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RESULTS OF APPLYING THE TETRAHEDRON METHOD FOR SYNCHRONOUS OBSERVATIONS OF ECHO I MADE DURING MAY AND JUNE 1963

The tetrahedron method presented during the National Seminar on Satellite Triangulation /Zieliński 1965/ Poznań, April 1964 and during the Scientific Conference in Riga - February 1965 /Pachelski, Zieliński 1965/ consists in determination of coordinates of the vector connecting two observing stations, from two pairs of synchronous observations of the satellite made at these stations and the distance between the two observed positions of satellite in the space. The method took its name from the geometric body formed by four directions to the satellite, the line connecting the observing stations and the chord of the orbit. The length of the chord may be computed on the basis of known orbital elements and observation moments. The accuracy analysis shows that adopting the mean errors of orbital elements of satellite ECHO I from the publication "Smithsonian Astrophysical Observatory Special Report No 159":

$$m_\omega = \pm 0°5 \quad \text{/ perigee argument/}$$
$$m_{\Delta\omega/1d} = \pm 0°1 \quad \text{/variation in the perigee argument over 24 hours/}$$
$$m_\Omega = \pm 0°01 \quad \text{/right ascension of ascending node/}$$
$$m_{\Delta\Omega/1\,rev.} = \pm 0°001 \quad \text{/variation in right ascension of the ascending node during one revolution/}$$
$$m_i = \pm 0°01 \quad \text{/inclination of the orbital plane/}$$
$$m_{M_o} = \pm 0°2 \quad \text{/mean anomaly at the epoch } T_o/$$
$$m_e = \pm 0.0001 \quad \text{/eccentricity/}$$
$$m_n = \pm 0.00005 \text{ rev/1 d. /diurnal mean motion/}$$
$$m_a = \pm 1 \text{ km} \quad \text{/semi-major axis/}$$

the length of the orbit chord corresponding to the change in the satellite position during 2 minutes, which amounts approximately to 800 km, can be determined with a mean error of $\pm$ 169 m.

Yet, computations made with the aid of the tetrahedron method show that the orbit chord may be determined with a markedly higher accuracy. And so, the length of the chord for the same moment was being calculated on the basis of two different sets of elements taken from the SAO Special Report, varying in epoch by 24 hours. These elements are partially independent of each other since each set was based on observations performed during 48 hours, this causing that the computation of elements for the next date a half of observational data was replaced. The differences between the lengths of chords computed in this way did not exceed 80 m. The following formulae /Gaposchkin 1964/ had been used for computations /chord length is denoted through l/:

1. Reduction of mean elements to the observation moments:

$$\omega = \omega_0 + \Delta\omega \cdot \Delta t$$
$$\Omega_s = \Omega_0 + \Delta\Omega \cdot \Delta t$$
$$i_s = i_0 + \Delta i \cdot \Delta t$$
$$e = e_0 + \Delta e \cdot \Delta t$$
$$n = n_0 + n' \cdot \Delta t$$
$$M = M_0 + n \cdot \Delta t$$

2. Computation of the true eccentric anomaly and of the latitude argument:

$$E = M + e \sin E$$

$$\sin v = \sqrt{\frac{1 - e^2}{1 - e \cos E}} \cdot \sin E$$

$$\cos v = \frac{\cos E - e}{1 - e \cos E}$$

$$u_s = \omega + v$$

3. Calculation and introduction of perturbation corrections taking into account the effect of the Earth oblateness:

$$\delta u = \frac{J}{p^2} \left[\frac{1}{2} \left\{ \left(-1 + \frac{7}{6} \sin^2 i \right) \sin(2\omega + 2v) + e \left[\left(-1 + \frac{5}{3} \sin^2 i \right) \sin(2\omega + v) \right. \right. \right.$$
$$\left. + \frac{1}{3} \left(-1 + \sin^2 i \right) \sin(2\omega + 3v) \right\} - \frac{1}{3} \left(-1 + \frac{3}{2} \sin^2 i \right) \left[\left(1 - \sqrt{1 - e^2} \right) \sin v \cos v \right.$$
$$\left. \left. + e^3 \frac{\sin v}{\left(1 + \sqrt{1 - e^2} \right)^2} \right] + \left(v - M + e \sin v \right) \left(-2 + \frac{5}{2} \sin^2 i \right) \right\} \right]$$

$$\delta r = \frac{J}{3p} \left[\left(-1 + \frac{3}{2} \sin^2 i \right) \left(1 - \frac{1 - e \cos E}{\sqrt{1 - e^2}} + \frac{e \cos v}{1 + \sqrt{1 - e^2}} \right) + \frac{1}{2} \cos(2\omega + 2v) \sin^2 i \right]$$

$$\delta \Omega = \frac{J}{p^2} \cos i \left[-\left(v - M + e \sin v \right) + \frac{1}{2} \left\{ \sin(2\omega + 2v) + e \left[\sin(2\omega + v) + \frac{1}{3} \sin(2\omega + 3v) \right] \right\} \right]$$

$$\delta i = \frac{1}{2} \sin i \left(\frac{J}{p^2} \cos i \right) \left[\cos(2\omega + 2v) + e \left\{ \cos(2\omega + v) + \frac{1}{3} \cos(2\omega + 3v) \right\} \right]$$

where

$$a = \left(\frac{k}{n^2} \right)^{1/3} \left[1 + \frac{1}{3} \frac{J}{p^2} \sqrt{1 - e^2} \left(-1 + \frac{3}{2} \sin^2 i \right) \right]$$

$$p = \left(\frac{k}{n^2} \right)^{1/3} \left(1 - e^2 \right)$$

$$k = 0.7537172 \cdot 10^5 \text{obr}^2 \text{mgm}^3 \text{d}^{-2}; \qquad J = 0.0660546 \text{ mgm}^2$$

$$\sin i = \sin i_s + \delta i \cos i_s$$
$$\cos i = \cos i_s - \delta i \sin i_s$$
$$\sin u = \sin u_s + \delta u \cos u_s$$
$$\cos u = \cos u_s - \delta u \sin u_s$$
$$r = a \left(1 - e \cos E \right) + \delta r$$

4. Computation of satellite coordinates:

$$x = r \cos u \cos \Omega - \sin u \sin \Omega \cos i$$
$$y = r \cos u \sin \Omega + \sin u \cos \Omega \cos i$$
$$z = r \sin u \sin i$$

5. Calculation of the length of chord:

$$l = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2}$$

Having the length l and the directions from the points A and B, we proceed to solution of the tetrahedron, using the formulae known from the analytical geometry and compute the coordinates of the vector AB and its length. From the published data of the experimental network of satellite triangulation three pairs of stations have been chosen for which several tetrahedrons could be generated, maintaining a two-minuts interval between observations, that is: Poznań - Riga /7 tetrahedrons/, Ushgorod-Riga /7/ and Nikolayev-Riga /5/. In Table 1 results of observations and the computed lengths or orbit chords are given. Table 2 contains mean values of coordinates and lengths of vectors found by means of solving each of the tetrahedrons listed in Table 1; Table 2 presents, moreover, their mean errors and also mean errors of individual observations. For comparison, coordinates and lengths of vectors calculated from geodetic coordinates referred to Hayford ellipsoid are given.

Table 2

		Δx	Δy	Δz	L
POZNAŃ – RIGA	1	−548.707	+288.688	+292.790	685.672
	2	−548.122	+290.055	+293.260	685.981
	3	−551.866	+287.922	+291.628	687.388
	4	−546.277	+293.059	+294.973	686.521
	5	−548.229	+289.066	+292.964	685.523
	6	−549.017	+289.112	+292.739	686.077
	7	−549.030	+289.125	+292.753	686.097
means		−548.750	+289.575	+293.015	686.180
errors of means		± 0.630	± 0.628	± 0.378	± 0.235
errors of individual observation		± 1.666	± 1.662	± 1.001	± 0.622
the geodetical collaboration		−548.165	+289.214	+292.920	685.515
USHGOROD – RIGA	8	−723.572	−180.591	+559.293	932.190
	9	−724.251	−180.595	+559.014	932.551
	10	−723.398	−181.098	+558.740	931.822
	11	−723.458	−180.829	+558.774	931.836
	12	−723.535	−180.881	+558.914	931.990
	13	−723.595	−180.900	+559.024	932.106
	14	−723.675	−180.943	+559.020	932.174
means		−723.641	−180.834	+558.968	932.096
errors of means		± 0.107	± 0.070	± 0.070	± 0.094
errors of individual observation		± 0.284	± 0.184	± 0.185	± 0.250
the geodetical collaboration		−723.535	−180.907	+558.809	931.932
NIKOLAYEV – RIGA	15	−514.844	−886.937	+683.295	1232.320
	16	−515.021	−887.159	+683.185	1232.493
	17	−514.521	−887.551	+682.855	1232.384
	18	−514.756	−887.084	+683.078	1232.269
	19	−515.045	−887.274	+683.175	1232.580
means		−514.837	−887.201	+683.118	1232.409
errors of means		± 0.096	± 0.103	± 0.074	± 0.057
errors of individual observation		± 0.214	± 0.231	± 0.166	± 0.127
the geodetical collaboration		−514.719	−887.144	+683.150	1232.336

The obtained results fully confirm the conclusions of the theoretical analysis and of testing computations on the accuracy of the segment l . As seen from Table 2, a marked difference in accuracy exists between the pair Poznań-Riga and the two other pairs of stations. This is to be explained by the situation of the direction connecting the stations with regard to the orbit plane at the observation moment. Directions Riga-Uzhgorod and Riga-Nikolayev are almost perpendicular to orbit plane while the direction Riga-Poznań is rather parallel to this plane. In the latter case, the tetrahedron formed by the observed directions with regard to the satellite has a very flattened shape and the observation errors have then a very strong effect upon results.

In the two remaining cases, the mean errors of individual observations amount to $\pm$ 250 m and $\pm$ 127 m respectively. Since these errors are the function of both the computation accuracy of segment l and the observation accuracy α and δ , it may be stated that they fall within the limits foreseen by the theory.

x x x

Literature

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2. J.B.Zieliński – "Zasada obliczenia triangulacji przestrzennej wykonanej za pomocą obserwacji synchronicznych na podstawie częściowej znajomości elementów orbity". Biul.P.O.S.S. 1965 Nr 13
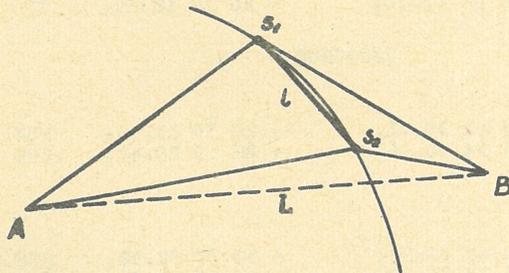3. E.M.Gaposohkin – "Differential orbit improvement", SAO Special Report 1964 Nr 161

Fig. 1

	alfa	delta	alfa	delta
	POZNAŃ		RIGA	

1
Date of obs. 6 months 2 d
23 16 20,	301 12 02.05,	+ 17 17 05.16,	286 22 51.78,	+ 10 08 28.52,
23 18 21,	318 04 38.54,	+ 18 25 17.88,	304 25 36.94,	+ 12 56 40.77,
1 = 777.179				

2
6 months 4 d
23 16 19,	303 55 28.56	+ 21 46 13.47,	288 06 59.09,	+ 14 20 08.57,
23 18 16,	320 27 22.78,	+ 20 42 48.33,	306 20 25.29,	+ 15 17 08.77,
1 = 750.801				

3
6 months 6 d
23 12 23,	264 14 02.71,	+ 21 39 12.20,	252 31 44.39,	+ 10 55 29.77,
23 14 24,	288 16 31.21,	+ 25 06 02.75,	272 17 52.72,	+ 15 21 48.48,
1 = 778.848				

4
6 months 6 d
23 14 24,	288 16 31.21,	+ 25 06 02.75,	272 17 52.72,	+ 15 21 48.48,
23 16 15,	307 46 13.99,	+ 24 16 59.69,	291 26 09.41,	+ 16 54 47.25,
1 = 712.856				

5
6 months 13 d
22 06 20,	215 20 56.61,	+ 18 22 31.45,	212 59 51.87,	+ 8 12 43.45,
22 08 17,	236 45 40.48,	+ 25 44 22.27,	228 39 55.00,	+ 13 13 35.25,
1 = 762.968				

6
6 months 13 d
22 08 17,	236 45 40.48,	+ 25 44 22.27,	228 39 55.00,	+ 13 13 35.25,
22 10 23,	265 07 26.83,	+ 29 03 38.44,	250 06 29.10,	+ 17 02 51.00,
1 = 817.855				

7
6 months 13 d
22 10 23,	265 07 26.83,	+ 29 03 38.44,	250 06 29.10,	+ 17 02 51.00,
22 12 16,	288 09 45.70,	+ 26 37 18.29,	270 57 54.09,	+ 17 29 49.46,
1 = 730.593				

	UZHGOROD		RIGA	

8
6 months 3 d
22 16 25,	297 41 55.71,	+ 20 59 13.42,	284 10 11.68,	+ 6 15 43.32,
22 18 25,	314 34 00.36,	+ 22 19 29.47,	299 55 33.30,	+ 9 43 00.53,
1 = 772.088				

9
6 months 4 d
23 16 19,	301 45 10.37,	+ 33 57 02.52,	288 06 59.09,	+ 14 20 08.57,
23 18 16,	322 35 31.00,	+ 30 56 38.06,	306 20 25.29,	+ 15 17 08.77,
1 = 750.801				

10
6 months 15 d
22 16 15,	322 24 31.61,	+ 17 18 43.40,	308 57 56.77,	+ 5 20 35.35,
22 18 16,	330 23 12.88,	+ 10 18 24.17,	318 32 00.75,	+ 0 44 53.67,
1 = 775.185				

11
6 months 17 d
22 04 20,	196 08 05.96,	+ 28 24 08.22,	209 50 48.71,	+ 12 09 47.04,
22 06 16,	215 30 47.82	+ 36 16 37.50,	226 42 55.50,	+ 15 05 37.70,
1 = 758.758				

12
6 months 17 d
22 06 16,	215 30 47.82,	+ 36 16 37.50,	226 42 55.50,	+ 15 05 37.70,
22 08 20,	246 42 51.52,	+ 40 30 56.18,	248 26 38.01,	+ 15 44 07.57,
1 = 807.193				

13
6 months 17 d
22 08 20,	246 42 51.52,	+ 40 30 56.18,	248 26 38.01,	+ 15 44 07.57,
22 10 23,	278 58 57.68,	+ 35 21 38.53,	270 07 38.25,	+ 13 00 34.27,
1 = 797.159				

14
6 months 17 d
22 10 23,
22 12 22,
1 = 768.319

278 58 57.68,	+ 35 21 38.53,	270 07 38.25,	+ 13 00 34.27,
300 27 23.44,	+ 25 34 30.65,	287 38 36.04,	+ 8 20 41.24,

NIKOLAYEV	RIGA

15
6 months 4 d
21 12 23,
21 14 24,
1 = 785.811

244 06 57.39,	+ 4 20 14.47,	252 34 04.22,	- 9 30 13.24,
263 39 41.73,	+ 14 23 40.71,	266 09 53.82,	- 3 13 05.48,

16
6 months 4 d
21 14 24,
21 16 20,
1 = 750.679

263 39 41.73,	+ 14 23 40.71,	266 09 53.82,	- 3 13 05.48,
283 57 30.32,	+ 21 21 13.39	279 48 14.44,	+ 2 07 28.04,

17
6 months 5 d
22 20 24,
22 22 15,
1 = 711.229

329 48 01.73,	+ 31 24 36.76,	317 09 36.43,	+ 12 30 43.85,
340 55 16.22,	+ 26 17 25.79,	327 37 29.69,	+ 11 08 04.17,

18
6 months 9 d
22 08 19,
22 10 24,
1 = 812.689

200 25 51.20,	+ 16 56 02.95,	222 23 59.43,	+ 1 47 40.98,
213 26 37.08,	+ 27 58 06.74,	238 16 20.78,	+ 8 02 18.67,

19
6 months 9 d
22 10 24,
22 12 16,
1 = 725.273

213 26 37.08,	+ 27 58 06.74,	238 16 20.78,	+ 8 02 18.67,
232 28 22.20,	+ 38 59 47.96,	255 53 37.00,	+ 12 56 54.46,