartość spłaszczenia elipsoidy Krasowskiego byłaby bliska $\frac{1}{297}$.

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S u m m a r y

The article outlines the principles of the Earth's figure determination by the astro-geodetic and gravimetric methods, indicating at the same time the weak points of the theory concerning these methods. The results of research for the Earth's figure parameters by the method of artificial satellite observations have been discussed and the numerical results have been given. The attempts of the joint adjustments of data obtained by three methods have been presented as well as the author's own conceptions connected with that problem: the introduction of results obtained by the satellite method into the geometrical and gravimetrical ones; the methods of joint adjustment different from that of Mr.W. Kaula.

The formulae have been derived and the calculation has been done in order to investigate the influence of the third spherical harmonic's P_3 presence on the result of determination of the ellipsoidal flattening from the arc measurement. Taking the above into account the corrected flattening of the Krassowski's ellipsoid is nearly equal to $\frac{1}{297}$.

1. INTRODUCTION

The methods being applied at present to the research of the Earth's figure give to the contemporary geodesist a rich variety of means to put through that task. Those means are the results both of the theory's and of the technics achievements. The methods can be roughly devided into four groups: the geometrical methods, the gravimetrical ones, the astronomical methods using the results of the observations of the artificial satellites, the astronomical methods which are taking advantage of some natural, astronomical events (according to some authors terminology the so-called cosmic geodesy). To the first group we wolud number first of all the triangulation and the trilateration, completed of course, by the astronomical and gravimetrical observations, the arc measurements, the astronomical levelling and so forth. Within the group of gravimetrical methods we find different ways of applying the results obtained from the absolute and relative determinations of gravity. As far as the third group is concerned - the one which is based on applying the results of the artificial satellites observations - we use many variants of the method of determining the geoid's parameters from the variations of the orbital motion of the satellite. The last group contains the methods which make use of the solar eclipses and star occultations. Those methods remain till now, within the sphere of experiments. The classification mentioned above is of course neither rigid nor strict; a number of methods combine features of the particular groups; moreover, we are trying to find the ways of applying the combined results of the different methods.

The methods and the theory of research have been developed in parallel with the evolution of the concept of the Earth's figure. When formerly one sought to determine the radius of the Earth's sphere and later on one was satisfied with the values of the equatorial radius and the ellipsoidal flattening, then this resulted not only from the modest technical means available, but also from the scantiness of the concept's stock, which as a matter of fact could develop only after a satisfactory amount of experiments has been accumulated and considered. The geoid's surface which has been searched and determined nowadays is more complicated than the sphere's surface in the same degree as are the contemporary methods of arc measurements in comparison with the Erathostenes method. It is necessary to say, that the determination of the Earth's surface for practical purposes, considered formerly often as the main task, becomes now merely a pretext for purely theoretical research. To draw up all maps of the world the Bessel's ellipsoid, calculated 120 years ago is sufficient and the Hayford's ellipsoid which is more than 50 years old, is even too good for that purpose. The greater are the merits of scientists which getting out of practicism continue the difficult and uphill works, the importance of which cannot be forecast.

To view better the means proposed by different methods and to compare their results - let's remind briefly the principles of each of them.

2. GEOMETRICAL METHOD

Under that general name we will understand all activities aiming at the geoid determination, connected with the measurements of geometrical elements - angles and lines - so, the astronomic determinations too.

What has to be understood under the concept of "geoid determination"?

Generally we can answer the question as follows: the geoid is determined when we can define the coordinates of its any arbitrary point in the system connected with another arbitrary point of the geoid and also in the system connected with definite points of the real, physical Earth's surface. In other terms: geoid will be defined when we can determine the connections between its arbitrary points and also betwen its points and the particular points of the physical Earth's surface. Evidently, like the contoured plan of a terrain's sector presents it always with some vertical and horizontal accuracy, analogically our image of the geoid will never be perfectly adequate with reality. So, every time, facing problems connected with geoid determination it is worth while to consider: what limit of accuracy has to be aimed at and how far is it reasonable to strive for details. In many cases for the theoretical purposes a less precise general picture is more valuable while in some cases just the details are of the most importance. To obtain such a general picture it is necessary to determine the ellipsoid from the arc measurements. The idea how to determine the Earth's dimensions from the measurement of the meridional arc's lenght is almost as old as geodesy itself. This idea has been developed and completed simultaneously with the evolution of geodesy, mathematics, physics and astronomy. Now, although the complicated character of the geoid's surface is known, the attempts for a better and more precise determination of the surface, known under the name of the Earth's ellipsoid, have not been discontinued. It has been so because the ellipsoid is a good base for further studies on "detail measurement". If we understand in the very way the ellipsoid's significance for the Earth's figure investigation, if we treat it not as much as the surface defining the real shape of the Earth's solid but rather as a reference surface on which we can draw more precisely the geoid's run, then it comes out that the question of the ellipsoid's dimensions isn't of the most importance. We are able to measure the distances between the physical Earth's surface and the ellipsoid of an arbitrary shape, - so even accepting some conventional, assumed values for the ellipsoid parameters we can obtain the correct idea of the geoid plotted on the ellipsoid. It concerns both, practical and theoretical questions.

Does it mean that we have to give up the attempts for the determination of the ellipsoid? Certainly not. Though during the last few years many results of research on that problem have been published, though we have a multitude of ellipsoids - yet we don't have a sufficiently good idea of the geoid, simply because great areas of the globe still remain unmeasured. Completing the new computations of the ellipsoid with the new results of measurements of areas being untouched so far - we obtain not only the new dimensions of the both semiaxes but also the new elements of orientation. Even knowing the correct ellipsoid's size we would have some difficulties in determining the geoid when the orientation's elements would be wrong. An ideal solution would be such a position of the ellipsoid with regard to the geoid when their centres of gravity as well as their axies of rotation would coincide. - We are quite far from fulfilling that condition. I. Fischer [22] estimates the precision of coincidence of two centres at $\pm 200\,m$, what is of course too much for the ambitions of the contemporary geodesy.

The basic idea of the arc measurement is to measure the segment of the meridian or parallel arc and to compare it with the astronomic measurements of differences in the geographical latitude or longitude of the extreme points of the arc. Here we have the elementary formulae of the method:

$$s_1 = \frac{a\,(\varphi_1' - \varphi_1)''}{\rho''}\left\{1 - \left[\frac{1}{4} + \frac{3}{4}\cos\,(\varphi_1 + \varphi_1')\right] e^2 - \ldots\right\}$$

$$s_2 = \frac{a\,(\varphi_2' - \varphi_2)''}{\rho''}\left\{1 - \left[\frac{1}{4} + \frac{3}{4}\cos\,(\varphi_2 + \varphi_2')\right] e^2 - \ldots\right\} \qquad (1)$$

$$S_1 = \frac{a \cdot \Delta\lambda_1''\cos\varphi_1}{\rho''}\left(1 + \frac{1}{2}e^2\sin^2\varphi_1 + \ldots\right)$$

$$S_2 = \frac{a \cdot \Delta\lambda_2''\cos\varphi_2}{\rho''}\left(1 + \frac{1}{2}e^2\sin^2\varphi_2 + \ldots\right) \qquad (2)$$

where

e - is the eccentricity of the meridian's ellipse;

a - the equatorial radius.

Formulae (1) concern the meridional measurement where s_1, s_2 are the length of the measured meridional segments and φ_1, φ_1', φ_2, φ_2' are the geographical latitudes of the ends of the segments. Formulae (2) concern the parallel measurements where S_1, S_2 are the lengths of two parallel arcs, $\Delta\lambda_1$, $\Delta\lambda_2$, are the differences in the geographical longitudes of the ends of the segments. We can compute the unknowns a and e^2 from each of those pairs of equations.

In this formulae there is a simplifing assumption - that $(\varphi' - \varphi)$ is considered as a sufficiently small quantity so that the function $S(\varphi)$ in this interval has a linear character. Thus, theoretically, it would be the most advantageous to measure the segments the shortest possible and the differences $(\varphi' - \varphi)$ adequately small (unfortunately, the relative error of the measurement would increase at the same time). As the limit of the ratio $\frac{\Delta s}{\Delta\varphi}$ is the inverse of meridian's curvature, we can state that the determination of the ellipsoid from the arc measurement consists in the measurement of the curvature of the meridional section of that surface. This concerns, to the same degree, the parallel measurement except, that we have to do there with the non central, parallel section.

The fact, that on measuring the ellipsoid we are not on its surface but on another surface, which we do not know, is the essential obstacle in the theoretical sense as well as the source of errors in the practice. So, the length of the base lines, measured on the geoid, does not fit to the length of the same lines on the ellipsoid. As the distances from the geoid to the ellipsoid oscillate within bounds of some scores of meters it may happen that the length error of the arc measured will be equal to 1:1 000 000. The problem of measurement of the angle values φ and λ is more serious. The local vertical deflections are reaching a dozen or so second of arc, whereas the gravimetrical corrections of the astronomical measurements are accurate to not more than 1". As it is, the principle

$$\sum(\xi^2 + \eta^2) = min$$

(ξ - meridional component of the vertical deflection, η - the east-west component, both gravimetrically corrected) accepted for the astro-geodetic determinations does not make for the adjustment of the measured values because the accuracy of their measurement is much greater but it leads to smoothing the irregularities of the fragment of the geoid and to find the ellipsoid somehow corresponding to the geoid in the measured area. ξ and η observed on a limited area are not distributed at random. These values are influenced by the distribution of the gravity anomalies within a radius of several hundreds kilometers and more weakly but still clearly within a radius of several thousands of kilometers. A wrong knowledge of the anomalies distribution in so distant terrains causes systematic errors of the corrections $\Delta\xi$ and $\Delta\eta$ for the whole measured region.

It is the weakest side of the arc measurements that they are limited to the continental terrains, while we can fore-expect a different run of the geoid on the oceanic areas. That question has been a long time the subject of controversy. Still before the end of the last century F. A. Słudski [24] maintained that the geoid ought to be situated over the ellipsoid on the oceans and under it on the lands. When Hayford [10] computed his ellipsoid he used the procedure based on the assumption that the geoid runs over the ellipsoid on lands and under it on oceans. Jeffreys [10] has come to believe that there isn't any systematic difference in that matter. The result obtained from observations of the artificial satellites shows that Słudski was right anyway. Evidently, having not concrete knowledge on this subject in was not possible to take it into account for the computations.

The still growing accuracy of the successive determinations is caused not only by the perfectionning of the technics and theory of measurements but also by the measurement of larger and larger territories. Hayford's ellipsoid has been determined from the measurements of the network which covers the North America's continent. Krassowski included into his computations the American and European triangulation together with the one performed in USSR where almost the whole European part of the country was covered by a net-

The more important results of the ellipsoid measurement

Table 1

Author	year	a	α^{-1}
Bougier Maupertuis	1738	6 397 300 m	216,8
Delambre	1800	6 375 653	334,0
Everest	1830	6 377 276	300,8
Bessel	1841	6 377 397	299,15
Clarke	1880	6 378 249	293,5
Helmert	1907	6 378 200	298,3
Hayford	1910	6 378 388	297,0
Heiskanen	1926	6 378 397	(297,0)
Krassowski	1946	6 378 245	298,3
Jeffreys	1948	6 378 099	297,1
Lederstreger	1951	6 378 298	(297,0)
Hough	1956	6 378 260	(297,0)
Fischer	1961	6 378 157 6 378 155	298,1 (298,3)

The number α^{-1} in brackets - means that it was not computed in that determination but was assumed in anticipation.

work and where the giant parallel arc from Ural to Pacific in the Asian part of USSR was measured. The last item of the table 1 presents the results as obtained by I. Fischer [5]. They are based on all the previous measurements as well as on the latest ones: the transafrican arc connected with the european network, the transamerican arc from Alasca to the southern extreme of South America with the additional chain from the Pacific to Atlantic Ocean through Brasil; the Near East arc which permits to connect the triangulation of India and Burma with the european network; the Japanese network attached through Korea and Manchuria to the USSR system; the intercontinental connexion by the trilateration method:North-America - Europa through Greenland, North America - South America through the West India islands.

The utilization of measurements of such large areas has a great influence on the accuracy of orientation of the determined ellipsoid - even greater one than on the size of the ellipsoid. From that point of view it is particularly important to connect the separately measured fragments of Earth.

The obtaining of the geoid's profile by the method of astronomic levelling is the "by-product" of the great arc's measurement. As it is a question of the future to extend the triangulation network all over the Earth's area and as, moreover, it is possible that some new methods will allow us to give up this expensive and laborious method of laying out the network, so we have to admit that the astronomic levelling is not a method which would bring about the cognition of the geoid.

3. GRAVIMETRICAL METHOD

The gravity measurements are a much younger branch of science than the geometrical ones. If we consider Galileo to be their inventor, then the gravity measurements are 2000 years younger than the first arc's measurement done by the Eratosthenes. So it is no wonder that the theory of the gravimetrical measurements is not yet completed. We witness the continued theoretical controversies; moreover, new conceptions and ideas are arising. The gravity anomaly is the basic element used by the gravimetry for the determination of the Earth's figure as well as for orientating the triangulation network and for the geological researches. The anomaly is defined very simply

$$\Delta g = g_o - g_c \tag{3}$$

g_o - observed, g_c - calculated.

But then the first complications begin. The value of the gravity acceleration, measured at the point of observations is not suitable itself to a direct comparison with g_o, which corresponds to zero-altitude above the sea level. The question of the proper method of reduction of the measured value g is still a subject of controversy. Different authorities have different opinions on that subject: Jeffreys, Graaf Hunter, Hirvonen - are the adherents of the free-air reduction, Heiskanen, Vening-Meinesz find the isostatic reduction holds good, while lately, the Rudzki's inversion reduction wins more and more adherents (L a m b e r t, M i c h a i l o w [7]). The introduction of one or another reduction indirectly define the level surface to which the measurements value will be reduced. g_c corresponds to a model surface, e.g. the ellipsoid, thus Δg is the difference between the specific acceleration of the model surface and that of the surface defined by the applied reduction. That latter surface mustn't always be the geoid. The appliance of one or another method of reduction has an essential influence on the results by changing the anomalies distribution. It has even been lately adopted that the results of the determination of the ellipsoid from the arc measurements are presented twice, according to the character of anomalies applied to the compute of the gravimetric vertical deflections. I. Fischer presents the results of one of the last measurements

$$\alpha = \frac{1}{297} \qquad a = 6378240 \pm 100 \text{ (free-air)}$$

$$a = 6378280 \pm 100 \text{ (isostatic)},$$

g_c is a value derived from the formula for the normal gravity acceleration

$$g_r = \gamma_e \left(1 + \beta \sin^2 \varphi - \varepsilon \sin^2 2\varphi\right), \tag{4}$$

γ_e - gravity value of the equator,
φ - geographical latitude,

coefficients β and ε depend on the physical constants of the Earth and among others on the ellipsoidal flattening α.

After J e f f r e y s [10]

$$\beta = \frac{5}{2} m - \alpha + \frac{15}{4} m^2 - \frac{17}{14} em,$$

$$\varepsilon = \frac{1}{8} e^2 - \frac{5}{8} em, \qquad m = m(M, \omega, a, \alpha).$$

The theory of the research on the Earth's figure is based on the application of those relations. By determining empirically the coefficients β and ε by means of multiple measurement at different points of the globe we can define the ellipsoidal flattening.

The more important results of the determination of the flattening α by the gravimetric method

T a b l e 2

Author	year	α^{-1}
Helmert	1884	299,25
Iwanow	1889	297,2
Bowie	1907	297,4
Helmert	1915	296,7
Heiskanen	1924	297,4
Heiskanen	1938	298,2
Niskanen	1945	297,8
Heiskanen	1957	297,4

Similarly as in the case of the arc measurements, the accuracy of the results given in the table 2 was growing simultaneously with the extension of measured territories. The gravimetric method has the unquestionable advantage over the geometrical measurements for it is applicable also to the oceanic parts of the Earth's surface. The new types of gravimeters used nowadays e.g. Graf's gravimeters as well as older types e.g. Vening-Meinesz pendulum apparatus allow a quick performing of measurement on the sea. That makes possible to cover our whole planet by uniform measurements. Now we are far from that very ideal and at the present the best known terrains are those where the oil is explored.

The second advantage of the gravimetrical method is that α can be determined irrespective of the ellipsoidal size, while in the geometrical methods α is functionally connected with

a. If we take into account[1] that measuring Δg with accuracy to 0,2 mg, at two points of different geographical latitudes, we obtain α accurate to the same order of values as obtained by the arc measurement between these two points, accurate do $\frac{1}{500000}$, - then we arrive at a conclusion that the advantage of the gravimetrical method for the determination of the geoid's figure is really considerable.

Let us consider now the "measurements od details" in respect of the research on the Erath's figure and of the possibilities offered here by the gravimetry. Here we have a possibility to obtain absolute values of the distances between ellipsoid and the geoid as well as absolute deflections of the vertical. The ellipsoid, with regard to which we are computing these values, has the absolute orientation - its center coincides with the Earth's gravity center - but its dimensions are unknown. There are two principal directions of proceeding. The one is to determine the distances between the geoid and the ellipsoid from the Stokes formula

$$N = \int_0^S F(\psi)\, \Delta g\, ds, \tag{5}$$

ψ - angular distance,
Δg - anomaly,
ds - element of surface,
S - the Earth's surface.

It is necessary to consider some characteristics of that formula which follow from its assumptions and derivation:

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[1] Substituting $e^2 = 2\alpha$ into (1) and then differentiating we obtain

$$\frac{ds}{d\alpha} = \frac{a(\varphi' - \varphi)}{\varrho''} \cdot \frac{3}{2} \sin 2\varphi,$$

so

$$d\alpha = \frac{3}{2} \frac{\varrho'' ds}{a(\varphi' - \varphi) \sin 2\varphi}.$$

Applying the (4) for two values φ and forming the difference we can obtain

$$\Delta g = \gamma_e\, \beta (\sin^2 \varphi' - \sin \varphi) = \gamma_e\, \beta \sin 2\varphi \sin(\varphi' - \varphi),$$

$$d\alpha = d\beta = \frac{d\Delta g}{\gamma_e \sin 2\varphi \sin(\varphi' - \varphi)};$$

If

$$\frac{\varrho'' ds}{a(\varphi' - \varphi)} = \frac{1}{500000}; \quad \varphi \approx 45^\circ \quad \varphi' - \varphi \approx 10^\circ$$

then

$$d\alpha = \frac{1}{750000} = 1{,}3 \times 10^{-6},$$

when $\Delta g = 0{,}2\ mgl$, $\sin(\varphi' - \varphi) \approx \frac{1}{5}$

$$d\alpha = d\beta = \frac{1}{978000} \approx 1{,}0 \times 10^{-6}.$$

1° the formula is valid on the assumption that all masses are situated inside the geoid. To fulfil that condition it is necessary to transform adequately the geoid as well as the measured anomalies,

2° that formula includes an integral to be extended all over the Earth's surface but it is known that the measurements do not cover the surface of the whole planet. Giving up the mathematical precision we omit the very distant territories, none the less the considered point has to be surrounded by the gravimetrically measured area with a radius of thousands kilometers.

The theoretical difficulties we meet with using the Stokes' formula as well as the difficulties in choosing the most satisfactory reduction are the reasons that we search for new conceptions solving the same problem. We can meet with new concepts such as co-geoid, terroid, telluroid, quasi-geoid, model-Earth, geop, spherop etc. However, it doesn't stop the substantial work on the determination of the geoid's run; on the contrary, we still obtain better and more interesting results. While the scientists - let us name them - the par excellence geodesists are forward to apply the Stokes' method, the geodesists having a bent for astronomy choose a different way which leads through series expansions of the spherical harmonics. Representing the known anomaly distribution by series of spherical harmonics we can express the distribution of the distances N_0 by the aid of the same series coefficients.

If

$$\Delta g = \Delta g\,(Y_{ik}),$$

then

$$N_c = N_0\,(Y_{ik})\,.$$

Hitherto that method remained under the shadow of the Stokes' method, for the knowledge of the anomaly distribution all over the Earth is here more important. How risky it is to extrapolate the data of the measured terrains into the unmeasured ones, is shown by the example of the Zhongolovich's geoid. He has determined on the ground of the known anomalies the coefficients of the expansion by spherical functions to the 8th order but the latter measurements do not agree with values extrapolated by the Zhongolovich's formula [7]. At present the method of the series expansion by spherical functions goes through its period of glory, for it is convenient for the research by means of the satellite methods. The coefficients of that series define also the potential function in the outer space. By those means we can pass from the potential field studied with the aid of the satellite observations, to the gravity anomalies and to the distances from geoid to ellipsoid. This method is also convenient for work combining the results of the gravimetric, geometrical and satellite methods.

From among the interesting studies on geoid's problem it's worth while to mention, next to the Zhongolovich's determination quoted above, the works of Hirvonen, Tanni and Heiskanen. There was lately (1961) computed, under the direction of Heiskanen, so called "Columbus geoid". The Stokes' method has been applied here. From the same data of observation Uotila determined the coefficients of the eight successive terms of expansion by the zonal spherical harmonics. It comes out that the term of the 3th order has a value approximate to that one given by O'Keefe as determined by the observation of the artificial satellites [6].

4. THE SATELLITE METHOD

This is the youngest of methods, which has arised but three years ago, yet has got ahead very quickly. At least several centers and a dozen or so of outstanding authorities are paying a particular attention to that problem.

Hitherto the applications of the artificial satellites to the research of the Earth's figure were based on the laws of dynamics ruling the Earth's gravity field. That's why several authors are using the term of "astronomical gravimetry". The principle of that method consists in representing the perturbations of the Keplerian motion of satellites by the function of the parameters characterising the Earth's gravity field. Of course, although the orbit's elements undergo perturbations, not all of them are equally fit for the determination of parameters. The most important and the most readily considered - especially for the Earth's flattening determination - is the change of the right ascension of an ascending node. But in some cases it is equally or even more convenient to consider other changes: those of the perigee argument, eccentricity, inclination. Elaborating the observations of the Vanguard satellite O'Keefe [22] applied the changes $\dot{\Omega}$ and $\dot{\omega}$ to the computing of the second and fourth harmonics as well as $\dot{e}, i, \dot{\Omega}, \dot{\omega}$, to the third and fifth ones.

The theory of the satellite's motion nearby the planet, which is due to the fact that artificial celestial bodies were created, is developing with a speed of an explosion. The best evidence is the quantity of publications: in the bibliography of the Bulletin of Polish Observations of the Artificial Satellites, in six successive issues there have beeen checked off 273 publications of the orbital problems of the Earth's artificial satellites, 71 of them being closely connected with geodesy. In such a situation any attempt for a sistematization in that domain, undertaken by one person is doomed to fail.

We will not consider the general basis of the problem, for it is discussed in the article of W.Opalski "Theoretical foundations of utilizing observations of artificial satellites for the investigation of the Earth figure" [23]. Instead, let us consider the advantages and the disadvantages of that method in comparison with the methods discussed before.

In the gravimetric method the measured value refers always to some particular point. And then, interpolating the results of those particular points we can form a general picture of the surface, ex.g. the map of the gravimetric anomalies for a given area. In the satellite method we measure - undirectly - the gravity field of the whole Earth. Every curve of the terrain, every stone, has its influence on the shape of the satellite's orbit. Therefore the problem of the unmeasured areas - so essential for the previous methods- disapears automatically. That points towards the important conclusions. The formula for the normal value of the gravity acceleration is derived for the ellipsoid. Its coefficients β and ε as well as the γ_e value are determined empirically from several observations in different places. But if those observations are not uniformly disposed all over the Earth's surface and the geoid deviates systematically from the ellipsoid, then we obtain values $\beta, \varepsilon, \gamma_e$ charged with the systematical errors,peculiar to the ellipsoid fitting the best the measured areas. In that case anomalies of a little value over a vast area will not be find out, it means that they will not be find out till a much larger area is measured where the excessive anomalies of the opposite sign will occur. The formula for the normal acceleration has to be corrected by introducing the flattening value α computed by the satellite method. γ_e has to be computed from the adjustment of the gravimetric measurements.

The structure of the Earth's gravity field is as complicated as the geoid's shape is, and it can be defined by a formula e.g. of expansion in spherical harmonics

$$U = \sum \frac{A_{nk}}{r^{n+1}} P_{nk} \quad (\phi, \lambda). \tag{6}$$

A measurement made in the satellite method is a determination of the variation of that or another orbit's element. That variation is a function of the whole gravity field and therefore of all terms of the series. It is comparatively easy to determine from that measurement the prime terms which have a great influence on the perturbations. The determination of the further terms depends on fulfilling the special orbital conditions, which allow, in the equations linking the variations of the orbit's elements with the coefficients of the formula for the potential, to eliminate some of the terms and to bring out the other ones. This is especially important in analysing the formula for U as a function of λ, where it is indispensable to apply the resonant orbits. It is expected that by applying the resonant orbit it will be possible to compute the coefficients for the tesseral and sectorial harmonics of the same order as for the zonal ones. "The distributive power" of that method is moreover conditioned by two factors: the accuracy of the observations and the adequate elimination of the perturbations caused by other factors. The satellites applied for geodetic purposes have been usually observed by the photographic method or by the radio method, making use of the interference effect. The formal accuracy of the photographical observations with cameras Baker-Nunn is really high: 2" in the angle, $0^s.001$ in time. It's necessary to remember that the observations are distributed irregularly in time and in space; not every circuit is observed, and several parts of the orbit are not being observed at all. Besides, one simplifies the reality by applying in computations the osculating or mean orbit which is that of Kepler —though in further considerations the variations of elements in time are introduced. The eccentricity variations are considered though the movement does not proceed along the ellipse, the variations of the orientation and inclination of the orbit's plane are considered though the satellite does not move in the plane. Those simplifications are unoffending when we have the rich and uniform observational material; they are rather dangerous when we have to do with a scarce number and disadvantageous distribution of the observations. When computing the unknown coefficients for the terms in the formula for the potential, we apply, as the known quantities, some orbit's elements, such as: eccentricity, semi-axis, inclination and mean motion. For instance after O'Keefe

$$\Delta i_3 = -\frac{3}{2}\,\frac{A_{30}\, e \cos i}{n\, a^6\, n'}\left[1-\frac{5}{4}\sin^2 i\right]\sin\omega ,$$

Δi_3 - effect of the third zonal harmonic term on orbit's inclination,
i - inclination of orbit,
A_{30} - coefficient of the third zonal harmonic,
e - eccentricity,
a - semi-major axis,
n - mean motion,
n' - motion of perigee,
ω - argument of perigee.

The formula can be written

$$\Delta i_3 = \frac{A_{30}}{n}\cdot Q$$

and calculating the derivative of A_{30} with regard to n, we obtain

$$\Delta i_3 = Q\,\frac{\partial A_{30}}{\partial n};$$

Proceeding to the relative errors, we obtain

$$\frac{\Delta A_{30}}{A_{30}} = \frac{\Delta n}{n}.$$

So, apart from the required accuracy of the determination of variations of the orbit's elements (here Δi_3), a precise knowledge of the absolute values of those elements is necessary.

In spite of the difficulties mentioned above, in practice a high accuracy is being obtained. K o z a i [18] gives the elements of the orbit of the satellite 1961 o 2 (Injun): $n = 13{,}86862 \pm 0{,}00002$ turn/1^d; $e = 0{,}008105 \pm 0{,}000012$; $i = 66^o{,}8146 \pm 0{,}0009$. O'K e e f e [22] uses for computation $\Delta\Omega/1^d$ with accuracy 10^{-5}, $\Delta\omega/1^d$ with accuracy 3×10^{-4}, what when computing the flattening α, allows to attain an accuracy of the order of 1 : 20000.

To consider the effects of the other factors perturbing the Keplerian motion of the satellite it is necessary to know those factors. It is quite simple, for instance, to introduce corrections of the attraction of the Sun and the Moon. The most troublesome is the atmosphere resistance and this is the main reason of the divergence between the results obtained by different authors. It is a factor which is dangerous mainly because of its variableness and even of its "unmathematicality". In fact, the highest layers of the atmosphere are not stabile. There appear vertical and horizontal currents as well as density variations caused by the alternations of seasons, of day and night and other unknown reasons. In such a situation it seems the most smart to determine the atmospheric effects simultaneously with the parameters of the gravity field. O'K e e f e [22] applies that method by using the variations of the semi-axis a of the orbit, which is of little sensitivity to the gravity factors, for tracking out the atmospheric effects. There remains an outstanding question: whether the accuracy of that method is sufficient, for the irregularity of the distribution of observations in time and space still rebounds here upon.

Different authors apply different symbols to define the Earth gravitational potential. The table 3 contains the coefficients of the formula

$$U = \frac{f \cdot M}{r}\left[1 - \sum_{n=2}^{\infty} J_n \left(\frac{a}{r}\right)^n P_n(\sin\phi)\right]. \tag{7}$$

These can be related to the Jeffreys coefficients J and D by the formulae

$$J = \frac{3}{2} J_2 \qquad D = -\frac{35}{8} J_4,$$

to those of O'Keefe

$$A_{n0} = -\frac{fM}{a^n} J_n$$

to those of Zhongolovich

$$c_{n0} = -J_n .$$

The flattening α can be determined from the formula (16)

$$\alpha = \frac{1}{2}(3J_2 + m)\left(1 + \frac{3}{4}J_2 + \frac{3}{28}m\right) + O(J_2^3). \tag{8}$$

The results in the table 3 are striking because of their coincidence, especially in the column J_2 and in the corresponding column α^{-1}. On the basis of these data it can be ascertaines that α^{-1} closes between 298,2 and 298,3. To assume this or another number in that rang, is a question of convention only, for the accuracy of the first place after the comma in the denominator of the flattening corresponds to the accuracy approx. 6,5 m in the difference $a-b$.

The value of J_3 can be regarded as established too - it is close to $-2{,}5\times10^{-6}$. On the other hand, in the columns J_4 and J_5 we find greater discrepancies than we could expect of the internal accuracies of the particular results given by the authors. So we have to treat with a reserve the last results presented by K o z a i [18] though they are derived from very rich observational data - i.e. observations of 31 satellites.

The calculation of the high order zonal harmonics only, has a somehow speculative character, for it is generally known that the gravity field is a function of φ as well as of λ. Hitherto there were not determinations of that sort and those

Results of the determinations of the Earth's gravity potential from observations of artificial satellites

Table 3

Author	satellites	J_2	J_3	J_4	J_5	J_6	J_7	J_8	α^{-1}
1958									
Buchar	Sputnik 2	+1085,2	-	-2,4	-	-	-	-	297,90
Merson King-Hele	Sputnik 2	+1084,0	-	-2,4	-	-	-	-	298,1
Jacchia	Sputnik 2 Vanguard 1	+1082,7	-	-2,1	-	-	-	-	298,28
1959									
Merson King-Hele	Sputnik 2 Vanguard 1	+1083,3	-	-1,0					298,2
Merson King-Hele	Sputnik 2 Vanguard 1 Explorer 4	+1083,0 (0,2)	-	-1,3 (0,2)	-	-0,1 (1,5)			298,2
Leoar Squires Bokels	Vanguard 1	+1082,2	-	-2,4	-	-2,4	-	-	298,32
O,Keefe Bokels Squires	Vanguard 1	+1082,5 (0,1)	-2,4 (0,3)	-1,7 (0,1)	-0,1 (0,1)	-	-	-	298,26
Kozai	Vanguard 1	+1082,1	-2,2 (0,1)	-0,9	-	-	-	-	298,34
1960									
King-Hele	Sputnik 2 Vanguard 1 Explorer 7	+1082,79 (0,15)	-	-1,4 (0,2)	-	-0,9 (0,8)	-	-	298,24
Zhongolovich	Sputnik 2 Sputnik 3 Sputnik 3R	+1083,3 (0,7)	-2 (3)	-4,1 (0,7)	-	-	-	-	298,17
Zhongolovioh	Sputnik 3 Sputnik 3R	+1083,2	-1,8	-4,2	-	-	-	-	298,18
1961									
Zhongolovioh	Sputnik 3	+1082,5	-0,9	-3,4	-	-	-	-	298,26
Kozai	Explorer 7 Vanguard 3 Vanguard 1	+1082,21 (0,04)	-2,29 (0,02)	-2,1 (0,1)	-0,23 (0,02)	-	-	-	298,31
Miohielsen	Transit 1B(R) Vanguard 1 Sputnik 4	+1082,7	-2,5	-1,7	+0,3	+0,7	-0,6	+0,1	298,25
Newton Hepfield Kline	Transit 2A Vanguard 1 Transit 1B	-	-2,36 (0,14)	-	-0,19 (0,10)	-	-0,28 (0,11)	-	-
Smith	Sputnik 3 Transit 1B Tiros 1	+1083,15 (0,2)	-	-1,4 (0,3)	-	-0,7 (0,6)	-	-	298,19
1962									
Shelkey		+1082,61 (0,05	-	-1,52 (0,08)	-	-0,73 (0,10)	-	-	298,26
Kozai	31 Satelitów	+1082,36 (0,06)	-2,57 (0,01)	-2,14 (0,08)	-0,06 (0,02)	+0,15 (0,12)	-0,47 (0,02)	-0,31 (0,02)	298,29 J_9=+0,11 (0,02)

which took place has the character of the investigations on the method rather than of a measurement of the gravity field.

The results of the researches on the ellipticity of the equator by the satellite method .

Table 4

Author	year	satellites	$\beta \times 10^{-5}$	$\lambda_x(E)$	$a-a'$
Izsak	1961	Vanguard 2 Vanguard 3	3,2 ± 0,3	-33°,2 ± 0,5	205 m
Kaula	1961	Vanguard 1	1,1 ± 0,5	-36 ±16	70 m
Newton	1961	Transit 4A	2,4 ± 0,5	-11 ± 6	219 m

β - flattening of the equatorial ellipse,
λ_x - geographical longitude of the major axis direction,
$a-a'$ - semi-axis difference.

All the three results given in the table 4 refer to one element only - the tesseral harmonic of the second order - this being synonymous with the determination of the ellipticity of the equator. The author's opinion is that en essential progress might not be expected before a realization of an resonant orbit.

5. COMPARISON AND COMBINATION OF THE RESULTS OF DIFFERENT METHODS

Recapitulating briefly the preceding considerations, we can state that:

a. The geometrical method permits us to define the size and the general shape of the Earth, but its applications are limited to the continents. The method of the astronomical levelling can not promise to perform the research of the geoid's run over all continents - in a short time, because of the difficult and expensive measurements. There have been considerable difficulties arising from the adequate orientating of every independent system as well as from the connection of different systems.

b. The gravimetric method enables us to determine the general as well as the detail shape of geoid, but it does not give us the possibility of precising the size of the geoid. The accurate applying of that method requires measurements to be done all over the globe's surface, what we are quite far from. Moreover there are still some questions unsolved in the theory of that method.

c. The satellite method enables an accurate determination of the general shape of the Earth's figure - without its dimensions. Every determination of that figure - even from the observation of one satellite - is being accomplished with the aid of the measurement concerning the whole Earth. Applying the satellites which be especially destinated for the purpose of investigation of the Earth's figure it will be possible to reduce the degree of generality of the figure being determined. None the less the detail researches of the geoid's run are inaccessible to that method.

In this situation it is a logical necessity to combine and complement the particular methods one another. The example of that is the introducing of gravimetrical corrections for the deflection of the vertical into the triangulation measurements. It is a kind of the unilateral service offered by one method for the benefit of perfectionning of the other one. A similar advantages can be taken of the satellite method.

The estimate of the values of anomalies of the unmeasured terrains presents a serious problem of gravimetry. The knowledge of the form of the surface being the generalized geoid - let us call it quasi-geoid - enables us to find those values approximately, none the less a good deal more accurately than before. Let us present [23] the gravity acceleration by the formula

$$g = g_0 \left[a_{oo} + \sum_{n=2}^{\infty} \sum_{s=0}^{n} \left(\frac{a}{r}\right)^n (a_{ns} \cos s\lambda + b_{ns} \sin s\lambda) P_{ns}(\sin\phi) \right] \quad (9)$$

assuming that the several first coefficients a_{ns} and b_{ns} $(n \leqslant n_1, s \leqslant n)$ are known from the observations of the artificial satellites. So, introducing the known values a_{ns} b_{ns} as constants into (9) and measuring g we obtain the observational equation with the unknown coefficients a_{ns} b_{ns} $(n_2 \geqslant n > n_1, s \leqslant n)$

$$g = g_0 \left(a_{oo} + \underset{\substack{n \leqslant n_1 \\ s \leqslant n}}{[A_{ns}]} \right) + \underset{\substack{n_1 < n \leqslant n_2 \\ s \leqslant n}}{[A_{ns}]}, \quad (10)$$

where $[A_{ns}]$ are the sums of spherical harmonics. The first sum consists of the known terms, the second one consists of the unknowns. The determined values of a_{ns} and b_{ns} for $n' > n_1$ will be realiable also for the unmeasured terrains.

A fuller knowledge of the shape of the surface representing the Earth's figure allows us to take a farther step into the theory of the arc measurements. Now we can try to determine the parameters of more a complicated surface, the general formula of which as well as some coefficients of the formula are known. That can be realized in practice be means of a suitable computing of the values ξ and η as the deflections between the vertical and the normal to that surface.

That knowledge can be also applied to a determination of the conventional surface of the ellipsoid. It is generally understood that the geodetic-astronomical measurements are always necessarily limited to a certain, larger or smaller, fragment of the whole Earth's surface. So, to reduce the resulting errors as much as possible - it is necessary when determining the ellipsoid, to take into account those regional deflections from the ellipsoidal shape of the Earth, which are already known. It doesn't come to the same thing as the introducing the gravimetrical corrections to the vertical deflection for these latter ones take account mainly of the local geoid's folding caused by masses which are placed comparatively near. Values ξ and η introduced into the adjustment, are

$$\xi = \varphi - B - \Delta\xi gr - \Delta\xi s ,$$

where

φ - astronomical latitude,
B - geodetic latitude,
$\Delta\xi gr$ - gravimetrical correction,
$\Delta\xi s$ - correction resulting from the regional folding of quasigeoid.

Such defining of the values η and ξ allows to apply the condition $\Sigma(\xi^2+\eta^2)=min$, so we have a smaller risk that the ellipsoid fitting the best the measured area would considerably differ from the ellipsoid appropriate to the whole Earth.

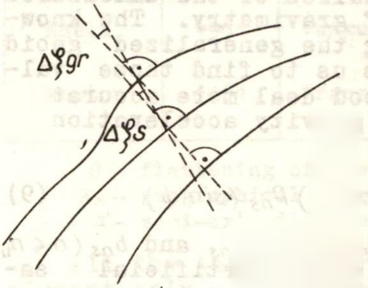

Fig. 1

Let us investigate, on the example of the Krassowski's ellipsoid, what is the effect of taking into account of the third spherical harmonics on the results of the arc measurements. The so called "O'Keefe's pear" which is defined by the formula (14) we are going to call surface Γ.

If the measurements, made of the surface Γ and reduced on the ellipsoid E give a defined value of the ellipsoid's flattening $\alpha = (\frac{1}{298,3})$, it follows that in a certain range of latitude (in that one where the measurements are done) the curvature of the meridian section of the surface Γ coincides with the curvature of the ellipsoid E.

Simplifing, we can imagine that „Γ" arises by the superposition of the third spherical harmonic P_3 on an ellipsoidal surface, which would be the correct, wanted ellipsoid which we search for and call E_1.

$$E_1 + P_3 = \Gamma$$

where

E_1, P_3, Γ - are the functions of the polar coordinates ϕ, λ, r.

Denoting by $\varkappa_E, \varkappa_\Gamma$ the curvatures of respective surfaces we have

$$\varkappa_{E_{40<\varphi<50}} = \varkappa_{\Gamma_{40<\varphi<50}}$$

so

$$\varkappa_E - \Delta\varkappa_{P_3} = \varkappa_{E_1} .$$

The ellipsoid's curvature is a function of its flattening α, at the point of the given latitude, so knowing the value of α_E we know $\varkappa_E$ too. We can also compute the increment of the curvature $\Delta\varkappa_{P_3}$ as resulted from the superposing of the surface P_3 on the ellipsoid. On computing the increment of the curvature as a function of α we find

$$\alpha_{E_1} = \alpha_E - \frac{d\alpha}{d\varkappa} \cdot \Delta\varkappa_{P_3} . \qquad (13)$$

To determine $\Delta\mathscr{X}p_3$ let us use the O'Keefe formula for the gravity potential

$$U=\frac{A_{00}}{r}+\frac{A_{20}}{r^3}p_2+\frac{A_{30}}{r^4}p_3+\frac{A_{40}}{r^5}p_4. \qquad (14)$$

When we assume $A_{30}=0$, we obtain a surface coming near in form to the ellipsoid (under the condition that there is definite functional relation between the coefficients A_{20} and A_{40}, according to the formulae given below)

$$U=\frac{A_{00}}{r}+\frac{A_{20}}{r^3}p_2+\frac{A_{40}}{r^5}p_4, \qquad (15)$$

where

$$A_{00}=f.M,$$

$$A_{20}=-\frac{2}{3}f.M.J.a^2,$$

$$A_{40}=\frac{8}{35}f.M.D.a^4$$

and J, D are Jeffrey's coefficients

$$J=\alpha-\frac{1}{2}m+\alpha\left(\frac{1}{2}m-\frac{1}{2}\alpha\right),$$

$$D=\frac{7}{2}\alpha^2-\frac{5}{2}m\alpha,$$

and

$$m=\frac{\omega^2a^3(1-\alpha)}{f.M}.$$

When $\alpha=\frac{1}{298,3}$ and $a=6,378245$, thus we obtain

$$m=+0,0034497$$
$$J=+0,0016234$$
$$D=+0,0000104.$$

To find the coefficients for the formula (15) let us write it in form of

$$\frac{A_{00}}{Ur}+\frac{A_{20}}{Ur^3}p_2+\frac{A_{40}}{Ur^5}p_4=1 \qquad (16)$$

and calling $\frac{A_{i0}}{U}=B_i$ we have

$$B_0=\frac{f.M}{U}, \qquad (17a)$$

$$B_2=-\frac{2}{3}B_0Ja^2, \qquad (17b)$$

$$B_4=\frac{8}{35}B_0Da^4. \qquad (17c)$$

B_0 can be found from the Jeffrey's formula

$$\frac{U}{f.M}=\frac{1}{a}\left(1-\frac{2}{3}Jp_2+\frac{8}{35}Dp_4\right). \qquad (18)$$

Assuming $\phi=0$, for the same values of α and a we obtain

$$B_0=6,374790$$

$$B_2 = -0{,}2806564$$
$$B_4 = +0{,}0250806\,.$$

Now we are going to compute the curvature twice - at first assuming $B_3 = 0$, at second when $B_3 = +0{,}003998$ which corresponds to the value of $A_{30} = +0{,}25$, determined by O'Keefe

$$\varkappa = \frac{r^2 + 2r'^2 - rr''}{(r^2 + r'^2)^{3/2}}. \tag{19}$$

We can find r' and r'' from the formula (14), which can be written

$$r^5 = B_0 r^4 + B_2 r^2 p_2 + B_3 r p_3 + B_4 p_4, \tag{20}$$

$$r' = \frac{B_2 r^2 p_2' + B_3 r p_3' + B_4 p_4}{5r^4 - 4B_0 r^3 - 2B_2 r p_2 - B_3 p_3} \tag{21}$$

and neglecting in the numerator the small terms of the order of r'^2

$$r'' = \frac{(2B_2 rr'p_2' + B_2 r^2 p_2'' + B_3 r p_4'') - (2_0 r^3 r' - 12 B_0 r^2 r' - 2B_2 r p_2' - B_3 p_3')r'}{5r^4 - 4B_0 r^3 - 2B_2 r p_2 - B_3 p_3}. \tag{22}$$

The Legendrian functions and their derivatives are expressed by the formulae

$$p_2 = \frac{3}{2}\sin^2\phi - \frac{1}{2};\quad p_2' = \frac{3}{2}\sin 2\phi;\quad p_2'' = 3\cos 2\phi, \tag{23a}$$

$$\left.\begin{aligned} p_3 &= \frac{1}{2}(5\sin^3\phi - 3\sin\phi);\quad p_3' = \frac{3}{2}\cos\phi\,(5\sin^2\phi - 1),\\ p_3'' &= \frac{3}{2}\sin\phi\,(10\cos^2\phi - 5\sin^2\phi + 1),\end{aligned}\right\} \tag{23b}$$

$$\left.\begin{aligned} p_4 &= \frac{1}{8}(35\sin^4\phi - 30\sin^2\phi + 3);\quad p_4' = \frac{5}{2}\cos\phi\,(7\sin^3\phi - 3\sin\phi)\\ p_4'' &= -\frac{5}{2}\sin^2\phi\,(7\sin^2\phi - 3) + \frac{15}{2}\cos^2\phi\,(7\sin^2\phi - 1).\end{aligned}\right\} \tag{23c}$$

Substituting the numerical values, we obtain for $\phi = 40^\circ$ and $\phi = 50^\circ$

	$\phi = 40^\circ$	$\phi = 50^\circ$
p_2	+0,1197638	+0,3802360
p_2'	+1,4772117	+1,4772117
p_2''	+0,5209446	-0,5209446
p_3	-0,3002206	-0,0252336
p_3'	+1,2247665	+1,8648426
p_3''	+4,6303469	+2,5252334
p_4	-0,4275346	-0,3190045
p_4'	-0,1326649	+1,3636726
p_4''	+8,4393692	+8,0052490

assuming $B_3 = 0$

r	6,373957	6,372157
r'	-0,0102094	-0,0101981
r''	-0,0035368	+0,0036604
$\varkappa$	+0,1569757	+0,1568428

assuming $B_3 = +0,003998$

r	6,373952	6,372157
r'	-0,0101905	-0,0101693
r''	-0,0034650	+0,0037000
$\varkappa$	+0,1569743	+0,1568419.

Thus we get $\Delta\varkappa_{B_3}$ for $\phi = 40^\circ$ equal to -14.10^{-7}

$\phi = 50^\circ$ equal to -9.10^{-7}

To find the increment of the curvature in the function of flattening we use the formula for the meridional curvature

$$\varkappa = \frac{(1-e^2\sin^2\varphi)^{3/2}}{a(1-e^2)}, \tag{24}$$

where e is the eccentricity of the meridional ellipse

$$e = \sqrt{\frac{a^2-b^2}{a^2}}.$$

Developing the numerator in a power series, we have

$$\varkappa = \frac{\left(1-\frac{3}{2}e^2\sin^2\varphi + \frac{3}{8}e^4\sin^4\varphi \ldots\right)}{a(1-e^2)}.$$

Differentiating and neglecting the terms of the order of e^4 we obtain

$$d\varkappa = \frac{e\left(2-3\sin^2\varphi\right)+\frac{3}{2}e^3\sin^4\varphi}{a(1-e^2)^2}\,de. \tag{25}$$

Because

$$e^2 = 2\alpha - \alpha^2,$$

and

$$de = \frac{1-\alpha}{e}\,d\alpha,$$

so

$$d\varkappa = \frac{(2-3\sin^2\varphi + 3\alpha\sin^4\varphi)(1-\alpha)\,d\alpha}{a(1-2\alpha)^2},$$

thus we obtain

$$\Delta\alpha = \frac{a(1-2\alpha)^2\Delta\varkappa}{(2-3\sin^2\varphi+3\alpha\sin^4\varphi)(1-\alpha)}. \tag{26}$$

Taking

$$\Delta\varkappa = -14.10^{-7} \quad \text{for} \quad \phi = +40^\circ$$

$$\Delta\varkappa = -9.10^{-7} \quad \text{for} \quad \phi = +50^{\circ}$$

and considering the difference

$$(\varphi - \phi)'' \approx \varrho'' \cdot \alpha \cdot \sin 2\varphi ,$$

we obtain for $\phi = 40^{\circ}$ $\Delta\alpha = -0,0000118$ and next, the corrected value of flattening

$$\alpha_{E_1} = 0,0033523 + 0,0000118 = 0,0033641 = \frac{1}{297,26} ;$$

for

$$\phi = 50^{\circ} \qquad \Delta\alpha = -0,0000243 ,$$

$$\alpha_{E_1} = 0,0033523 + 0,0000243 = 0,0033766 = \frac{1}{296,16} .$$

The variation $\Delta\varkappa(\rho_3)$ caused by the presence of the term ρ_3, changes with φ differently than the variation $\Delta\varkappa(\alpha)$. For $\phi = 50^{\circ}$ the value $\Delta\varkappa$ smaller than for $\phi = 40^{\circ}$ gives a greater $\Delta\alpha$. Taking the above into consideration we can estimate that the ellipsoid which would be obtained from the same measurements from which the Krassowski's ellipsoid has been determined, but taking into account the term ρ_3 - would have the flattening near to $\frac{1}{297}$.

That conclusion concerns only the results of the measurements and computations from which the Krassowski's ellipsoid has been determined. Generally taken, the satellite determinations show that the Earth's flattening is quite near to the very value of $\frac{1}{298,3}$, the one which is given by Krassowski. We can admit that the harmony is being due to the systematical and accidental errors of the geometrical method. We can also admit that in this connection the value of the semi-axis a requires the correction too.

A part from those examples of the unidirectional reaction of one method upon the other one, we have to do with the mutual, two and three-sided correlations. Before the satellite methods appeared, Heiskanen and Vening-Meinesz [7] had presented the idea of creating an uniform world geodetic system based on the astronomical, geodetic and gravimetrical observations. The principal idea of that project was to use the gravimetrically determined absolute deflections of the verical as corrections for the astronomical coordinates. Following that way, I. Fischer [3] and later on W. Kaula [13] determined the size of the Earth as well as the mutual relations of the three great triangulate systems: American, Euro-Asian and Far-Eastern.

In 1961, Kaula [15] attempted to find the parameters of the geoid and the coordinate system common for the different continental datums on the basis of data obtained from the three mentioned methods. As a result of that work he got: the expression of the shape of the geoid in form of an expansion by spherical harmonics up to the 8th degree, the value of the equatorial radius a and finally the correlated positions of three main geodetic systems. The course of action has been as follows: the results obtained from the different methods have been formulated in form of 196 "observations" subject to adjustment. 122 conditions have been introduced into adjustment and as a result of that, the corrections for all of the "observations" as well as 11 unknowns have been obtained by the generalized method of least squares. "Observations" have been formed as follows: the gravimetrical data were applied to compute 81 coefficients of the spherical harmonics up to the 8th order. These 81 coefficients made up the gravimetrical observations.

The astro-geodetic data were applied to compute the distances N for the unitary areas $10^{\circ} \times 10^{\circ}$ (on the terrains covered by measurements). Then the differences ΔN between the adjacent squares have been formed and thus we got the next 106 "observations". Further three "observations" were obtained from the matching of geoid heights along the Amur River. The last 6 "observations" were: the secular motions of the eccentricity of the satellite's 1958 β (Vanguard 1) orbit and the secular motion of the node of satellite's 1957 β (Sputnik 2) orbit. The additional eleven unknowns were: the equatorial radius of the Earth's ellipsoid a, the ellipsoid's flattening and nine coordinates of the origins of the three mentioned geodetic systems (three coordinates for each of them).

The conditions have been formulated as follows: 106 conditions for the astrogeodetic ΔN, which have to be equal to the differences computed from the spherical harmonics; 6 conditions for the variations of the parameters of the satellite orbits, which have to be equal to the variations being due to the harmonics; 3 conditions for the Amur geoid match; 6 conditions for the harmonics of the low orders (10, 11, 21, 20) which have been omitted in the computation of ΔN; 1 condition connecting the value a with N_{gr} and N_{astr}.

The results obtaines by Kaula are following

$$a = 6378163 \pm 15$$
$$\alpha^{-1} = 298{,}24 \pm 0{,}01$$
$$\gamma_e = 978043{,}6 \pm 1{,}0 \ mgl.$$

Zonal harmonics according to the symbols used in the table

$$J_2 = +1082{,}61 \pm 0{,}06 \times 10^{-6}$$
$$J_3 = - \ 2{,}05 \quad 10 \times 10^{-6}$$
$$J_4 = - \ 1{,}43 \quad 06 \times 10^{-6}$$
$$J_5 = + \ 0{,}08 \quad 11 \times 10^{-6}$$
$$J_6 = + \ 0{,}20 \quad 05 \times 10^{-6}$$

But still such a method of "one kettle" may rise some objections. For instance, Kaula sets on the same level the results of the gravimetrical measurements covering about 20 of the globe s surface and the results of satellite observations being the most representative for the whole Earth. Further on, the interdependence transmits the errors from the weakest items (e.g. the Far-Eastern network covering an area where the geoid s surface is more folded) to the values which can be determined independently, with a high accuracy, as, for instance, the flattening from the satellite motion.

Therefore it seems reasonable, to introduce a hierarchy of observations, according, to the possibilities of each of the methods. First of all, taking into account the observations of the artificial satellites we can determine the general geoid's shape (the quasi-geoid). The response successive terms of the series expansion - is conditionned by the actual possibilities of the technics, observation and calculation. For instance, at present, the J_3 value is known with such a high accuracy, that it can be considered as a constant in adjusting the gravimetrical measurements. Assuming that special "gravimetrical" satellites will be launched it can be expected that we will be able to determine with a sufficient accuracy the harmonics of the 5th order (functions of ϕ as well as of λ). Introducing those values as constants into the computations, we can determine, according to the formula (10), the coefficients of terms of higher orders, even from incomplete gravimetrical data. Here we can introduce the conditions for N to be determined by the gravimetrical and astrogeodetic methods, what will bear only upon the adjustment of the observation errors. Having the shape of surface determined, we are able to find its size by the determination of the ellipsoid's parameters (according to the formula (12)) or of a complicated high order surface.

Such a proceeding would be in line both with the geodetic principle "from generality to details", and with the other one recommending that we ought not spoil the accurate observations by the less accurate ones.

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НОВЫЕ МЕТОДЫ И РЕЗУЛЬТАТЫ ИССЛЕДОВАНИЯ ФИГУРЫ ЗЕМЛИ

К р а т к о е с о д е р ж а н и е

В настоящей статье автор припоминает в общих чертах основы определения фигуры Земли методами: астрономо-геодезическим и гравиметрическим с обращением внимания на слабые места теории этих методов. В дальнейшем рассмотрены результаты исканий параметров фигуры Земли методом наблюдения искусственных спутников и приведены численные результаты. Представлены попытки об-

щего уравнивания данных, полученных тремя методами и приведены связанные с этой проблемой собственные концепции автора : введение в геометрический и гравиметрический метод результатов, полученных методом искусственных спутников и различный от принятого В. Каулем способа, способ общего уравнивания.

Введены формулы и выполнен расчет для исследования влияния наличия третьей шаровой гармоники P_3 на результат определения сжатия из измерения градуса. Вычислено, что при учете этой поправки величина сжатия эллипсоида Красовского была бы близкой $\frac{1}{297}$.