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Janusz B. Zieliński

APPLICATION OF THE RADIUS-VECTOR OF ARTIFICIAL SATELLITE AS LENGTH MEASURE FOR GEODETIC PURPOSES

Barbara Kołaczek, Jan Bieniewski

NEW PROGRAMME OF LATITUDE OBSERVATIONS AT JOZEFOSLAW

Jan Bieniewski

OBSERVATION OF A PAIR OF BRIGHT STARS USING THE HORREBOW -TALCOTT METHOD

Jan Bieniewski

TABLES FOR COMPUTATION OF AZIMUTH FROM POLARIS OBSERVATIONS USING THE HOUR ANGLE METHOD

Jan Bieniewski

CONTROL OF INSTRUMENTAL ERRORS OF THE ZENITH TELESCOPE

WYDAWNICTWA POLITECHNIKI WARSZAWSKIEJ • WARSZAWA 1968

SPIS RZECZY - СОДЕРЖАНИЕ - CONTENTS

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ZASTOSOWANIE PROMIENIA WODZĄCEGO SZTUCZNEGO SATELITY ZIEMI JAKO MIARY DŁUGOŚCI DLA CELÓW GEODEZYJNYCH

Wykorzystując trzecie prawo Keplera, jest możliwe wyznaczenie wielkiej półosi orbity na podstawie zaobserwowanego okresu obiegu.

Jeżeli znany jest także mimośród orbity, mamy możność obliczenia długości promienia wodzącego satelity na dowolny moment, nawet jeżeli pozostałe elementy znane są tylko w przybliżeniu.

Niniejsza praca poświęcona jest przede wszystkim zbadaniu stopnia dokładności jaki może być osiągniety przy wyznaczeniu promienia wodzącego.

Zakładając, że mimośród orbity jest bliski zera, wysokość perigeum 1000 + 3000 km oraz, że satelita ma mały stosunek powierzchni (masa, promień wodzący może być obliczony, jak wykazano w rozdz. VI, z dokładnością 10^{-5}.

Niedostateczna znajomość współczynników harmonik tesseralnych ziemskiego pola grawitacyjnego jest główną przeszkodą w osiągnięciu wyższej dokładności.

Pozostałe źródła perturbacji, takie jak: błędy współczynników harmonik zonalnych, opór atmosfery, ciśnienie światła, przyciąganie Słońca i Księżyca wpływają w sposób o wiele mniej znaczący.

Podany jest także sposób obserwacji i obliczeń eliminujący wpływ błędów stacji obserwacyjnych na wyznaczenie okresu.

Rozdział VIII zawiera niektóre sposoby wykorzystania znanej długości promienia wodzącego dla wyznaczenia współrzędnych punktów na powierzchni Ziemi w układzie współrzędnych zorientowanych zgodnie z kierunkiem osi obrotu Ziemi i płaszczyzną równika.

CHAPTER I

CONCEPTION OF THE METHOD AND ITS MAIN DEPENDENCES

We know from the experience of natural sciences that the time intervals may be measured with a considerably higher accuracy than the distances. Clocks that are able to maintain during several weeks their rate constancy of the order of 10^{-9}, do not belong any more to exceptions today, while the geodetic measurements of an accuracy of 10^{-6} represent, again and again, a complicated technical problem. That is why, numerous attempts are being made with a view to replacing the direct method of distance measurement by time measurement methods, which is, in a given physical phenomenon, the function of length of the segment involved. Among these methods range the measurements of distances made with the help of radio and light waves propagating with the known speed.

The astronomy knows another phenomenon presenting a strictly determined relation between the time and the distance - this is the undisturbed Kepplerian motion in which the semi-major axis of the orbit a is related to the period of revolution P by the third Law of Kepler

$$\frac{a^3}{P^2} = \text{const} . \qquad (1.1)$$

The leading idea of the present study is to show the attempts of utilizing this dependence. Of course, artificial satellites will be called into play here, for their orbits have dimensions comparable to those of the Earth and may be shaped by the man in order to attain optimal conditions. In addition, the artificial satellites first of all remain under the influence of the gravitation field of the Earth, this allowing for the other essential feature of this method to be stated, i.e. the possibility of referring the involved measurements to the center of the Earth's mass.

One of the main tasks of the contemporary higher geodesy is to obtain possibilities of determining homogeneous coordinates of points on the whole Earth's surface in a system whose position with regard to the fundamental physical elements of the Earth body - its mass center and axis of revolution - would be known. A particular and the most advantageous case would be offered by a system whose origin were to coincide with the center of the Earth's mass, and one of its axis - with the Earth's revolution axis. An additional condition should be that the scale of such a World Geodetic System be the same everywhere.

The fulfillment of the above conditions by means of methods of the classical geodesy proves to be difficult not only because of the distance between the different continents or the separate geodetic systems or the necessity of gathering an immense quantity of observational data, but also because the computation and adjustment of extensive triangulation networks require the adoption of one or another hypothesis concerning the inner structure of the Earth. The appearance of satellite methods have opened new and vast possibilities. Using the satellite geometric methods, such as satellite triangulation,we perform the measurements of directions in the system of astronomical coordinates - the declination and the right ascension - it means, in such a system whose orientation with regard to the axis of the Earth's revolution, the instantaneous or the mean one, is exactly known. In the dynamic methods,the center of the Earth's mass is directly involved. A short review and the dicussion of results obtained by this method are presented in Chapter III. The work done so far does not, of course, exhaust all possibilities offered by satellite methods. It is, therefore, desirable to proceed to the improvement of the existing and the search for new methods in this domain.

The method proposed in this study is different from methods applied so far, which are described in Chapter III. In those methods, the quantities serving as starting points for further computations and ultimate results were always the instantaneous three-dimensional coordinates of the satellite. Comparision, adjustment and transformation of these coordinates led to the final result represented by the geocentric coordinates of the observing station. In our method, the only basic element will be the radius-vector. Its length changes much more slowly - especially in orbits of small eccentricity - than the coordinates which vary literally with a cosmic velocity. The idea of this method has sprung up in 1963, after Poland had taken the engagement to count the ephemerides of the satellite Alouette (1962 Beta Alfa 1) for the socialist countries participating in the multilateral scientific cooperation. The project for the respective observations,described in Chapter VIII, was submitted to the Conference of Observers of Artificial Earth Satellites in Moscow, 1963. However, the specificity of this kind of research work is, that it cannot be performed on a small scale; if we want to obtain reliable scientific results, the collaboration of a great number of observing and computing agencies is required, this being possible only within the framework of an international cooperation. Before undertaking and, the more, before initiating such a collaboration, a thorough analysis of the whole problem is indispensable. During the course of the work,this problem has evolved to the present - a more general - form.

In this paper the application of the theory of radius-vector is being treated in broad outlines. Among the projects mentioned in Chapter VIII only one possesses a full analysis of accuracy, namely, the project for which we succeeded in using the substitutional observational data: the synochronous observations of Echo I. The main topic is being devoted to the theory itself and to the problem: which perturbations ought to be taken into account and what is the accuracy with which we are able to compute them?

Let us now go over to a brief discussion of the theory.

An element characterizing the linear dimensions of the orbit is its semi-major axis a. We shall determine it by measuring the period of revolution or - what comes to the same - the mean motion n. Introducing the mean motion, we can write down the third Kepler's Law

$$n^2 \cdot a^3 = \mu \ , \qquad (1.2)$$

where

$n = \frac{2\pi}{P}$.

In this formula μ designates the mass of the Earth (the case being confined to the motion of artificial satellites) multiplied by the gravitation constant. Let us first see what effect the error of the period measurement will have on the accuracy of a.

Differentiating (1.2) we have

$$d_a = \frac{\mu P\, dP}{6\, a^2 \pi^2} . \qquad (1.3)$$

Assuming that dP = $0^s\!.01$, this being not at all an exorbitant accuracy for observing possibilities, we obtain for orbits at altitudes from 1000 to 3000 km

$$d\,a = 7\ m .$$

It follows from the ulterior analysis as well as from numerous publications (for instance: Satellite Orbital Data SAO Spec. Rept.), that the accuracy of period equal to $0^s\!.01$, this corresponding to the accuracy n : 10^{-6}, would be rather the inferior limit of accuracy. Whereas, the error: 7 m corresponding also to the relative accuracy: 10^{-6}, is from the geodetic point of view quite satisfactory. A further reduction of the value of d a will give no effects becasue it would be below the level of observation errors.

The quantity μ is also encumbered with a certain error. The latter has the following bearing upon the semi-axis

$$d\,a = \frac{a \cdot d\mu}{3\mu} . \qquad (1.4)$$

Assuming that μ is equal to 398603 $km^3 sec^{-2}$ (see Chapter V) and that d μ is equal to 3 $km^3 sec^{-2}$, we obtain for

$$H = 1000\ km,\ da = 18\ m$$

$$H = 3000\ km,\ da = 23\ m .$$

The quantity μ and its accuracy is of a particular importance for the present theory. It defines the scale of the system or of the geometrical construction which will be considered in some concrete case. Anyway, the error of μ will appear as a systematic error not bringing about local deformations of a given construction. We derive the radius-vector from semi-major axis using the formula

$$r = a(1 - e \cdot \cos E) . \qquad (1.5)$$

This brief discussion leads to the conclusion that the accuracy we have a chance of attaining by this method is of the order of 10^{-6}, this amounting to 7 m - 10 m, according to the altitude of orbit. And so, we shall adopt the following rule with regard to the need of introducing perturbation corrections and their accuracy: corrections will be considered negligible, if they give a perturbation smaller than one meter in radius-vector, while they will be considered sufficiently precise, if the accuracy is better to one meter.

However, not all types of orbits and, naturally, not all types of satellites can be chosen for this method. In order to avoid rapid changes in the length r, we shall utilize orbits with small eccentricities of the type ANNA Ib and ALOUETTE, those being smaller than 0,01. For possible elimination of disturbing effects of the atmosphere, we shall fix the inferior limit of the altitude to 1000 km over the Earth's surface and the upper limit - to 3000 km. Further, we admit that the satellite to be dealt with is of the type called heavy vehicle, of a low area/mass ratio, similar to ANNA or ALOUETTE.

For computing the length of the radius-vector, we are going to adopt the following assumption: we shall admit that we have at our disposal an appropriate set of observations well located in space and time, permitting for the mean elements of the orbit to be computed with satisfactory accuracy (in accordance with the definition given in Chapter II), as well as their variations. Thus, we cease dealing with all the long-period perturbations, since they would be contained in variations of mean elements. Only the short-period and diurnal perturbations will need to be taken into account. It seems purposeful to indicate here that we are using for the computation of perturbations caused by consecutive harmonics of the terrestrial potential of gravitation, the values of coefficients of harmonics determined from satellite observations. Yet we do by no means enter in this way the "vicious circle", since those determinations are mainly based on observed long-period variations, which are greater than short-period variations computed here.

CHAPTER II

BASIC FORMULAE AND DEFINITIONS OF THE THEORY OF MOTION OF ARTIFICIAL EARTH SATELLITES

A full presentation of the theory of motion of the artificial Earth satellite in all possible approaches overruns the scope of this work and provides rather material for a fairly voluminous handbook. Hence, we are going to confine ourselves to formulae and definitions that are required for the subject discussed here. The motion of satellite proceeds in the field of action of various forces that are shaping its trajectory. The most important among them is the force of gravitation which comes mainly from the attraction exerted by our globe.

We shall assume that the mass of the satellite is negligibly small; further, we shall not take into account its dimensions nor the motion around its center.

Let us adopt a coordinate system x, y, z, whose origin coincides with the center of the Earth mass.

In a system defined in this way, the equations of motion have the form

$$\left.\begin{aligned} m\frac{d^2 x}{dt^2} &= Fx \\ m\frac{d^2 y}{dt^2} &= Fy \\ m\frac{d^2 z}{dt^2} &= Fz \ , \end{aligned}\right\} \qquad (2.1)$$

where

Fy, Fy, Fz - are the vector components of the force acting on the satellite and

m-is the mass of satellite.

If we limit ourselves to the force of attraction of the Earth and introduce the function

$$U = k\int\frac{dM}{r} \ ; \qquad (2.2)$$

where

dM - is the element of the Earth's mass,

k - is the constant of gravitation,

$r - (x^2 + y^2 + z^2)^{1/2}$;

and if we consider the mass of the satellite to be negligibly small as compared to the mass of the Earth, the equations of motion (2.1) will take the shape

$$\left.\begin{aligned}\frac{d^2x}{dt^2} &= \frac{\partial U}{\partial x}\\ \frac{d^2y}{dt^2} &= \frac{\partial U}{\partial y}\\ \frac{d^2z}{dt^2} &= \frac{\partial U}{\partial z}\ .\end{aligned}\right\} \tag{2.3}$$

The function U will be called the terrestrial potential of gravitation.

The examination of the motion of the satellite consists in the solution of those equations through their integration either by the analytical or the numerical method. This solution comprises six integration constants; they may be either initial coordinates and velocities at the moment t_o, or orbital elements. Let us now assume that the function U has a simple form:

$$U = \frac{\mu}{r}\ , \tag{2.4}$$

where

$\mu = k \cdot M.$

This means that the whole mass of the attracting body (Earth) is concentrated at one point - at the center of the mass. The integration of equations of motion lead then to the three Kepler's Laws while the integration constants will consist of six orbital elements. Such a motion will be termed Keplerian motion, and all deviations from this motion will be called p e r t u r b a t i o n s. The Kepler's Laws that will be often referred to in the present paper, have, as we know, the following form:

1° The trajectory of the body attracted in space (satellite, for instance) it a conical curve (an ellipse - in the case of an Earth satellite), whereas the attracting body (the center of the Earth - in our case) is in the focus.

To this corresponds the equation

$$r = \frac{a(1 - e^2)}{1 + e \cos v} , \qquad (2.5)$$

which is an equation of ellipse. The meaning of symbols will be explained below.

2° The areas of sectors traced by the radius-vector are proportional to the time during which they have been traced

$$\frac{A}{t - t_o} = c , \qquad (2.6)$$

A - area of sector

c - constant.

3° The cube of semi-major axis of orbit divided by the square of period is a constant quantity proportional to the quantity μ

$$\frac{a^3}{P^2} = \frac{\mu}{4\pi^2} \quad \text{or} \quad a^3 n^2 = \mu \quad . \qquad (2.7)$$

In the above formulae there appear some elements (which will be used hereinafter) that correspond to Keplerian elements applied to motion of planets:

Ω - right ascension of the ascending node

i - inclination of the orbital plane to the equator

ω - argument of perigee

v - true anomaly

e - eccentricity of the ellipse

a - semi-major axis of the ellipse.

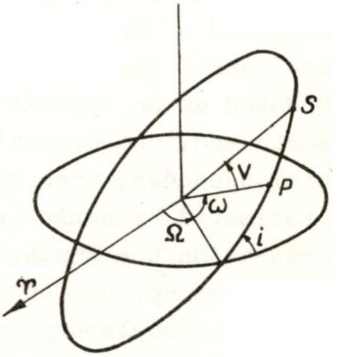


Fig. 1

A different choice of integration constants is, naturally, possible; in such a case, the orbital elements will be differently defined. Now and then this might be necessary, e.g., for near zero values of eccentricity e when the angle ω

becomes indetermined, or for i being near zero when Ω becomes undetermined.

Apart from the above elements we are going to use some other elements or quantities being functions of the afore-mentioned, and so:

u - argument of latitude $= \omega + v$

r - radius-vector $= a\,(1-e\cos E)$ (2.8)

P - anomalistic period $=\sqrt{\dfrac{a^3 \cdot 4\pi^2}{\mu}}$

n - mean motion $= \dfrac{2\pi}{P}$

M - mean anomaly $= n \cdot t$

E - eccentric anomaly related to the mean anomaly by the Keplerian equation

$$E = M + e \sin E \tag{2.9}$$

and with the true anomaly - by means of the formulae

$$\mathrm{tg}\,\frac{v}{2} = \sqrt{\frac{1+e}{1-e}} \cdot \mathrm{tg}\,\frac{E}{2}\,. \tag{2.10}$$

The real motion of an artificial Earth satellite differs, however, quite distinctly from the Keplerian motion - considerably more than motions of planets, for instance. The greatest perturbations are provoked by the very fact that not only the mass of our globe is not concentrated at its center but that it does neither represent a uniform sphere nor even a globe composed of homogeneous spherical layers. In reality,the Earth has rather the form of an ellipsoid and even this should be treated as an approximation. In more accurate approaches the Earth and, consequently, the field of its gravitation potential dependent upon the distribution of the mass should be approximated by series expansions, as shown in Chapter V.In this connection the Kepler's orbit can serve us merely as an auxiliary concept not actualized in reality. A Keplerian orbit which is osculatory to the real trajectory of the satellite at a given moment t, i.e. an orbit such as would develop if at that moment the disturbing forces suddenly disappeared,will be called o s c u l a t i n g o r b i t. So, we can imagine that the real orbit of a satellite consists of infinitesimal segments of osculating orbits, or that it is similar to the Keplerian orbit, yet with varying elements. Changes of these elements will be termed perturbations of elements.

Among the perturbations, we shall distinguish the followings ones: s e c u l a r perturbations progressively varying with time and p e r i o d i c perturbations. The latter are divided into: short-period perturbations occuring during one period of revolution, diurnal perturbations appearing during the day of the orbital plane (the period of time during which the Earth turns with regard to the orbital plane about 360 degrees), and l o n g - p e r i o d perturbations with a period equal to the period of change in the perigee argument about 2π.

In our practice with satellites, we have to do - next to the concept of the osculating orbit - with the mean elements. Different authors (Zhongolovich and Pellinen [1962], Tchebotariev [1963], Gaposchkin [1964]) are using this designation for different quantities, hence a precise definition appears to be necessary. We are going to adopt the definition given by Zhongolovich and used also by Gaposchkin, which seems to be the most convenient in our case. The p e r t u r b a t i o n c o r r e c t i o n will mean a correction including only short-period and diurnal perturbations. The mean element will be equal to the osculating element less the perturbation correction, i.e.

$$\varepsilon_o = \varepsilon_m + \delta_\varepsilon \ , \tag{2.11}$$

where

ε_o - osculating element

ε_m - mean element

δ_ε - perturbation correction.

That way, the changes of mean elements are cumulating secular variations produced both by the gravitation field of the Earth and any other possible cause.

Also the term "period of revolution" has various meanings.

Zhongolovich [1960c] defines the anomalistic period as an interval of time between two subsequent passages of the satellite through the perigee. Kozai [1959] uses an expression for the mean motion, defining by the same period

$$n = \left\{ \mu \cdot a^{-3} \left[1 - \frac{1}{3} \frac{C_{20}}{p^2} (1 - \frac{3}{2} \sin^2 i) \cdot \sqrt{1-e^2} \right] \right\}^{1/2} , \qquad (2.12)$$

where

a - is semi-axis of mean orbit,

p - orbit parameter;

thus, Kozai is including into it also the secular variation of the mean anomaly in epoch. For our purposes, the anomalistic period will signify the period of the osculating orbit in a given epoch, connected with the semi-major axis of that orbit by the equation

$$n^2 a^3 = \mu .$$

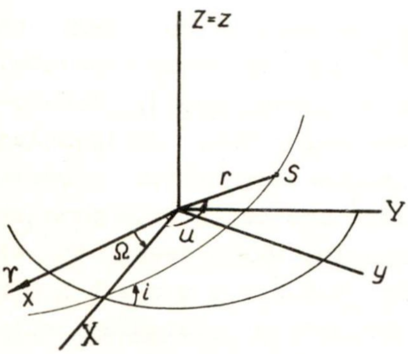


Fig. 2

We shall present also formulae for coordinates and velocities of the satellite in the function of orbital elements.

In the coordinate system X Y Z, with the origin at the center of the Earth's mass, with the Z-axis coinciding with the Earth's revolution axis, and with the X-axis directed to the ascending node of the orbit, we have the formulae

$$\left. \begin{aligned} X &= r \cdot \cos u \\ Y &= r \cdot \cos i \cdot \sin u \\ Z &= r \cdot \sin i \sin u . \end{aligned} \right\} \qquad (2.13)$$

Since

$$\left. \begin{aligned} x &= X \cdot \cos \Omega - Y \cdot \sin \Omega \\ y &= Y \cdot \cos \Omega + X \cdot \sin \Omega \end{aligned} \right\} \qquad (2.14)$$

the x-axis being in this system directed to the equinox. If we use instead of u the true anomaly v, then the formulae (2.15) will have the form [Deutsch, 1963, p. 3]

$$\left.\begin{aligned} x &= r\left[P_1 \cos v + P_2 \sin v\right] \\ y &= r\left[Q_1 \cos v + Q_2 \sin v\right] \\ z &= r\left[R_1 \cos v + R_2 \sin v\right] \end{aligned}\right\} \quad (2.16)$$

where

$$\left.\begin{aligned} P_1 &= \cos\omega \cos\Omega - \sin\omega \cos i \sin\Omega \\ P_2 &= -\sin\omega \cos\Omega - \cos\omega \cos i \sin\Omega \\ Q_1 &= \cos\omega \sin\Omega + \sin\omega \cos i \cos\Omega \\ Q_2 &= -\sin\omega \sin\Omega + \cos\omega \cos i \cos\Omega \\ R_1 &= \sin\omega \sin i \quad R_2 = \cos\omega \sin i. \end{aligned}\right\} \quad (2.17)$$

Substituting in (2.16) dependences

$$r \cos v = a(\cos E - e) \quad (2.18)$$

$$r \sin v = a\sqrt{1 - e^2} \sin E \quad (2.19)$$

we obtain

$$\left.\begin{aligned} x &= a\left[P_1 (\cos E - e) + P_2\sqrt{1 - e^2} \sin E\right] \\ y &= a\left[Q_1 (\cos E - e) + Q_2\sqrt{1 - e^2} \sin E\right] \\ z &= a\left[R_1 (\cos E - e) + R_2\sqrt{1 - e^2} \sin E\right] \end{aligned}\right\} \quad (2.20)$$

hence after differentiation

$$\left.\begin{aligned} \dot{x} &= \frac{a^2 n}{r}\left[- P_1 \sin E + P_2\sqrt{1-e^2} \cos E\right] \\ \dot{y} &= \frac{a^2 n}{r}\left[- Q_1 \sin E + Q_2\sqrt{1-e^2} \cos E\right] \\ \dot{z} &= \frac{a^2 n}{r}\left[- R_1 \sin E + R_2\sqrt{1-e^2} \cos E\right]. \end{aligned}\right\} \quad (2.21)$$

CHAPTER III

RECENT RESULTS OF ORBITAL METHODS

The possibilities of using artificial satellite observations for the determination of geocentric coordinates of points on the Earth's surface were known and had been theoretically worked out from the very moment of launching the first satellites. A detailed analysis of those possibilities was given by Veis [1960] who discussed in his work the different variants of solutions, including the purely geometric ones (synchronous observations) and presented formulae and an accuracy analysis. As to orbital methods, the following two had been discussed more broadly:

1° Simultaneous adjustment of orbital elements, their variations and coordinates of observing stations;

2° Preliminary computation of orbital elements from observations carried out by a great number of stations over a longer period and, then, utilization of those data for computing coordinates of a certain limited number of stations, using observations performed during a markedly shorter period.

Proposing the method 1°, Veis is avare of the difficulties connected with a simultaneous calculation of so many unknowns which, in addition, are correlated and do not form well conditioned equations. No wonder, therefore, that the application in practice of this solution appeared to be a difficult task, more difficult than the determination of the oblateness of the Earth, for instance. It is not long ago (in 1963) since the first results of such determinations were published. We are going to present here the results derived by Kaula [1963 a, 1963 b], Izsak [1964] and Veis [1964] - in order to see more clearly the possibilities and the inconveniences of the methods applied.

All those papers are based on observations performed with the help of Baker-Nun cameras belonging to the network of the Smithsonian Astrophysical Observatory. In addition, the coordinates of the same points had been determined, this facilitating the comparision of results. Kaula and Izsak are applying in their work the method 1^{o}. This method, for all the difficulties bound with it, is attractive because it permits to find the parameters of the Earth's gravitation field whose function are the variations of orbital elements. Fully realizing the difficulties of this method, Kaula assails his task with the help of the richly equipped mathematical apparatus. In the issue, tesseral harmonics up to the sixth order and zonal harmonics up to the seventh order have been determined, in addition to corrections for the origins of geodetic systems involving the respective observing stations. The six systems are:

The North-American Datum - extended over the area of the Central America and the northern part of South America. - 4 stations

The European Datum - extended over Africa and the South - West Asia - 4 stations.

The Japanese, the Australian and the Havaiian Datums - one station for each system.

Kaula does not give corrections of coordinates for individual stations, that could be compared with results obtained by the other two authors. We shall calculate the coordinates of stations by means of adding to the initial coordinates the coorections proper for a given Datum. In his first work [1963 a], Kaula is utilizing 2546 observations of three satellites, in his second work [1963 b] - 10996 observations of five satellites. It is interesting to note that both versions of Kaula have very similar results which are differring, however, from the two other solutions. Izsak proceeds analogically to Kaula, using even the same program for the evaluation of the effect of tesseral harmonics. Still, he does not compute corrections for geodetic systems, but for coordinates of the individual stations. Izsak is utilizing 15191 observations of 10 satellites. Apart from coordinates, Izsak determines 19 tesseral harmonics to the 6th order, presuming that the values of zonal harmonics are known from the results obtained by Kozai [1962].

The work of Veis is based on very rich observational data including 46538 observations of 14 satellites. The way of calculation applied by him is a modification of the method 2^o, consisting in a two-stage development. During the first stage, the orbital elements had been computed on the base of approximate coordinates of the station (only observations made by the station B-N had been taken into account). During the second stage, the corrections for coordinates of the station have been calculated on the basis of deviations between the positions found from the elements and the observed ones (O-C). As compared with the above solutions, a dissimilarity is to be seen especially in the approach to the gravitation effects. The orbit computational program contained only short-period perturbations caused by the oblateness of the Earth as well as the effects of the Moon and the Sun. All the other long-period variations have been determined from observations, using the least-squares method, and registered as variations of mean elements. The influence of tesseral harmonics, producing short-period alterations, had not been taken into account; the author assumed that with such a great quantity of observations this effect will have an accidental character. Finally, the geocentric coordinates of the station have been determined, and from them - the displacements of the respective Datums with regard to the geocentric system. As an additional result, Veis derived the value of the semi-major axis of the terrestrial ellipsoid and its flattening, as well as the contours of the geoid with regard to such an ellipsoid.

The Table 1, contains a comparision (differences) of results given in all four papers: KI - Kaula [1963 a], KII - Kaula [1963 b], I - Izsak, V - Veis. Coordinate accuracies for particular points correspond to the mean geometric accuracy of the three coordinates, in agreement with the rule adopted by Veis

$$m = \sqrt{m_x \cdot m_y \cdot m_z} \; .$$

An analysis of the Table 1 leads to certain conclusions. Primo: It confirms the capacity of the orbital method for achieving accuracies of the order of some tens of meters in geocentric coordinates, in other terms - the satellite orbit is a sufficiently good measuring "instrument" securing the necessary accuracy rates for the needs of the Higher Geodesy.

Table 1

Comparison of the results of coordinate determinations

by Kaula, Izsak and Veis

	mv	Δx	Δy	Δz	m_{KI}	Δx	Δy	Δz	m_{KII}	Δx	Δy	Δz	m_I
1. Organ Pass	± 15	- 32	- 8	- 45	±15	- 39	- 16	- 46	± 6	+ 1	+25	+60	± 5
2. Olifantsfontein	± 10	+ 36	- 24	+ 67	±21	+ 38	- 30	+ 62	± 5	+ 17	+53	- 3	± 5
3. Woomera	± 7	- 42	+ 65	+ 42	±38	- 56	+ 70	+ 54	± 7	+ 18	- 6	- 1	± 4
4. S. Fernando	± 20	+ 38	+ 64	+ 74	±21	+ 41	+ 59	+ 69	± 5	+ 63	-36	-28	± 9
5. Tokyo	± 18	-137	- 42	- 63	±18	-140	- 64	- 59	± 4	- 6	-37	-64	± 7
6. Naini Tal	± 30	- 23	- 53	- 52	±21	- 21	- 47	- 57	± 5	- 6	+27	+ 2	±20
7. Arequipa	± 15	- 37	+ 11	+ 2	±15	- 43	+ 2	+ 1	± 6	- 52	+26	- 8	± 8
8. Shiraz	± 20	+ 5	- 20	- 3	±21	+ 7	- 26	- 7	± 5	+ 33	-47	+54	± 7
9. Curacao	± 10	+ 15	+ 9	- 47	±15	+ 9	0	- 48	± 6	- 51	-33	-15	± 8
10. Jupiter	± 12	- 38	- 4	- 54	±15	- 45	- 12	- 45	± 6	- 16	+11	+10	± 6
11. Villa Dolores	± 12	-133	-236	+ 25	±32	-124	-172	+ 40	± 9	- 2	-15	+19	± 7
12. Mani	± 22	- 34	- 29	-191	±39	- 69	- 53	-161	± 16	+ 35	-63	+54	± 6

Secundo: The numbers in columns designated with the letter m, characterize in a certain way the correctness of the method and of the solution. And so, errors contained in the column m_{KI} are of the same order as errors given by Veis in the column m_V, although the number of observations utilized in the second case was almost 20 times higher. Errors of m_{KII} are distinctly smaller than those of m_V (some 2-3 times), in spite of the superior number of observations used for the latter determination. This seems to show that a simultaneous determination of many unknowns (it means: next to coordinates also gravitation parameters) allows for achieving a much better inner agreement than the handling with a smaller number of unknowns. On the contrary, the differences Δx, Δy, Δz surpass in all three columns markedly, and in some cases even many times, the given mean square errors. Thence it appears that the accuracy evaluations in m_{KI}, m_{KII} and m_I are too optimistic. Moreover, a comparision of harmonics $C_{m\ n}$ (Table 3) determined simultanecusly, shows that also there the agreement is rather problematic. It seems that the results obtained by Veis - based on the greatest number of observations - are nearer the truth, and that the accuracies given by him more actual.

Another very interesting example of the application of the orbital method is presented by Anderle and Oesterwinter [1963]. It is a report on Doppler's observations of the satellite Anna IB, carried out by six stations on the territory of the United States of America. There was a possibility of comparing the obtained results with the accurate triangulation network existing in the USA. An amazingly high accuracy has been achieved. The lengths of chords between particular points calculated from geodetic coordinates and observations of the satellite ANNA differred within the limits from 1 m to 23 m (the lengths of chords being 600-3500 km). There was only one point whose geodetic coordinates appeared to be inaccurate, and whose position has been corrected accordingly.

The presented examples show the existing great possibilities of the orbital method which ought to be developed and improved.

CHAPTER IV

COMPUTATION OF THE ORBIT AND ITS ACCURACY

The modes of computing orbits of artificial satellites from observations of directions - and only such will be here dealt with - are merely slightly modified methods of the classical astronomy, used for the calculation of orbits of minor planets and comets. We shall not discuss here the approximate methods, such as the Gauss's or Laplace's methods applied for the determination of approximate elements on the basis of several observations. But let us give more attenion to the method destined for the most accurate computation of elements from the observational data available, the so-called improvement of orbits. The principle of this method consists in expressing corrections to the observed coordinates α and δ by the function of corrections to the orbital elements

$$\Delta c = \sum \frac{\partial c}{\partial \epsilon_i} \cdot \Delta \epsilon_i \ , \tag{4.1}$$

where

Δc is the correction to the coordinate

ϵ_i - the orbital element.

That way are forming the observation equations in which the free terms are the differences between the observed coordinates and the computed from the approximate elements (o-c), the unknowns - corrections to those approximate elements. Adopting the observed coordinates α and δ and introducing the threedimensional coordinate system x,y,z, we obtain

$$\Delta\alpha = \sum \frac{\partial\alpha}{\partial x} \cdot \frac{\partial x}{\partial \epsilon_i} \cdot \Delta\epsilon_i + \sum \frac{\partial\alpha}{\partial y} \cdot \frac{\partial y}{\partial \epsilon_i} \cdot \Delta\epsilon_i$$

$$\Delta\delta = \sum \frac{\partial\delta}{\partial x} \cdot \frac{\partial x}{\partial \epsilon_i} \cdot \Delta\epsilon_i + \sum \frac{\partial\delta}{\partial y} \cdot \frac{\partial y}{\partial \epsilon_i} \cdot \Delta\epsilon_i + \sum \frac{\partial\delta}{\partial z} \cdot \frac{\partial z}{\partial \epsilon_i} \cdot \Delta\epsilon_i \tag{4.2}$$

It results from the very geometry of the orbit that the accuracy in its determination depends not only on the accuracy and the number of observations but also on their distribution.

A precise determination of the ascending node will not be possible if the observations are concentrated in the proximity of $u = 90^o$ or 270^o; nor shall we be able to determine properly the eccentricity if only a small arc of orbit is being observed. With that, the distribution of observations depends mainly upon the visibility conditions, and so, the altitude of the orbit, its orientation with regard to the direction of the Sun. Yet for our purposes, we shall assume that the observations are uniformly distributed at least on the arc of orbit $= 180^o$. According to the analysis given by Sotchilina [1963] in her paper devoted to the accuracy of the determination of orbits, in such a case all elements will be found with an accuracy of the same order. Sotchilina calculates the coefficients of weight of the particular unknowns Q_{ii} as follows

$$\left.\begin{aligned} Q\,(M_o) &= \frac{9}{4}\frac{1}{Na^2} \\ Q\,(n) &= \frac{75}{4}\frac{1}{Na^2t^2} \\ Q\,(\Omega) &= \frac{2}{Na^2} \\ Q\,(i) &= \frac{2}{Na^2} \\ Q\,(ax) &= \frac{1}{2Na^2} \\ Q\,(ay) &= \frac{1}{2Na^2} \end{aligned}\right\} \qquad (4.3)$$

where

N = number of observations,

t = observation period expressed in days, $a_x = e\cos\omega$, $a_y = e\sin\omega$.

The accuracy of the respective elements is calculated with the help of formulae

$$m_{\epsilon i} = m_o \cdot \sqrt{Q_{ii}} \qquad (4.4)$$

$$m_o = \sqrt{\frac{\sum^{N}[(\varrho \cdot \cos\delta\Delta\alpha)^2 + (\varrho\Delta\delta)^2]}{2N \cdot 6}}, \qquad (4.5)$$

where

ϱ - is the topocentric distance of the satellite.

Adopting the accuracy of observations as being

$$m_\delta = \cos\delta\, m_\alpha = 2'' ,$$

this corresponding to

$$m_\delta = \varrho\cos\delta\, m_\alpha = a \cdot 10^{-6}$$

we shall have

$$m_o = a \cdot 10^{-6}\sqrt{\frac{2\ N}{2\ N-6}} \cong a \cdot 10^{-6} .$$

If we take $N = 100$ and $t = 10^d$, we obtain

$$\left.\begin{aligned} m_{Mo} &= 1{,}5 \cdot 10^{-7} \text{ rad} \\ m_n &= 0{,}5 \cdot 10^{-7} \text{ rad/d} \\ m_\Omega &= 1{,}3 \cdot 10^{-7} \text{ rad} \\ m_i &= 1{,}3 \cdot 10^{-7} \text{ rad} \\ m_{ax} &= 0{,}3 \cdot 10^{-7} \\ m_{ay} &= 0{,}3 \cdot 10^{-7} \\ m_e &= m_{ax} = m_{ay} \end{aligned}\right\} \qquad (4.6)$$

since

$$de = da_x \cos\omega + da_y \sin\omega \qquad (4.7)$$

$$m_e^2 = m_{ax}^2 (\cos^2\omega + \sin^2\omega) . \qquad (4.8)$$

It results from the above development that it is possible to attain a very high accuracy of the order of 10^{-7}in the determination of orbital elements-provided that the observations are properly distributed and their number and accuracy being as assumed.

Still, we did not take here into account a factor having an essential bearing, it means, the errors of observing station coordinates. This factor does not appear in problems of the classical astronomy, because of the great distances between the observer and the body observed. The question is different with artificial satellites; having to deal here with the geocentric motion, it is necessary to know the geocentric coordinates of observing stations.

In order to evaluate the effect of errors of observing station coordinates, let us start with the basic equations determining the position of the satellite in the Cartesian geocentric system, on the ground of the known topocentric coordinates α and δ

$$\left.\begin{aligned} x &= X + \varrho \cdot \cos(\alpha - \Omega) \cdot \cos\delta \\ y &= Y + \varrho \cdot \sin(\alpha - \Omega) \cdot \cos\delta \\ z &= Z + \varrho \cdot \sin\delta \end{aligned}\right\} \qquad (4.9)$$

where

x,y,z = coordinates of satellite,

X,Y,Z = coordinates of observing stations,

α,δ = spherical coordinates observed,

ϱ = topocentric radius-vector,

Ω = right ascension of the ascending node

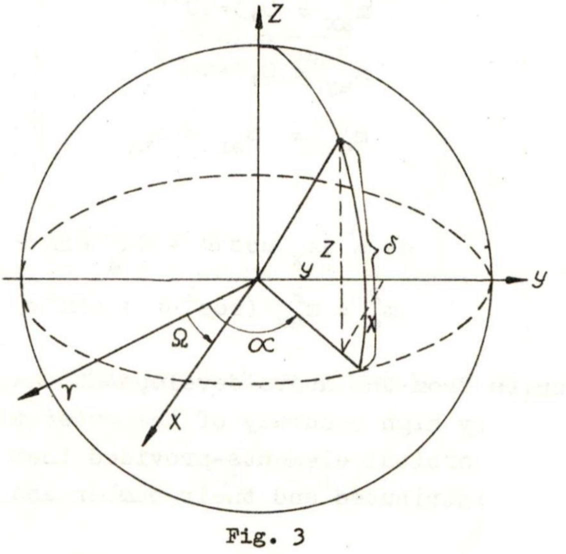


Fig. 3

assuming that the axis x coincides with the line of nodes. Coordinates of the observing point are expressed by formulae

$$\left.\begin{aligned} X &= R \cdot \cos\varphi' \cdot \cos(s - \Omega) \\ Y &= R \cdot \cos\varphi' \cdot \sin(s - \Omega) \\ Z &= R \cdot \sin\varphi' \end{aligned}\right\} \qquad (4.10)$$

where

s = sidereal time of the observing station

φ = geocentric latitude

R = radius vector of the observing station.

We may now write total differentials of equation (4.9) (treating Ω as constant):

$$\left.\begin{aligned} dx - dX &= -\varrho\cos(\alpha-\Omega)\cdot\sin\delta\, d\delta - \varrho\sin(\alpha-\Omega)\cdot\cos\delta\, d\alpha + \\ &\quad + \cos(\alpha-\Omega)\cdot\cos\delta\, d\varrho \\ dy - dY &= -\varrho\sin(\alpha-\Omega)\cdot\sin\delta\, d\delta + \varrho\cos(\alpha-\Omega)\cdot\cos\delta\, d\alpha + \\ &\quad + \sin(\alpha-\Omega)\cdot\cos\delta\, d\varrho \\ dz - dZ &= +\varrho\cos\delta\, d\delta + \sin\delta\, d\varrho \end{aligned}\right\} \quad (4.11)$$

where from we can find $d\alpha$ and $d\delta$

$$\cos\delta\, d\alpha = -\frac{\sin(\alpha-\Omega)}{\varrho}(dx - dX) + \frac{\cos(\alpha-\Omega)}{\varrho}(dy - dY) \quad (4.12)$$

$$d\delta = \frac{-\cos(\alpha-\Omega)\sin\delta}{\varrho}(dx - dX) +$$

$$- \frac{\sin(\alpha-\Omega)\cdot\sin\delta}{\varrho}(dy - dY) + \frac{\cos\delta}{\varrho}(dz - dZ).$$

These equations serve in the so-called orbit improvement process as observation equations in which the left-hand member represents a free term, assuming that $dX = dY = dZ = 0$, and that dx, dy and dz are functions of orbital elements. But dX, dY, dZ being in reality $\neq 0$, their effect will burden the free terms of equations (4.12). Let us rewrite those equations in the following manner

$$\begin{aligned} \varrho\cos\delta\, d\alpha + (\Delta)_\alpha &= -\sin(\alpha-\Omega)\, dx + \cos(\alpha-\Omega)\, dy \\ \varrho\, d\delta + (\Delta)_\delta &= -\cos(\alpha-\Omega)\sin\delta\, dx - \sin(\alpha-\Omega)\cdot\sin\delta\, dy + \cos\delta\, dz \end{aligned} \quad (4.13)$$

$(\Delta)_\alpha$ and $(\Delta)_\delta$ will be the effect of errors of observing station coordinates on the particular equations, taking the form of

$$(\Delta)_\alpha = -\sin(\alpha-\Omega)\,dX + \cos(\alpha-\Omega)\,dY \qquad (4.14)$$

$$(\Delta)_\delta = -\cos(\alpha-\Omega)\sin\delta\,dX - \sin(\alpha-\Omega)\sin\delta\,dY + \cos\delta\,dZ.$$

If the coordinates of stations are determined by astronomical methods, we may expect the error to reach ± 500 m, owing to local deflections of the vertical. Admitting $(\alpha-\Omega) = 315^\circ$ for the first of equations (4.14), and 225° - for the second one, $\delta = 45^\circ$, and $\varrho = 1200$ km (of an orbit of the type of Alouette, Anna, Echo I),

we obtain

$$\left.\begin{aligned} \frac{(\Delta)_\alpha}{\varrho\cos\delta} &= \frac{2\cdot 0{,}71\cdot 500}{1200000\cdot 0{,}71}\cdot 3438' \cong 2{,}'6 \\ \frac{(\Delta)_\delta}{\varrho} &= \frac{0{,}71\cdot(0{,}71+0{,}71+1)\cdot 500}{1200000}\cdot 3438' = 2{,}'4 \end{aligned}\right\} \qquad (4.15)$$

So, we can see that - when unfavorable circumstances occur - this effect may be very important, exceeding many times the observation accuracy. It will be, to a certain degree, attenuated by the fact that the coefficients: $\sin(\alpha-\Omega)$, $\cos(\alpha-\Omega)$ $\sin\delta$, co δ will take their values in a way close to an accidental one; and, what more, dX, dY are not constants, they being dependent upon the angle s (vide formula 4.10).

From the theoretical point of view the expressions $(\Delta)_\alpha$ and $(\Delta)_\delta$ do not concentrate the total effect of coordinate errors of observing stations, for both ϱ and the coefficients including $(\alpha-\Omega)$ and δ are functions of these coordinates

$$\text{ex.g.: } \varrho = \sqrt{(x-X)^2 + (y-Y)^2 + (z-Z)^2}\,. \qquad (4.16)$$

This remainder will however be insignificant, and may be neglected in practice. Let us assume that the ϱ will be found with an error of ± 600 m, this giving - when $\varrho = 1200$ km - a relative accuracy of 1/2000. It is with this accuracy that the free terms will be defined and the unknowns determined. Yet, the unknowns dx, dy, dz being of the order of hundreds of meters, their determination error will be within the range of one meter; this will be, in the same scale, carried over to corrections of elements.

The same situation is with reference to the trigonometric coefficients.

So, there is still the problem, how to proceed in order to eliminate or, at least, to reduce this unfavorable influence. One of the measures could be the utilization of observations of the greatest possible number of stations, so that the errors of station positions take an accidental character. Not always will this, however, be realizable. In such a case, another course of action ought to be applied.

We may anticipate that it will be impossible to eliminate the effect under consideration from certain elements - either by computations or by use of special observing methods. Such are the elements which define the orientation of the orbit: the right ascension of the ascending node, the inclination and the argument of the perigee. Those elements depend directly on the coordinate system chosen. The remaining three elements do not depend on the choice of the system, and it may therefore be presumed that we might succeed in eliminating the error effect of station positions wholly or, at least, partially. We shall now show that this is possible for the mean motion and, consequently, for the semi-major axis of the orbit. Let us assume that we have a set of observations made in such a way that at each station the satellite had been observed several times during different passes. We shall consider two of such observations carried out at the same station. Let us set for each of them the equations (4.13), subtracting them by members accordingly. Then we shall have

$$\varrho_2 \cos\delta_2 \, d\alpha_2 + (\Delta)\alpha_2 - \varrho_1 \cos\delta_1 \, d\alpha_1 - (\Delta)\alpha_1 =$$

$$= -\sin(\alpha_2 - \Omega_2) dx_2 + \cos(\alpha_2 - \Omega_2) dy_2 + \sin(\alpha_1 - \Omega_1) dx_1 - \cos(\alpha_1 - \Omega_1) \, dy_1$$

$$\varrho_2 d\delta_2 + (\Delta)\delta_2 - \varrho_1 d\delta_1 - (\Delta)\delta_1 = -\cos(\alpha_2 - \Omega_2) \cdot \sin\delta_2 dx_2 + \tag{4.17}$$

$$- \sin(\alpha_2 - \Omega_2) \sin\delta_2 dy_2 + \cos\delta_2 dz_2 + \cos(\alpha_1 - \Omega_1) \sin\delta_1 dx_1 +$$

$$+ \sin(\alpha_1 - \Omega_1) \sin\delta_1 dy_1 - \cos\delta_1 \, dz_1$$

Taking into account the formulae (4.14), we may write

$$(\Delta)\alpha_2 - (\Delta)\alpha_1 = -\sin(\alpha_2-\Omega_2)dX_2 + \cos(\alpha_2-\Omega_2)\, dY_2 +$$
$$+ \sin(\alpha_1-\Omega_1)\, dX_1 - \cos(\alpha_1-\Omega_1)\, dY_1$$

$$(\Delta)\delta_2 - (\Delta)\delta_1 = -\cos(\alpha_2-\Omega_2)\sin\delta_2 dX_2 - \sin(\alpha_2-\Omega_2)\sin\delta_2 dY_2 +$$
$$+ \cos\delta_2\, dZ_2 + \cos(\alpha_1-\Omega_1)\sin\delta_1 dX_1 + \qquad (4.18)$$
$$+ \sin(\alpha_1-\Omega_1)\sin\delta_1 dY_1 - \cos\delta_1\, dZ_1 .$$

The effect of errors of observing station coordinates will be eliminated when the above expressions = 0. Let us first consider the second equation (4.18), as being more simple. Here an adequate condition will be provided by $\delta_1 = \delta_2 = 0$. Since $dZ_1 = dZ_2$, the right-hand member becomes = 0. This is also a necessary condition because, if $\delta_1 \neq \delta_2$, then the coefficient of dZ is $\neq 0$, and if $\delta_1 = \delta_2 \neq 0$, then the coefficients of dX and dY will always be $\neq 0$.

Thus, we may deduce from this that for an utilization of the equation 4.17.2, as an observation equation for the improvement of the orbit, it is necessary to have observations carried out during the satellite pass through the topocentric equator of the observing station. Let us assume that this condition has been strictly fulfilled; then the equation 4.17.2 will take the following form

$$\varrho_2 d\delta_2 - \varrho_1 d\delta_1 = dz_2 - dz_1 . \qquad (4.19)$$

Substituting for dz the formulae connecting them with the corrections of elements in the form given by Sotchilina [1963] we shall have

$$dz_2-dz_1 = (y_2-y_1)\, di_o + (A_{z2}-A_{z1})\, dU_o +$$
$$+ (B_{z2}-B_{z1})\, dax + (C_{z2}-C_{z1})\, day + (A_{z2}\cdot t_z-A_{z1}t_1)\, d\, n_o +$$
$$+ (A'_{z2}-A'_{z1})\, dn'_o + (A''_{z2}-A''_{z1})\, dn''_o . \qquad (4.20)$$

In this equation all pairs of parameters designated with indices t_1 and t_2 will consist of elements differring slightly from each other, axcept for t_1 and t_2. Especially for the Keplerian orbit it will be as follows

$$\varrho_2\, d\delta_2 - \varrho_1 d\delta_1 = A_z\, (t_2 - t_1)\, dn\ . \qquad (4.21)$$

It is clearly seen from the equation (4.21) that this method helps to determine accurately only the mean motion and its derivatives from precisely measured observation moments. That is why, the following course of action seems to be advisable:

1. Accomplishing the computations according to the normal procedure, applying the formulae given by Sotchilina and utilizing the whole set of observations;
2. Computation - with the help of the formula (4.19) - of corrections $d\bar{n}_o$, dn'_o, dn''_o, assuming di_o, dU_o, da_x, da_y to be $= 0$, and utilizing only such observations which are adapted to the present method;
3. Return to the above formulae and observations, handling now $\bar{n}_o$, n'_o, n''_o as constants, and correcting i_o, U_o, a_x, a_y;
4. Repeted use of the formula (4.19) a.s.o. until an adequate convergence is attained.

The application of the above method, that is, the use of the equation (4.17.2) is possible only with satellites with great orbital inclinations. In order to utilize the equation (4.17.1), let us first examine the relationship occurring between the coordinates X,Y,Z connected with the system of astronomical coordinates α and δ and the system connected with the rotating Earth, which will be termed ξ, η, ζ (the axis ξ in the plane of the Greenwich meridian)

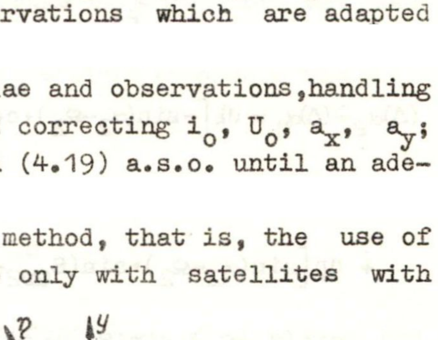


Fig. 4

$$\left.\begin{aligned} X &= \xi \cdot \cos(S_{Gr} - \Omega) - \eta \cdot \sin(S_{Gr} - \Omega) \\ Y &= \xi \cdot \sin(S_{Gr} - \Omega) + \eta \cdot \cos(S_{Gr} - \Omega) \\ Z &= \zeta \end{aligned}\right\}, \qquad (4.22)$$

where

S_{Gr} - the Greenwich sidereal time,

analogically

$$\left.\begin{aligned} dX &= d\xi \cdot \cos(S_{Gr} - \Omega) - d\eta \cdot \sin(S_{Gr} - \Omega) \\ dY &= d\xi \cdot \sin(S_{Gr} - \Omega) + d\eta \cdot \cos(S_{Gr} - \Omega) \\ dZ &= d\zeta \,. \end{aligned}\right\} \quad (4.23)$$

Substituting it in (4.18.1), we obtain

$$\begin{aligned} (\Delta)\alpha_2 - (\Delta)\alpha_1 = &-\sin(\alpha_2 - \Omega_2)\left[d\xi \cdot \cos(S_{Gr2} - \Omega_2) - d\eta \cdot \sin(S_{Gr2} - \Omega_2)\right] + \\ &+ \cos(\alpha_2 - \Omega_2)\left[d\xi \cdot \sin(S_{Gr2} - \Omega_2) + d\eta \cdot \cos(S_{Gr2} - \Omega_2)\right] + \\ &+ \sin(\alpha_1 - \Omega_1)\left[d\xi \cdot \cos(S_{Gr1} - \Omega_1) - d\eta \cdot \sin(S_{Gr1} - \Omega_1)\right] + \\ &- \cos(\alpha_1 - \Omega_1)\left[d\xi \cdot \sin(S_{Gr1} - \Omega_1) + d\eta \cdot \cos(S_{Gr1} - \Omega_1)\right]; \end{aligned} \quad (4.24)$$

$$\begin{aligned} (\Delta)\alpha_2 - (\Delta)\alpha_1 = d\xi\big[&-\sin(\alpha_2 - \Omega_2) \cdot \cos(S_{Gr2} - \Omega_2) + \cos(\alpha_2 - \Omega_2) \cdot \sin(S_{Gr2} - \Omega_2) + \\ &+ \sin(\alpha_1 - \Omega_1) \cdot \cos(S_{Gr1} - \Omega_1) - \cos(\alpha_1 - \Omega_1) \cdot \sin(S_{Gr1} - \Omega_1)\big] + \\ + d\eta\big[&\sin(\alpha_2 - \Omega_2) \cdot \sin(S_{Gr2} - \Omega_2) + \cos(\alpha_2 - \Omega_2) \cdot \cos(S_{Gr2} - \Omega_2) + \\ &- \sin(\alpha_1 - \Omega_1) \cdot \sin(S_{Gr1} - \Omega_1) - \cos(\alpha_1 - \Omega_1) \cdot \cos(S_{Gr1} - \Omega_1)\big]; \end{aligned}$$

$$\begin{aligned} (\Delta)\alpha_2 - (\Delta)\alpha_1 &= d\xi\left[+\sin(\alpha_2 - S_{Gr2}) + \sin(\alpha_1 - S_{Gr1})\right] + \\ &+ d\eta\left[\cos(\alpha_2 - S_{Gr2}) - \cos(\alpha_1 - S_{Gr1})\right]; \end{aligned} \quad (4.25)$$

The condition for the expression (4.25) to be = 0 is

$$\alpha_1 - S_{Gr1} = \alpha_2 - S_{Gr2} \,, \quad (4.26)$$

which will be satisfied when the observations are carried out on the same hour circle, and especially in the meridian. Neither in this case, when the equation (4.17.1) is being accepted as an observation equation, can all elements be accurately determined, because some differences of quantities close to each other appear in coefficients. Thus, the above mode of successive approximation ought to be also applied here. We should, further, define the accuracy of conditions to be satisfied for observations of the satellite in the meridian and in the equator, so as to make the above method utilizable. Suppose, we want $\frac{(\Delta)\alpha_2 - (\Delta)\alpha_1}{\varrho}$ and $\frac{(\Delta)\delta_2 - (\Delta)\delta_1}{\varrho}$ to be $< 1''$.

Considering $dx = dy = dz = 500$ m, $\varrho = 1200$ km

$$\alpha - S_{Gr} = 0 ,$$

we obtain

$$\left|(\alpha_2 - S_{Gr2}) - (\alpha_1 - S_{Gr1})\right| < 40' .$$

And so, if we intend to make observations near the meridian, they have to be within the range of $\pm 20'$ of the hour angle. For the second case, considering the most unfavorable conditions $\alpha_1 - \Omega_1 = 45^o$, $\alpha_2 - \Omega_2 = 225^o$, we obtain the following limitations:

$$\frac{\delta_1 + \delta_2}{2} < 30'$$

$$\left|\delta_2 - \delta_1\right| < 1^o .$$

By rendering the mean motion independent of errors of the station position, we also make the orbital eccentricity partially independent of these errors. This is important, for the accuracy of the orbital eccentricity is significant for the computation of the radius-vector.

According to Brouwer and Clemence [1961], page 236, we have:

$$\left.\begin{aligned} \frac{\partial x}{\partial e} &= Hx + K\dot{x} \\ \frac{\partial y}{\partial e} &= Hy + K\dot{y} \\ \frac{\partial z}{\partial e} &= Hz + K\dot{z} , \end{aligned}\right\} \qquad (4.27)$$

where

$$H = -\cos v\,, \quad K = \frac{2\sin v}{n} \quad \text{for} \quad e \cong 0\,.$$

The part of the derivative with the coefficient H will introduce the effect of the position errors, but the other part with the coefficients K will already be disburdened of this effect. So it seems that the determination accuracy of e will be lower than that of n, yet higher than of the remaining elements.

The afore-said evaluations are confirmed by inner accuracies of elements, published in SAO. Rept. For instance: satellite 1960 Iota 2 (Echo I - Rocket)

$$\frac{m_n}{n} \text{ from } 3 \cdot 10^{-8} \text{ to } 2 \cdot 10^{-7}$$

$$m_e \text{ from } \cdot 10^{-6} \text{ to } 5 \cdot 10^{-6}.$$

Satellite 1960 Beta Mu 1 (Anna I B)

$$\frac{m_n}{n} \text{ from } 10^{-7} \text{ to } 3 \cdot 10^{-7}$$

$$m_e \text{ from } 5 \cdot 10^{-6} \text{ to } 10^{-5}\,.$$

Satellite 1960 Alpha Delta 1 (Midas 4)

$$\frac{m_n}{n} \text{ from } 2 \cdot 10^{-7} \text{ to } 10^{-6}$$

$$m_e \text{ from } 2 \cdot 10^{-6} \text{ to } 5 \cdot 10^{-6}\,.$$

This seems to allow to state that - having an adequate number of well-distributed observations and an appropriate observational program, - we are able to determine two of the elements we are particularly interested in, that is, n and e - with an accuracy of the order of 10^{-6}, at least.

CHAPTER V

CONSTANTS OF THE EARTH'S GRAVITY FIELD

According to the recommendation of the Commission VII on Celestial Mechanics of the International Astronomical Union we are going to use the following formula for the Earth's gravity potential in the external space

$$U = \frac{\mu}{r}\left[1 + \sum_{n=1}^{\infty}\sum_{m=0}^{m=n}\left(\frac{R}{r}\right)^n P_{nm}(\sin\beta)(C_{nm}\cos m\lambda + S_{nm}\sin m\lambda)\right]; \qquad (5.1)$$

where

$\mu = k \cdot M$ - the gravity constant multiplied by the Earth's mass

r - the distance from the center of mass

R - equatorial radius of the Earth

C_{nm}, S_{nm} - numerical coefficients

P_{nm} - spherical functions of Legendre's associated polynomials expressed by the general formula:

$$P_{nm}(x) = (1 - x^2)^{\frac{m}{2}} \frac{d^m}{dx^m}\left(\frac{1}{2^n \cdot n} \cdot \frac{d^n(x-1)^n}{dx^n}\right). \qquad (5.2)$$

The functions P_{nm} written in explicit form to the degree 4,4 as well as to the degree 5,0 and 6,0 are presented in the Chapter VIII. According to the formula (5.1) the notion "gravity potential" will mean the potential produced by the attraction, except for the influence of the Earth's rotation, which will be omitted. Let us dwell upon the physical significance of parameters C_{nm} and S_{nm}. For this purpose, a short recapitulation of the derivation of the formula 5.1 may prove useful.

We know from the analysis the theorem [Gruszinsky 1963, page 203] concerning the below function expanded in series of Legendre's polynomials

$$\frac{1}{r} = \frac{1}{\sqrt{R^2 + \varrho^2 - 2R\varrho\cos\theta}}, \qquad (5.3)$$

where the denotations of the symbols r, R, ϱ and Θ are the same as in Fig. 5.

$$\frac{1}{\varrho} = \sum_{n=0}^{\infty} \frac{R^n}{r^{n+1}} \cdot P_n (\cos \Theta) , \qquad (5.4)$$

where

$P_n = P_{no}$ - the Legendre's polynomial of the degree n.

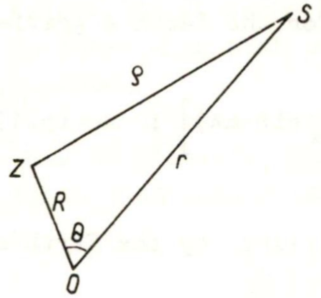


Fig. 5

In such a case, the gravitational potential of the material particle with a mass dM will have the following form

$$dU = dM \cdot \sum_{n=0}^{\infty} \frac{R^n}{r^{n+1}} P_n (\cos \Theta) \qquad (5.5)$$

and the potential of a body with the mass M

$$U = \sum_{n=0}^{\infty} \int_0^m \frac{R^n}{r^{n+1}} P_n (\cos \Theta) \, d\, m . \qquad (5.6)$$

Let us apply now another theorem, which is expressed by the formula

$$P_n (\cos \Theta) = P_n (\cos \psi) P_n(\cos\psi') +$$

$$+ 2 \sum_{m=1}^{n} \frac{(n-m)!}{(n+m)!} P_{nm} (\cos\psi) P_{nm}(\cos\psi') \cos m(\lambda - \lambda') \qquad (5.7)$$

where

$$\cos \Theta = \cos\psi \cos\psi' + \sin\psi \sin\psi' \cdot \cos(\lambda - \lambda') . \qquad (5.8)$$

If we collocate the points Z,O and S in the system of spherical coordinates ϱ, β, λ and the origin of the system at the point O, denoting $90 - \beta$ by ψ, then we obtain the dependence (5.8). Let the coordinates ψ and λ correspond to the point S, and ψ' and λ' - to the point Z. Then substituting to the formula (5.6) we have

$$U = \frac{M}{r} + \frac{1}{r^2} \int R \Big[P_{10}(\cos\psi) P_{10}(\cos\psi') +$$

$$+ P_{11}(\cos\psi) P_{11}(\cos\psi')(\cos\lambda \cos\lambda' + \sin\lambda \sin\lambda') \Big] + \qquad (5.9)$$

$$+\frac{1}{r^3}\int R^2\Big[P_{20}(\cos\psi)P_{20}(\cos\psi')+\frac{1}{3}P_{21}(\cos\psi)P_{21}(\cos\psi')(\cos\lambda\cos\lambda'+$$

$$+\sin\lambda\sin\lambda')+\frac{1}{12}P_{22}(\cos\psi)P_{22}(\cos\psi')(\cos 2\lambda\cos 2\lambda'+$$

$$+\sin 2\lambda\sin 2\lambda')\Big]d_M+\ldots \qquad (5.9)$$

If the integration is extended over a sphere with a radius equal to the equatorial radius of the Earth R_e, then $\frac{R_e n}{n+1}$ can be put before the integral sing, as well as the functions of variables ψ and λ.

We shall obtain

$$U=\frac{M}{r}\Bigg\{1+\frac{R_e}{2r}\Big[P_{10}(\cos\psi)\int P_{10}(\cos\psi')d_M+$$

$$+P_{11}(\cos\psi)(\cos\lambda)\int P_{11}(\cos\psi')\cos\lambda' d_M+\sin\lambda\int P_{11}(\cos\psi')\sin\lambda' d_M\Big]+$$

$$+\frac{1}{3}\left(\frac{R_e}{r}\right)^2\cdot\Big[P_{20}(\cos\psi)\cdot\int P_{20}(\cos\psi')d_M+$$

$$+\frac{1}{3}P_{21}(\cos\psi)(\cos\lambda\int P_{21}(\cos\psi')\cos\lambda' d_M+ \qquad (5.10)$$

$$+\sin\lambda\int P_{21}(\cos\psi')\sin\lambda' d_M)-\frac{1}{12}P_{22}(\cos\psi)(\cos 2\lambda\int P_{22}(\cos\psi')\cos 2\lambda' d_M+$$

$$+\sin 2\lambda\int P_{22}(\cos\psi')\sin 2\lambda\, d_M)\Big]+\ldots\Bigg\}.$$

As seen from the above, the coefficients C_{nm} and S_{nm} are integrals of three variables R, ψ' and λ' extended over the sphere with the radius R_e, whereat

$$dM=dV\cdot\sigma, \qquad (5.11)$$

where

dV - the element of volume,

σ - the function defining the density.

The integrands are the associated Legendre functions P_{nm} $(\cos\psi')$ multiplied respectively by $\cos n\lambda$ or $\sin m\lambda$.

We shall now consider more closely the coefficients C_{10}, C_{11}, S_{11} and C_{21}, S_{21}. Let us choose the coordinate system x, y, z, assuming that both this system and the system R, ψ, λ have their origins at the center of the Earth's mass. Thus, we have the dependences

$$R \cos \psi = Z , \quad R \sin \psi \cos \lambda = X, \quad R \sin \psi \sin \lambda = Y , \tag{5.12}$$

$$\int_M x \, d M = 0 , \quad \int_M y \, d M = 0 , \quad \int_M z \, d M = 0 . \tag{5.13}$$

Referring to the formula (5.10) it can be seen that the coefficients

$$R_e \cdot C_{10} = \int R \, P_{10} \, (\cos\psi') \, dM = \int R \cos\psi' \, dM = 0 \tag{5.14}$$

$$R_e \cdot C_{11} = \int R \, P_{11}(\cos\psi')\cos\lambda' \, dM = \int R \sin\psi'\cos\lambda \, dM = 0$$

$$R_e \cdot S_{11} = \int R \, P_{11}(\cos\psi')\sin\lambda' dM = \int R \sin\psi'\sin\lambda' dM = 0 \tag{5.15}$$

$$R_e^2 \cdot C_{21} = \int R^2 P_{21}(\cos\psi')\cos\lambda' \, dM = \frac{3}{2}\int R^2 \sin 2\psi' \cos\lambda' dM =$$

$$= 3\int R^2 \sin\psi'\cos\psi'\cos\lambda' \, dM = 3\int z \, x \, d M \tag{5.16}$$

$$R_e^2 \, S_{21} = 3\int z \, y \, d M .$$

For the latter two coefficients we have obtained integrals expressing the so called m o m e n t s o f d e v i a t i o n s o r p r o d u c t s o f i n e r t i a, which become zeros when the axis of the body's axial symmetry coincides with the axis of the coordinate system, this being the case here.

So, in virtue of the definition of the coordinate system, the terms with subscripts 10, 11, 21 are equal to zero.

The constants appearing in the formula (5.1) have been many a time determined with the help of different methods. A review of those determinations will allow to ascertain to what extend they may be at present considered complete. We shall refrain from the application of weighted arithmetical means, because the evaluation of the accuracy of different inferences proves difficult. Apart from accidental errors of the material observed, also systematic errors, characteristic for a given method, come into play here. Thence the difficulty and even the impossibility of establishing weights necessary for the averaging.

As a measure of dispersion of the various results we will adopt their range, i.m. the difference between the greatest and the least of values, obtained by different methods with approximately the same theoretical exactness. The value corresponding to the middle of the range will be recognized as being the most probable.

The first of the constants appearing in Equation (5.1) - the quantity μ denoting the Earth's mass multiplied by the Gauss gravitational constant - is of an essential importance for the present subject because, as indicated in Chapter I,it is upon it that depends the absolute accuracy in the determination of the length of the radius-vector. On the other hand, the error of the adopted value of μ bears only upon the scale of the given geometric construction (for instance,the triangulation network), causing no local deformations. Thanks to this, it is possible - by the comparision with measurements of another type - to proceed to the determination anew, with the view of perfecting the value of μ.

For the determination of μ, the method theoretically best fitting our purpose consists in executing a direct measurement of the distance Earth-to-Moon and in establishing this way the linear dimensions of the lunar orbit. We may then derive μ from the formula

$$\mu = \frac{n^2 (1 + \beta)^3 a^3}{1 + \frac{M_M}{M_E}} ;$$

where

n - the mean motion,

a - the semi-major axis,

β - its solar perturbation,

M_M, M_E - masses of the Moon and the Earth respectively.

Such an inference is almost entirely independent of geodetic measurements on the Earth's surface, since the error of geocentric coordinates of the observing station on the Earth is relatively small as compared with the distance measured. This error may be estimated to be approximately ± 200 m, this being the very accuracy of the distance measurement itself. But a source of errors is the unsufficient knowledge of the Moon's shape or, more strictly, of its radius in the direction of the Earth, in addition to a too low accuracy of the ratio M_M/M_E. The radius of the lunar limb amounts to 1737,85 km [B a l d - w i n 1949] the estimates of the radius in the direction to the Earth show differences ranging between 1738 and 1740. Assuming that the surface of the Moon is approximate to the equipotential and substituting the moments of intertia computed from the lunar libration, K a u l a [1963 c] finds the Moon's radius to be 1738,7 in the direction to the Earth.

The M_M/M_E ratio was lately determined from the Earth's motion about the center of the Earth-Moon masses, on the ground of observations of the minor planet Eros [R a b e 1950, D e- l a n o 1950]. The results derived oscillated within the limits of from 1/81.22 to 1/81.38. The most recent determinations from radio observations carried out with the help of the cosmic probe of Mariner II lead to the result of 1/(81.3015 ± ± 0.0033). [K a u l a 1963 c]. Yet, as there was question of a single observation it would be rather difficult to adopt this result as binding. A certain progress was attained after reiterated calculation of the Eros observations, taking into account the new measurements of the astronomical unit, based on radar observations of Venus. The results obtained range between 1/81.26 and 1/81.36. The above divergences demonstrate that the accuracy of this method is at present still lower than of other methods. Yet, there is no doubt that within the nearest years both the figure of the Moon and the M_M/M_E ratio will be precisely determined by means of astronautical methods, and in consequence also the quantity μ. According to the method described observations have been made in the United States [Y a p l e e and others 1959]. Their results are given and discussed by K a u l a [1963 c and d].

All other results contained in Table 2 are depending in one or another way upon the geodetic measurements carried out on the surface of the Earth. To these classical methods belongs the determination of the quantity μ from the absolute measurements of the Earth's acceleration

$$\mu = a_e^2 \gamma_e \left[1 + \frac{2}{3} m - f - \frac{15}{14} m f - \frac{1}{294} m f^2 - 0 (f^4)\right], \quad (5.18)$$

where

a_e - equatorial radius of the Earth

γ_e - equatorial acceleration

f - oblateness

$m = \frac{\omega^2 \cdot a_e}{\gamma_e}$

ω - angular velocity.

Table 2

Method	Author	f·M $km^3\ sek^{-2}$
Radar measurement of the Earth-to-Moon distance	Yaplee and others (1963)	398605.7
Geodetic measurements	Kaula (1961)	398602.0
	Kaula + Uotila (1962)	398604.3
	Fischer (1962)	398604.0
Motion of the Moon and geodetic measurements	Fischer (1962) + O'Keefe + Anderson	398605.7
Photographical observations of artificial satellites	Kaula (1963) 1960 Iota 2	398603.7
	Kaula (1963) 1961 Alpha Delta 1	398599.3
Compilation	NASA	398603.2
Compilation	Michajłow (1964)	398603

The Table 2 presents two results achieved by this method: the one obtained by Kaula [1961], based on a combined adjustment of the triangulation and of the gravimetrical data: the other one grounded on the determination of γ_e, made by Uotila [1962], a_e being computed by Kaula.

Also the method based on the measurement of the lunar parallax is to a large extent dependent on the accuracy we can produce for the dimensions of the Earth, since the above measurement consists in a determination of the Earth-to-Moon distance by measuring the parallactic angle from the known base on the Earth. Such observations were made during a couple of years at the beginning of the 20th century at Greenwich and on the Cape. They are now being reduced again by F i s c h e r [1962], using the latest results of the traingulation which connects at present those two remote points. The result of this work is shown in Table 2, item 4. The method of occultation of stars by the Moon, given by O'Keefe and A n d e r s e n [1962], and described in the Polish literature by K o ł a c z e k [1963], may be successfully applied to the determination of μ - if we consider the radius of the Earth to be a known quantity. Using the observations made by O'Keefe and the results obtained by Fischer concerning the dimensions of the Earth, K a u l a [1963 d] defined the value of μ (Table 2, item 5).

Also observations of artificial satellites have been used fot the same purpose - on the basis of the Kepler's Law and on the assumption of the known dimensions of the Earth. The work done by K a u l a [1963 a and b] in this domain, is presented in Table 2, items 6 and 7. A more detailed discussion of it may be found in Chapter III.

In addition, attempts of utilizing the Doppler-observations of satellites have been made by Anderle and Oesterwinter (in the afore-mentioned study [1963]), as well as the observations of the rocket Mariner II. In this method the scale of the system - which is being transformed afterwards by means of the Kepler's Law, into the quantity μ - issues from independent distance measurements performed with the help of the Doppler method. The potential source of errors lies here in the atmospherical refraction deforming the radio-signal path, and in the observation technique. Unfortunately, the publications available do not permit for a more comprehensive opinion to be formed on this subject. That is why, we are omitting here the results obtained.

For comparative purposes the values of μ such as adopted for computations at research centers, in United States and in Soviet Union will be given. They stand exactly in the middle of the range designated by the remaining data contained in Table 2. This seems to allow to adopt in the present work the following value of μ

$$398603 \pm 3 \text{ km}^3 \text{ sek}^{-2} .$$

As far as the values of zonal harmonics of higher degrees are concerned, undoubtedly the best ones - we may even say - the only good values have been achieved thanks to observations of artificial satellites. The reason is simple: insufficiency of observational data which could be used for the same purpose by the gravitional method and the astronomical levelling method. The comparision of possibilities of those three methods was made by the author in one of his earlier publications [Z i e l i ń s k i 1963]. While the gravimetrical and geodetic data always concern only certain fragments of the Earth, the observed satellite perturbations are reflecting the influence of the whole Earth's solid with its various irregularities. That is the reason for which the latest publications pertaining to geodetic data [F i s c h e r 1961] and to gravimetrical data [U o t i l a 1963] adopt the values of the flattening and the harmonics C_{20}, C_{30} and C_{40}, and do not consider them to be unknowns. It does not mean, however, that the latter data are unquestionable. There exist in the satellite method many error sources which do not permit to solve this problem simply at once. We see, for instance that - when computing a certain finit number of terms - the effect of the next terms is supposed to be equal to zero this does not, however, correspond to the reality. Another discrepancy is caused by the errors of geocentric coordinates of stations and by the disturbing action of other factors, mainly atmospherical. This explains why the results given in Table 3, column C_{20} (Appendix 2) differ one from another.

It can be seen from that list that all C_{20} are included within the range from 1082.2 to 1083.3, and if we omit the results obtained by Zhongolovich, based on observations of close satellites, the upper limit will amount to 1083.15.

Thereupon, we shall adopt for our computations

$$C_{20} = -\ 1082.7 \pm 0.5\ .$$

A similar review of the values C_{30} shows that they range between 2.29 - 2.59, without counting the result of K a u l a [1961 a], as differring distinctly from the remaining ones.

Thus, in accord with the principle adopted, we shall have

$$C_{30} = +\ 2.45 \pm 0.15\ .$$

Considering the values C_{40} we obtain (omitting the Zhongolovich's result) the range from 1,03 to 2.1, adopting

$$C_{40} = +\ 1.60 \pm 0.50$$

and for C_{50} (omitting the Kaula's result 1961) - the range from + 0.07 to + 0.23, adopting

$$C_{50} = +\ 0.15 \pm 0.08\ .$$

For the next values we shall obtain

$$C_{60} = 0.00 \pm 0.70$$

$$C_{70} = +\ 0.10 \pm 0.40$$

$$C_{80} = -\ 0.04 \pm 0.30$$

$$C_{90} = -\ 0.20 \pm 0.30\ .$$

The present task might have been dealt with somewhat differently by placing more reliance in the latest results based on a greater number of observations; then the coefficients adopted here would certainly be nearer the reality.However,the purpose of this work is not to find the most probable values of C_{nm}, S_{nm}, but to examine their effect when the extremal possible values will be adopted. For this reason, the method applied here seems to be adequate.

As to the tesseral harmonics, the comparitive data are here much more scarce, but the dispersion greater. The possibilities of the satellite method are in this case much more limited and become comparable to the possibilities of the gravimetrical method. In consequence of the Earth's rotation about its axis, the tesseral harmonics do not produce distinct - as do the zonal harmonics - long-period effects in the variations of orbital elements. The effects of errors of station coordinates and of observational irregularities appear here still more strongly. And so, in order to be able to obtain correct results, we ought to have at our disposal a richer documentation than available at present.

For calculations, data will be used - averaged in the same manner as before

$C_{22} = +1.15 \pm 0.70$ $\quad S_{22} = -1.25 \pm 1.00$

$C_{31} = +1.55 \pm 1.50$ $\quad S_{31} = +0.25 \pm 0.90$

$C_{32} = +0.20 \pm 0.20$ $\quad S_{32} = -0.03 \pm 0.15$

$C_{33} = +0.072 \pm 0.135$ $\quad S_{33} = +0.211 \pm 0.128$

$C_{41} = -0.16 \pm 0.52$ $\quad S_{41} = -0.01 \pm 0.45$

$C_{42} = -0.25 \pm 0.43$ $\quad S_{42} = -0.05 \pm 0.32$

$C_{43} = +0.084 \pm 0.074$ $\quad S_{43} = -0.006 \pm 0.032$

$C_{44} = +0.004 \pm 0.0012$ $\quad S_{44} = +0.011 \pm 0.015.$

C H A P T E R VI

PERTURBATIONS PRODUCED BY THE EARTH'S GRAVITY FIELD

Since the attraction of the Earth is so most important of the forces acting upon the artificial satellite, the perturbations produced by the Earth's gravity field evidently belong to the greatest disturbances; hence, their qualitative and quantitative evaluation will be of a fundamental significance for the present subject matter. Such evaluation will be made by using the numerical integration method. For this purpose, we are going to derive formulae for equations of satellite motion, taking into account the farther terms of the Earth's gravitational potential up to C_{60} for zonal harmonics and to

C_{44}, S_{44} - for tesseral harmonics. Those formulae were given by K o t c h i n a [1962] in a somewhat different form without derivations; here they have been derived again for checking purposes.

Since we are interested only in short-period and diurnal perturbations, the integration will be performed over the interval of a single revolution of the satellite for zonal harmonics and over 24 hours - for tesseral harmonics.

As demonstrated in Chapter II, the differential equations of satellite motion in rectangular coordinates have the form:

$$\left.\begin{aligned} \frac{d^2x}{dt^2} &= \frac{\partial U}{\partial x} \\ \frac{d^2y}{dt^2} &= \frac{\partial U}{\partial y} \\ \frac{d^2z}{dt^2} &= \frac{\partial U}{\partial z}, \end{aligned}\right\} \qquad (6.1)$$

where U is the gravitational potential defined by the formula (5.1). We write it now in an explicit form, confining ourselves to the terms 60 in zonal harmonics and to 44 - in tesseral ones (and remembering that C_{10}, C_{11}, S_{11}, C_{21}, S_{21} are equal to 0)

$$\begin{aligned} U = {} & U_{00} + U_{20} + U_{22} + U_{30} + U_{31} + U_{32} + U_{33} + \\ & + U_{40} + U_{41} + U_{42} + U_{43} + U_{44} + U_{50} + U_{60} \end{aligned} \qquad (6.2)$$

where the particular terms are respectively

$$\left.\begin{aligned} U_{00} &= \frac{\mu}{r} \\ U_{20} &= \frac{\mu}{r}\left(\frac{R}{r}\right)^2 C_{20}\cdot P_{20}(\sin\beta) \\ U_{22} &= \frac{\mu}{r}\left(\frac{R}{r}\right)^2 (C_{22}\cos 2\lambda + S_{22}\sin 2\lambda)\cdot P_{22}(\sin\beta) \\ U_{30} &= \frac{\mu}{r}\left(\frac{R}{r}\right)^3 C_{30}\cdot P_{30}(\sin\beta) \end{aligned}\right| \qquad (6.3)$$

$$
\left.
\begin{aligned}
U_{31} &= \frac{\mu}{r}\left(\frac{R}{r}\right)^3 (C_{31}\cos\lambda + S_{31}\sin\lambda)\cdot P_{31}(\sin\beta)\\
U_{32} &= \frac{\mu}{r}\left(\frac{R}{r}\right)^3 (C_{32}\cos 2\lambda + S_{32}\sin 2\lambda)\cdot P_{32}(\sin\beta)\\
U_{33} &= \frac{\mu}{r}\left(\frac{R}{r}\right)^3 (C_{33}\cos 3\lambda + S_{33}\sin 3\lambda)\cdot P_{33}(\sin\beta)\\
U_{40} &= \frac{\mu}{r}\left(\frac{R}{r}\right)^4 C_{40}\cdot P_{40}(\sin\beta)\\
U_{41} &= \frac{\mu}{r}\left(\frac{R}{r}\right)^4 (C_{41}\cos\lambda + S_{41}\sin\lambda)\cdot P_{41}(\sin\beta)\\
U_{42} &= \frac{\mu}{r}\left(\frac{R}{r}\right)^4 (C_{42}\cos 2\lambda + S_{42}\sin 2\lambda)\cdot P_{42}(\sin\beta)\\
U_{43} &= \frac{\mu}{r}\left(\frac{R}{r}\right)^4 (C_{43}\cos 3\lambda + S_{43}\sin 3\lambda)\cdot P_{43}(\sin\beta)\\
U_{44} &= \frac{\mu}{r}\left(\frac{R}{r}\right)^4 (C_{44}\cos 4\lambda + S_{44}\sin 4\lambda)\cdot P_{44}(\sin\beta)\\
U_{50} &= \frac{\mu}{r}\left(\frac{R}{r}\right)^5 C_{50}\cdot P_{50}(\sin\beta)\\
U_{60} &= \frac{\mu}{r}\left(\frac{R}{r}\right)^6 C_{60}\cdot P_{60}(\sin\beta).
\end{aligned}
\right\}
\quad (6.3)
$$

Let us now introduce the coordinate system x,y,z, the origin of which is at the center of the Earth's mass, the z-axis coincides with the rotation axis, being orientated in the direction of the vernal equinox.

Thereupon

$$
\left.
\begin{aligned}
x &= r\cos\beta\sin(\lambda + s)\\
y &= r\cos\beta\cos(\lambda + s)\\
z &= r\sin\beta,
\end{aligned}
\right\}
\quad (6.4)
$$

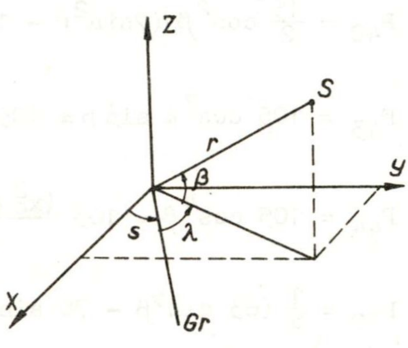


Fig. 6

where

s - ist the sidereal Greenwich time.

The functions of P_{nm} will be presented in their explicit form, as functions of the angle β; and further - as functions of the coordinates x, y, z, substituting

$$\sin\beta = \frac{z}{r} \; ; \qquad \cos\beta = \frac{\sqrt{x^2 + v^2}}{r} \tag{6.5}$$

$$P_{20} = \frac{1}{2}\,(3\sin^2\beta - 1) = \frac{1}{2}\left(\frac{3z^2}{r^2} - 1\right)$$

$$P_{22} = 3\cos^2\beta = 3\,\frac{x^2 + y^2}{r^2}$$

$$P_{30} = \frac{1}{2}\,(5\sin^3\beta - 3\sin\beta) = \frac{1}{2}\left(\frac{5z^3}{r^3} - \frac{3z}{r}\right)$$

$$P_{31} = \frac{3}{2}\cos\beta\,(5\sin^2\beta - 5) = \frac{3}{2}\,\frac{\sqrt{x^2+y^2}}{r}\left(\frac{5z^2}{r^2} - 1\right)$$

$$P_{32} = 15\cos^2\beta\,\sin\beta = 15\,\frac{(x^2 + y^2)z}{r^3}$$

$$P_{33} = 15\cos^3\beta = \frac{15(x^2 + y^2)^{3/2}}{r^3}$$

$$P_{40} = \frac{1}{8}\,(35\sin^4\beta - 30\sin^2\beta + 3) = \frac{1}{8}\left(\frac{35z^4}{r^4} - \frac{30z^2}{r^2} + 3\right) \tag{6.6}$$

$$P_{41} = \frac{1}{2}\cos\beta\,(35\sin^3\beta - 15\sin\beta) = \frac{5}{2}\,\frac{\sqrt{x^2 + y^2}}{r^2}\left(7\,\frac{z^2}{r^2} - \frac{3z}{r}\right)$$

$$P_{42} = \frac{15}{2}\cos^2\beta\,(7\sin^2\beta - 1) = \frac{15}{2}\,\frac{(x^2 + y^2)}{r^2}\left(7\,\frac{z^2}{r^2} - 1\right)$$

$$P_{43} = 105\cos^3\beta\,\sin\beta = 105\,\frac{(x^2 + y^2)^{3/2}\,z}{r^4}$$

$$P_{44} = 105\cos^4\beta = 105\,\frac{(x^2 + y^2)^3}{r^4}$$

$$P_{50} = \frac{1}{8}\,(63\sin^5\beta - 70\sin^3\beta + 15\sin\beta) = \frac{1}{8}\left(\frac{63z^5}{r^5} - \frac{70z^3}{r^3} + \frac{15z}{r}\right)$$

$$P_{60} = \frac{1}{16}\,(231\sin^6\beta - 315\sin^4\beta + 105\sin^2\beta - 5) =$$
$$= \frac{1}{16}\left(\frac{231\,z^6}{r^6} - \frac{315\,z^4}{r^4} + \frac{105\,z^2}{r^2} - 5\right).$$

We have to express, moreover, the sines and cosines of the angle λ by the coordin a tes x,y,z, and the angle s.

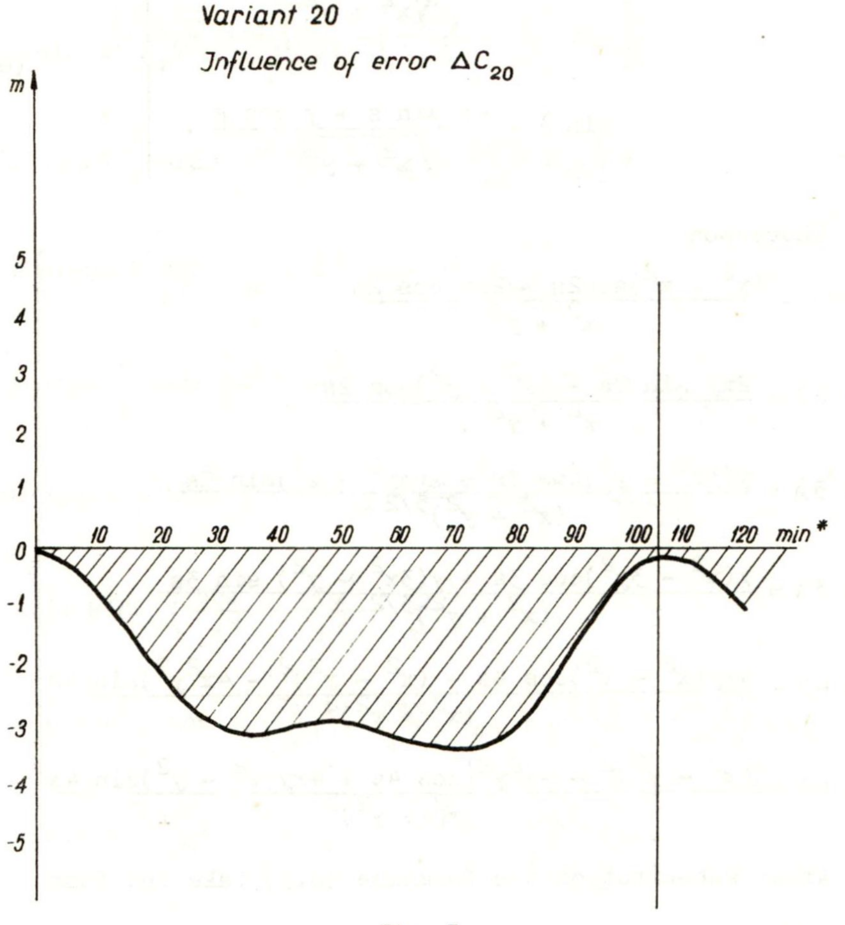


Fig. 7

From the Fig. 6, we have

$$\xi = x \cos s + y \sin s$$
$$\eta = -x \sin s + y \cos s \quad (6.7)$$

but

$$\xi = \sqrt{x^2 + y^2} \cdot \cos \lambda$$
$$\eta = \sqrt{x^2 + y^2} \cdot \sin \lambda$$

hence

$$\left.\begin{aligned} \cos\lambda &= \frac{x\cos s + y\sin s}{\sqrt{x^2+y^2}} \\ \sin\lambda &= \frac{-x\sin s + y\cos s}{\sqrt{x^2+y^2}} \,. \end{aligned}\right\} \quad (6.8)$$

Thereupon

$$\left.\begin{aligned} \sin 2\lambda &= \frac{(y^2-x^2)\sin 2s + 2xy\cos 2s}{x^2+y^2} \\ \cos 2\lambda &= \frac{2xy\sin 2s + (x^2-y^2)\cos 2s}{x^2+y^2} \\ \sin 3\lambda &= \frac{y(3x^2-y^2)\cos 3s + x(3y^2-x^2)\sin 3s}{(x^2+y^2)^{3/2}} \\ \cos 3\lambda &= \frac{x(x^2-3y^2)\cos 3s + y(3x^2-y^2)\sin 3s}{(x^2+y^2)^{3/2}} \\ \sin 4\lambda &= \frac{4xy(x^2-y^2)\cos 4s - (x^2-y^2)^2 - 4x^2y^2)\sin 4s}{(x^2+y^2)^2} \\ \cos 4\lambda &= \frac{[(x^2-y^2)^2 - 4x^2y^2]\cos 4s + 4xy(x^2-y^2)\sin 4s}{(x^2+y^2)^2} \end{aligned}\right\} \quad (6.9)$$

After substitution the formulae (6.3) take the form

$$\begin{aligned} U_{00} &= \frac{\mu}{r} \\ U_{20} &= \frac{1}{2}\mu R^2 C_{20}\left(\frac{3z^2}{r^5} - \frac{1}{r^3}\right) \\ U_{22} &= 3\mu R^2\Big[\cos 2s\Big(C_{22}\,\frac{(x^2-y^2)}{r^5} + \frac{2xy}{r^5}S_{22}\Big) \\ &\quad + \sin 2s\left(C_{22}\,\frac{2xy}{r^5} - \frac{(x^2-y^2)}{r^5}S_{22}\right)\Big] \\ U_{30} &= \frac{1}{2}\mu R^3 C_{30}\left(\frac{5z^3}{r^7} - \frac{3z}{r^5}\right) \end{aligned} \qquad (6.10)$$

$$U_{31} = \frac{3}{2}\mu R^3 \left\{ \cos s\left[\left(\frac{5z^2x}{r^7} - \frac{x}{r^5}\right)C_{31} + \left(\frac{5z^2y}{r^7} - \frac{y}{r^5}\right)S_{31}\right] + \right.$$

$$\left. + \sin s\left[\left(\frac{5z^2y}{r^7} - \frac{y}{r^5}\right)C_{31} - \left(\frac{5z^2x}{r^7} - \frac{x}{r^5}\right) S_{31}\right]\right\}$$

$$U_{32} = 15\mu R^3\left[\cos 2s\left(\frac{(x^2-y^2)z}{r^7}\, C_{32} + \frac{2xyz}{r^7}\, S_{32}\right) + \right.$$

$$\left. + \sin 2s \left(\frac{2xyz}{r^7}\, C_{32} - \frac{(x^2 - y^2)z}{r^7}\, S_{32}\right)\right]$$

$$U_{33} = 15\mu R^3\left[\cos 3s \left(\frac{x(x^2-y^2)}{r^7}\, C_{33} + \frac{y(3x^2 - y^2)}{r^7}\, S_{33}\right) + \right.$$

$$\left. + \sin 3s \left(\frac{y(3x^2 - y^2)}{r^7}\, C_{33} - \frac{x(x^2 - 3y^2)}{r^7}\, S_{33}\right)\right]$$

$$U_{40} = \frac{1}{8}\mu R^4 C_{40}\left(\frac{35z^4}{r^9} - \frac{30z^2}{r^7} + \frac{3}{r^5}\right)$$

$$U_{41} = \frac{5}{2}\mu R^4\left\{ \cos s\left[\left(\frac{7z^3x}{r^9} - \frac{3zx}{r^7}\right) C_{41} + \left(\frac{7z^3y}{r^9} - \frac{3z\,y}{r^7}\right)S_{41}\right] + \right.$$

$$\left. + \sin s\left[\left(\frac{7z^3y}{r^9} - \frac{3y}{r^7}\right)C_{41} - \left(\frac{7z^3x}{r^9} - \frac{3zx}{r^7}\right) S_{41}\right]\right.$$

$$U_{42} = \frac{15}{2}\mu R^4\left\{\cos 2s\left[\left(\frac{7z^2(x^2 - y^2)}{r^9} - \frac{(x^2-y^2)}{r^7}\right)C_{42} + \right.\right.$$

$$+\left(\frac{14xyz^2}{r^9} - \frac{2xy}{r^7}\right)S_{42}\Big] + \sin 2s\left[\left(\frac{14xyz^2}{r^9} - \frac{2xy}{r^7}\right)C_{42} + \right.$$

$$\left.\left. -\left(\frac{7z^2(x^2 - y^2)}{r^9} - \frac{(x^2 - y^2)}{r^7}\right)S_{42}\right]\right\}$$

$$U_{43} = 105\mu R^4\left[\cos 3s\left(\frac{xy(x^2-3y^2)}{r^9}C_{43} + \frac{yz(3x^2-y^2)}{r^9}S_{43}\right) + \right.$$

$$\left. + \sin 3s\left(\frac{yz(3x^2-y^2)}{r^9}C_{43} - \frac{xz(x^2-3y^2)}{r^9}S_{43}\right)\right]$$

$$U_{44} = 105\mu R^4\left[\cos 4s\left(\frac{(x^2-y^2)^2-4x^2y^2}{r^9}C_{44} + \frac{4xy(x^2-y^2)}{r^9}S_{44}\right) + \right.$$

$$\left. + \sin 4s\left(\frac{4xy(x^2-y^2)}{r^9}C_{44} - \frac{(x^2-y^2)^2-4x^2y^2}{r^9}S_{44}\right)\right] \qquad (6.10)$$

$$U_{50} = \frac{1}{8}\mu R^5 C_{50}\left(\frac{63z^5}{r^{11}} - \frac{70z^3}{r^9} + \frac{15z}{r^7}\right)$$

$$U_{60} = \frac{1}{16}\mu R^6 C_{60}\left(\frac{231z^6}{r^{13}} - \frac{315z^4}{r^{11}} + \frac{105z^2}{r^9} - \frac{5}{r^7}\right).$$

We now write equations of the type of 6.1 for zonal harmonics, that is, U_{no}, keeping in mind that

$$\left.\begin{aligned}
&\frac{\partial r}{\partial x} = \frac{x}{r}; \quad \frac{\partial r}{\partial y} = \frac{y}{r}; \quad \frac{\partial r}{\partial z} = \frac{z}{r};\\
&\frac{\partial U_{00}}{\partial x} = -\frac{\mu x}{r^3}\\
&\frac{\partial U_{20}}{\partial x} = \frac{3}{2}\mu R^2 C_{20}\left(\frac{-5xz^2}{r^7} + \frac{x}{r^5}\right)\\
&\frac{\partial U_{30}}{\partial x} = \frac{5}{2}\mu R^3 C_{30}\left(\frac{-7xz^3}{r^9} + \frac{3xz}{r^7}\right)\\
&\frac{\partial U_{40}}{\partial x} = \frac{15}{8}\mu R^4 C_{40}\left(\frac{-21xz^4}{r^{11}} + \frac{14xz^2}{r^9} - \frac{x}{r^7}\right)\\
&\frac{\partial U_{50}}{\partial x} = \frac{21}{8}\mu R^5 C_{50}\left(\frac{-33xz^5}{r^{13}} + \frac{30xz^3}{r^{11}} - \frac{5xz}{r^9}\right)\\
&\frac{\partial U_{60}}{\partial x} = \frac{7}{16}\mu R^6 C_{60}\left(\frac{-429xz^6}{r^{15}} + \frac{495xz^4}{r^{13}} - \frac{135xz^2}{r^{11}} + \frac{5x}{r^9}\right)
\end{aligned}\right\} \qquad (6.11)$$

$$\left.\begin{aligned}
\frac{\partial U_{00}}{\partial y} &= -\frac{\mu y}{r^3} \\
\frac{\partial U_{20}}{\partial y} &= \frac{3}{2}\mu R^2 C_{20}\left(\frac{-5yz^2}{r^7} + \frac{y}{r^5}\right) \\
\frac{\partial U_{30}}{\partial y} &= \frac{5}{2}\mu R^3 C_{30}\left(\frac{-7yz^3}{r^9} + \frac{3yz}{r^7}\right) \\
\frac{\partial U_{40}}{\partial y} &= \frac{15}{8}\mu R^4 C_{40}\left(\frac{-21yz^4}{r^{11}} + \frac{14yz^2}{r^9} - \frac{y}{r^7}\right) \\
\frac{\partial U_{50}}{\partial y} &= \frac{21}{8}\mu R^5 C_{50}\left(\frac{-33yz^5}{r^{13}} + \frac{30yz^3}{r^{11}} - \frac{5yz}{r^9}\right) \\
\frac{\partial U_{60}}{\partial y} &= \frac{7}{16}\mu R^6 C_{60}\left(\frac{-429yz^6}{r^{15}} + \frac{495yz^4}{r^{13}} - \frac{135yz^2}{r^{11}} + \frac{5y}{r^9}\right)
\end{aligned}\right\} \quad (6.12)$$

$$\left.\begin{aligned}
\frac{\partial U_{00}}{\partial z} &= -\frac{\mu z}{r^3} \\
\frac{\partial U_{20}}{\partial z} &= \frac{3}{2}\mu R^2 C_{20}\left(\frac{3z}{r^5} - \frac{5z^3}{r^7}\right) \\
\frac{\partial U_{30}}{\partial z} &= \frac{1}{2}\mu R^3 C_{30}\left(\frac{-35z^4}{r^9} + \frac{30z^2}{r^7} - \frac{3}{r^5}\right) \\
\frac{\partial U_{40}}{\partial z} &= \frac{5}{8}\mu R^4 C_{40}\left(\frac{-63z^5}{r^{11}} + \frac{70z^3}{r^9} - \frac{15z}{r^7}\right) \\
\frac{\partial U_{50}}{\partial z} &= \frac{3}{8}\mu R^5 C_{50}\left(\frac{-231z^6}{r^{13}} + \frac{315z^4}{r^{11}} - \frac{105z^2}{r^9} + \frac{5}{r^7}\right) \\
\frac{\partial U_{60}}{\partial z} &= \frac{7}{16}\mu R^6 C_{60}\left(\frac{-429z^7}{r^{15}} + \frac{693z^5}{r^{13}} - \frac{315z^3}{r^{11}} + \frac{35z}{r^9}\right).
\end{aligned}\right\} \quad (6.13)$$

The derivatives of tesseral harmonics may be represented by the general formula

$$\frac{\partial U_{nm}}{\partial q} = k_{nm}\,\mu R^n\Big[\cos ms(Pq_{nm}C_{nm} + Qq_{nm}S_{nm}) + \sin ms(Qq_{nm}C_{nm} - Pq_{nm}S_{nm})\Big] \qquad (6.14)$$

where

q-is identical to x or y or z.

The values of k_{nm} are as follows

$$k_{22} = 3\,,\quad k_{31} = \frac{3}{2}\,,\quad k_{32} = 15\,,\quad k_{33} = 15\,,\quad k_{41} = \frac{5}{2}\,,$$

$$k_{42} = \frac{15}{2}\,,\quad k_{43} = 105\,,\quad k_{44} = 105\,;$$

whereas the respective Pq_{nm} and Qq_{nm}

$$Px_{22} = \frac{-5(x^2-y^2)x}{r^7} + \frac{2x}{r^5}\,; \qquad Qx_{22} = \frac{-10x^2y}{r^7} + \frac{2y}{r^5}$$

$$Px_{31} = \frac{-35x^2z^2}{r^9} + \frac{5(x^2+z^2)}{r^7} - \frac{1}{r^5}; \qquad Qx_{31} = \frac{-35xyz^2}{r^9} + \frac{5xy}{r^7}$$

$$Px_{32} = \frac{-7xz(x^2-y^2)}{r^9} + \frac{2xz}{r^7}\,; \qquad Qx_{32} = \frac{-14x^2yz}{r^9} + \frac{2yz}{r^7}$$

$$Px_{33} = \frac{3(x^2-y^2)}{r^7} - \frac{7x^2(x^2-3y^2)}{r^9}\,; \qquad Qx_{33} = \frac{-7xy(3x^2-y^2)}{r^9} + \frac{6xy}{r^7}$$

$$Px_{41} = \frac{-63x^2z^3}{r^{11}} + \frac{7z(3x^2+z^2)}{r^9} - \frac{3z}{r^7}; \qquad Qx_{41} = \frac{-63xyz^3}{r^{11}} + \frac{21xyz}{r^9}\,;$$

$$Px_{42} = \frac{-63xz^2(x^2-y^2)}{r^{11}} + \frac{7x(x^2-y^2+2z^2)}{r^9} - \frac{2x}{r^7}\,; \qquad (6.15)$$

$$Qx_{42} = \frac{-126x^2yz^2}{r^{11}} + \frac{14y(x^2+z^2)}{r^9} - \frac{2y}{r^7}\,;$$

$$Px_{43} = \frac{-9x^2z(x^2-3y^2)}{r^{11}} + \frac{3z(x^2-y^2)}{r^9}; \qquad Qx_{43} = \frac{-9xyz(3x^2-y^2)}{r^{11}} + \frac{6xyz}{r^9}\,;$$

$$Px_{44} = \frac{-9x((x^2-y^2)^2-4x^2y^2)}{r^{11}} + \frac{4x(x^2-3y^2)}{r^9};$$

$$Qx_{44} = \frac{-36x^2y(x^2-y^2)}{r^{11}} + \frac{4y(3x^2-y^2)}{r^9}$$

$$Py_{22} = \frac{-5y(x^2-y^2)}{r^7} - \frac{2y}{r^5}; \qquad Qy_{22} = \frac{-10xy^2}{r^7} + \frac{2x}{r^5};$$

$$Py_{31} = \frac{-35xyz^2}{r^9} + \frac{5xy}{r^7}; \qquad Qy_{31} = \frac{-35y^2z^2}{r^9} + \frac{5(y^2+z^2)}{r^7} - \frac{1}{r^5};$$

$$Py_{32} = \frac{-7yz(x^2-y^2)}{r^9} - \frac{2yz}{r^7}; \qquad Qy_{32} = \frac{-14xy^2z}{r^9} + \frac{2xz}{r^7};$$

$$Py_{33} = \frac{-7xy(x^2-3y^2)}{r^9} - \frac{6xy}{r^7}; \qquad Qy_{33} = \frac{-7y^2(3x^2-y^2)}{r^9} + \frac{3(x^2-y^2)}{r^7};$$

$$Py_{41} = \frac{-63xyz^3}{r^{11}} - \frac{21xyz}{r^9}; \qquad Qy_{41} = \frac{-63y^2z^3}{r^{11}} + \frac{7z(3y^2+z^2)}{r^9} - \frac{3z}{r^7};$$

$$Py_{42} = \frac{-63yz^2(x^2-y^2)}{r^{11}} + \frac{7y(x^2-y^2-2z^2)}{r^9} + \frac{2y}{r^7};$$

$$Qy_{42} = \frac{-126xy^2z^2}{r^{11}} + \frac{14x(y^2+z^2)}{r^9} - \frac{2x}{r^7};$$

$$Py_{43} = \frac{-9xyz(x^2-3y^2)}{r^{11}} - \frac{6xyz}{r^9}; \qquad Q_{43} = \frac{-9y^2z(3x^2-y^2)}{r^{11}} + \frac{3z(x^2-y^2)}{r^9};$$

$$Qy_{44} = \frac{-36xy^2(x^2-y^2)}{r^{11}} + \frac{4x(x^2-3y^2)}{r^9}; \tag{6.17}$$

$$Py_{44} = \frac{-9y((x^2-y^2)^2 - 4x^2y^2)}{r^{11}} - \frac{4y(3x^2-y^2)}{r^9};$$

$$Pz_{22} = \frac{-5z(x^2-y^2)}{r^7}; \qquad Qz_{22} = \frac{-10xyz}{r^7};$$

$$Pz_{31} = \frac{-35xz^3}{r^9} + \frac{15xz}{r^7}; \qquad Qz_{31} = \frac{-35yz^3}{r^9} + \frac{15yz}{r^7};$$

$$Pz_{32} = \frac{-7z^2(x^2-y^2)}{r^9} - \frac{x^2-y^2}{r^7}; \qquad Qz_{32} = \frac{-14xyz^2}{r^9} + \frac{2xy}{r^7};$$

$$Pz_{33} = \frac{-7xz(x^2-3y^2)}{r^9}; \qquad Qz_{33} = \frac{-7yz(3x^2-y^2)}{r^9};$$

$$Pz_{41} = \frac{-63xz^4}{r^{11}} + \frac{42xz^2}{r^9} - \frac{3x}{r^7}; \qquad Qz_{41} = \frac{-63yz^4}{r^{11}} + \frac{42yz^2}{r^9} - \frac{3y}{r^7};$$

$$Pz_{42} = \frac{-63z^2(x^2-y^2)}{r^{11}} + \frac{21z(x^2-y^2)}{r^9}; \qquad Qz_{42} = \frac{-126xyz^3}{r^{11}} + \frac{42xyz}{r^9};$$

$$Pz_{43} = \frac{-9xz^2(x^2-3y^2)}{r^{11}} + \frac{x(x^2-3y^2)}{r^9} ;$$

$$Qz_{43} = \frac{-9yz^2(3x^2-y^2)}{r^{11}} + \frac{y(3x^2-y^2)}{r^9}$$

$$Pz_{44} = \frac{-9z((x^2-y^2) - 4x^2y^2)}{r^{11}}; \quad Qz_{44} = \frac{-36xyz(x^2-y^2)}{r^{11}} ; \quad (6.17)$$

The numerical methods have gained in significance especially after the introduction of electronic computers.A number of new methods have arisen, adapted to the new possibilities,optimalizing the processes of calculus and programming. Among them is the method given by C. Runge in 1895, improved by W. Kutta in 1901 and adapted to the mechanical computation by S. Gill in 1951. A full description of the Runge-Kutta-Gill method, including the derivation of formulae and the logical scheme of the program is presented in the R o m a n e l l i ' s work [1962]. Those formulae are constituting a simple computation scheme. Suppose that we have n + 1 differential equations of the first order in the form

$$y_i'(x) = f_i(y_o(x), y_1(x) \dots y_n(x)) \quad (6.18)$$

whereat

$$y_o'(x) = f_o = 1 \quad \text{or} \quad y_o(x) = x \quad (6.19)$$

and boundary values

$$y_i(x_o) = y_{io} . \quad (6.20)$$

The indice j will denote the number of the iteration, that is

$$j = 1,2,3,4 .$$

For i = 0,1,2,... n, we calculate

$$y_{ij}' = k_{ij} = f_i(y_{o,j-1}, y_{1,j-1} \dots y_{n,j-1}). \quad (6.21)$$

For $j = 1$, $y_{i,o} = y_i(x_o)$ so this is either the initial value (for the first step) or y_i computed in the preceding step.

Next

$$y_{ij} = y_{i,j-1} + h\left[a_j(k_{ij} - b_j q_{i,j-1})\right] \qquad (6.22)$$

$$q_{ij} = q_{i,j-1} + 3\left[a_j(k_{ij} - b_j q_{i,j-1})\right] \qquad (6.23)$$

where

$$\left.\begin{array}{lll} a_1 = \frac{1}{2} & b_1 = 2 & c_1 = \frac{1}{2} \\ a_2 = 1 - \sqrt{\frac{1}{2}} & b_2 = 1 & c_2 = 1 - \sqrt{\frac{1}{2}} \\ a_3 = 1 + \sqrt{\frac{1}{2}} & b_3 = 1 & c_3 = 1 + \sqrt{\frac{1}{2}} \\ a_4 = \frac{1}{6} & b_4 = 2 & c_4 = \frac{1}{2} \end{array}\right\} \qquad (6.24)$$

h - value of the step.

We iterate the calculation cycle (6.21), (6.22),(6.23) for $j = 2,3,4$. Having the values of $y_{i,4}$ we are proceeding to the next step, substituting them for $y_{i,o}$.

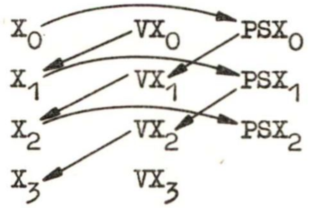
In our case, we have to integrate the following equations:

$$\left.\begin{array}{l} k_o = \frac{dt}{dt} = 1 \\ k_1 = \frac{dx}{dt} = VX \\ k_2 = \frac{dv}{dt} = VY \\ k_3 = \frac{dz}{dt} = VZ \\ k_4 = \frac{d^2x}{dt^2} = \frac{dVX}{dt} = PSX \\ k_5 = \frac{d^2y}{dt^2} = \frac{dVY}{dt} = PSY \\ k_6 = \frac{d^2z}{dt^2} = \frac{dVZ}{dt} = PSZ\ . \end{array}\right\} \qquad (6.25)$$

Initial data: X_0, Y_0, Z_0, VX_0, VY_0, VZ_0.

Since the motion equation is an equation of the second order, it has to be integrated twice; the first equation will give the velocity, the second one - the coordinates.

This is shown in the scheme:



Yet, not the coordinates by themselves are of interest for our subject matter, but the lenght of radius-vector or, more precisely, the variations of it produced by one or another term of the formula for the gravity potential.

With a view to investigating the perturbations caused by a definite term of the n-degree and the m-order from the formula (6.2), we always kept three terms: 00, 20 and the examined term nm, equating the remining ones to zero. In a gravitational field determined in such a way, the motion of satelllite has been computed twice for two different values of coefficients C_{nm}, S_{nm}. There were substituted either the values of harmonics differing one from another by the error value according to definition in Chapter V or zero was substituted for the first pair and the most probable values for the second pair of coefficients C_{nm}, S_{nm}. In this manner the coordinates of the satellite were obtained in the same system, but in two different potential fields. The differences in position is determined by a certain vector $\bar{d}\left[dx,dy,dz\right]$, coordinates of which being next transformed into the nonorthogonal system $\left[dr,du,dw\right]$. The axes of the latter system are defined as follows: r - direction of the radius vector, u - direction of the velocity vector, w - transversal direction selected in such a way that the three vectors dr,du,dw have the same orientation as dx, dy,dz.

It has been adopted that the integration step is equal to one sidereal minute, this corresponding approximately to 1/100 of the revolution period. On account of the circular orbit, the constant integration step has been accepted.

A few words should also be devoted to the characteristics of the computer with the help of which the calculus has been carried out.

It is a GIER machine made in Danemark executing some 10.000 floating-point operations per second. Calculations are carried out with an accuracy to 29 digits in the binary system, this corresponding to approximately $8\frac{1}{2}$ decimal digits, that means that the roundings appear when the number is greater than $2^{29} = 536\,870\,912$. The programming is performed in the ALGOL 60 language, so it is relatively easy to be mastered. The programs are universal and may be used by others at different computation centers. For this reason, the author is enclosing herewith the copies of programs elaborated and utilized by him.

For performing the integration process, a programme termed RKG had been established. The following input data were being introduced into machine: number of the variant computed, two differing values of coefficient C_{nm}, two values for S_{nm}, number of steps, coefficient k_{nm} and m (as taken from the formula 6.14), semimajor axis in megameters, exxentricity, inclination, argument of perigeum and R.A. of node in degree. The geophysical constants were used

$$\mu = 398603 \text{ km}^3\text{sec}^{-2}$$

$$C_{20} = 0.0010827$$

$$R = 6.378165 \text{ Mgm.}$$

The programme is conceived so that the fragment computing PX,QX,PY,QY,PZ,QZ, is replaceable with appropriatenes to the evaluated variant. Most of calculations were performed for the orbit: a = 7.398 Mgm, e = 0.0025, $i = 80^\circ\!.5$, $\omega = 0$, $\Omega = 0$ (satellite Alouette). The computations were made according to the following sequence:

1) Variant 20 - effect of error of the term C_{20} amounting to $\Delta C_{20} = 5.10^{-7}$ Fig. 7.
2) Variant 30 - effect of the term $C_{30} = +245.10^{-8}$
3) Variant 30 - effect of error of the term C_{30}: $\Delta C_{30} = 15.10^{-8}$ Fig. 8
4) Variant 40 - effect of the term $C_{40} = +16.10^{-7}$

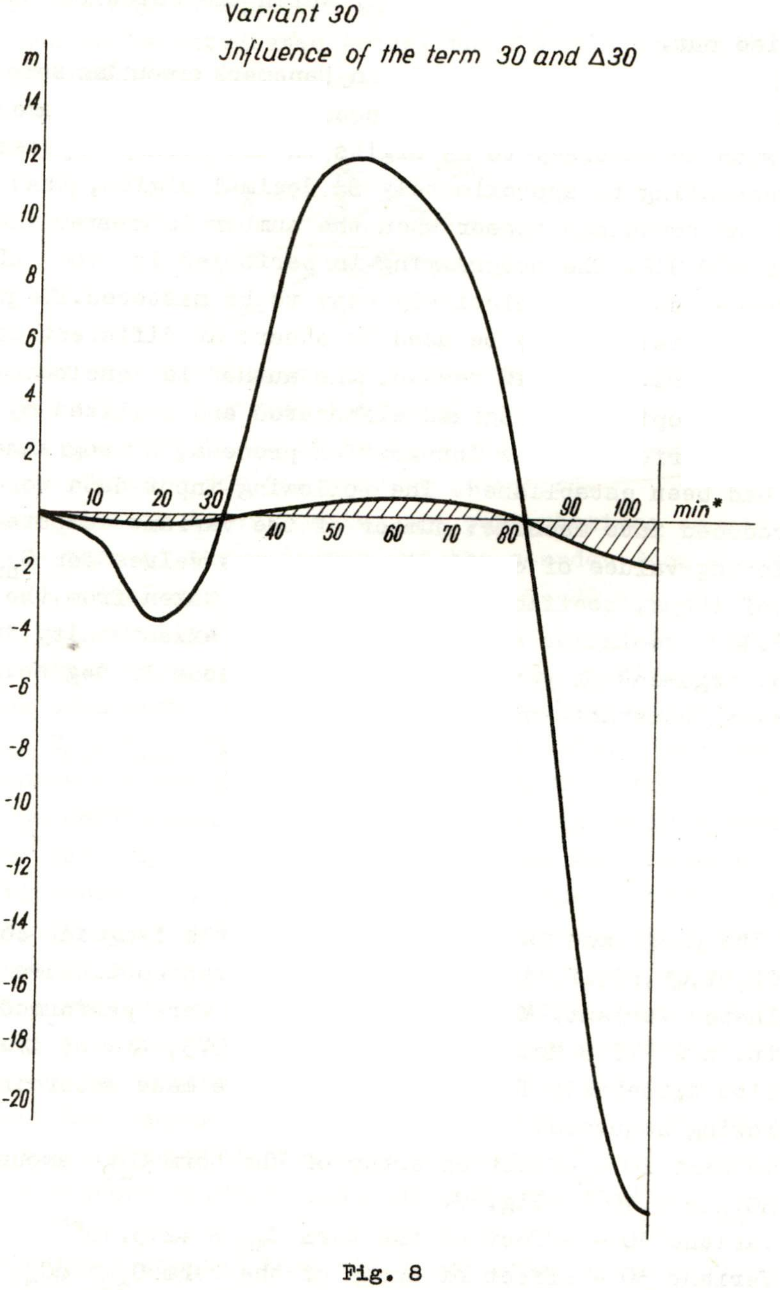

Variant 30
Jnfluence of the term 30 and Δ30
m
14
12
10
8
6
4
2
-2
-4
-6
-8
-10
-12
-14
-16
-18
-20
10
20
30
40
50
60
70
80
90
100
min*

Fig. 8

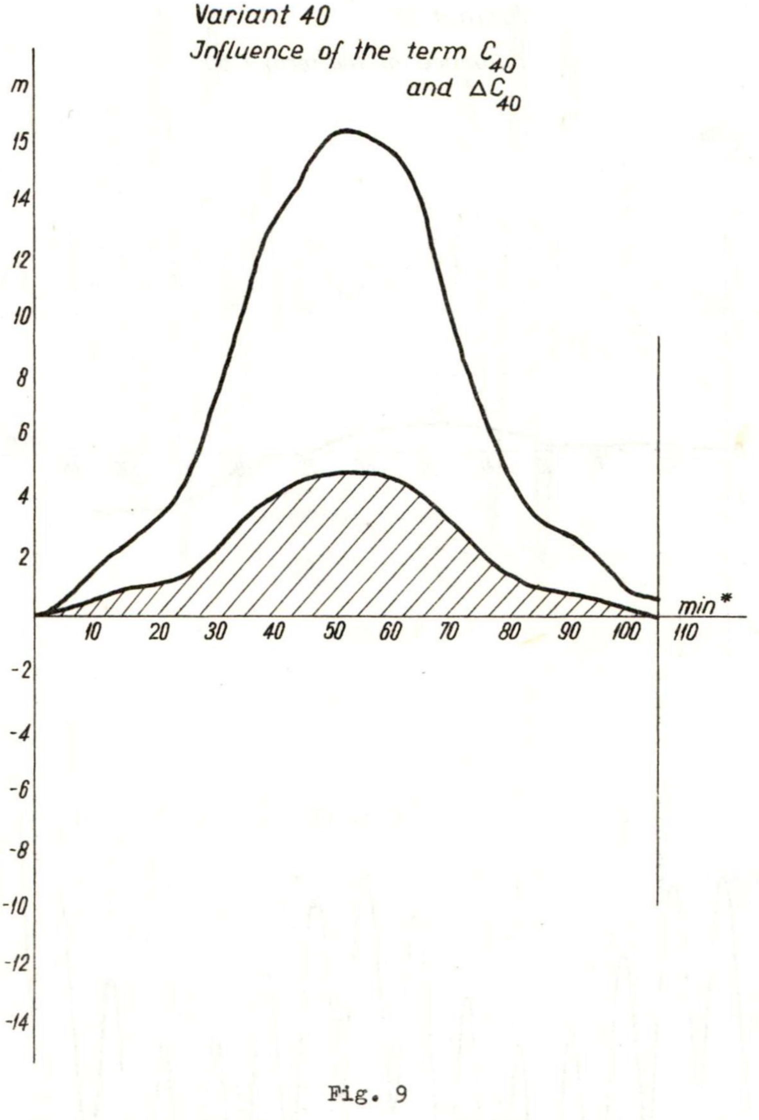


Fig. 9

5) Variant 40 - effect of error of the term $C_{40} = 5 \cdot 10^{-7}$ Fig.9
6) Variant 50 - effect of the term $C_{50} = +15 \cdot 10^{-8}$ Fig. 10
7) Variant 22 - effect of the terms $C_{22} = +115 \cdot 10^{-8}$ and $S_{22} = -125 \cdot 10^{-8}$ Fig. 11.
8) Variant 22 - effect of errors of terms C_{22} and S_{22} amounting to $C_{22} = 70 \cdot 10^{-8}$, $\Delta S_{22} = 100 \cdot 10^{-8}$.

Variant 50

Jnfluence of the term C50

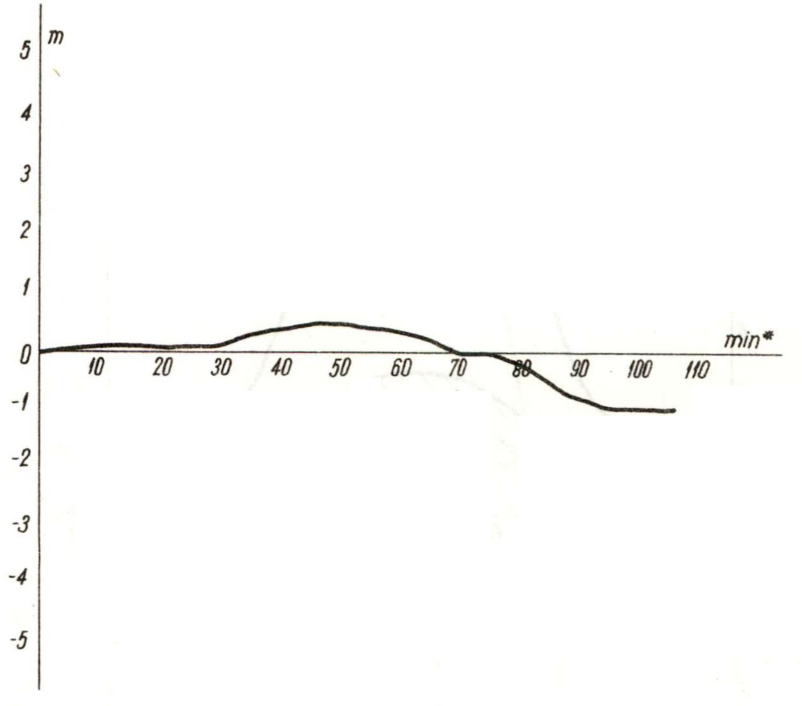


Fig. 10

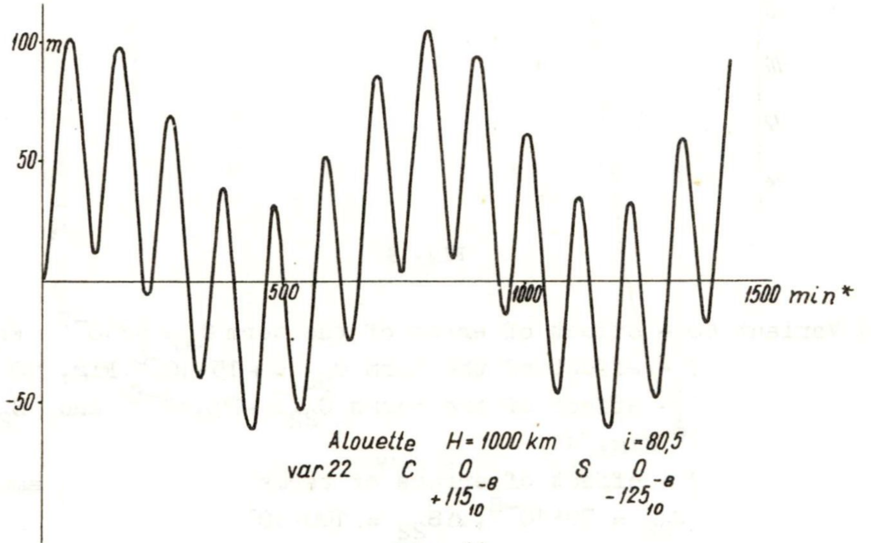


Fig. 11

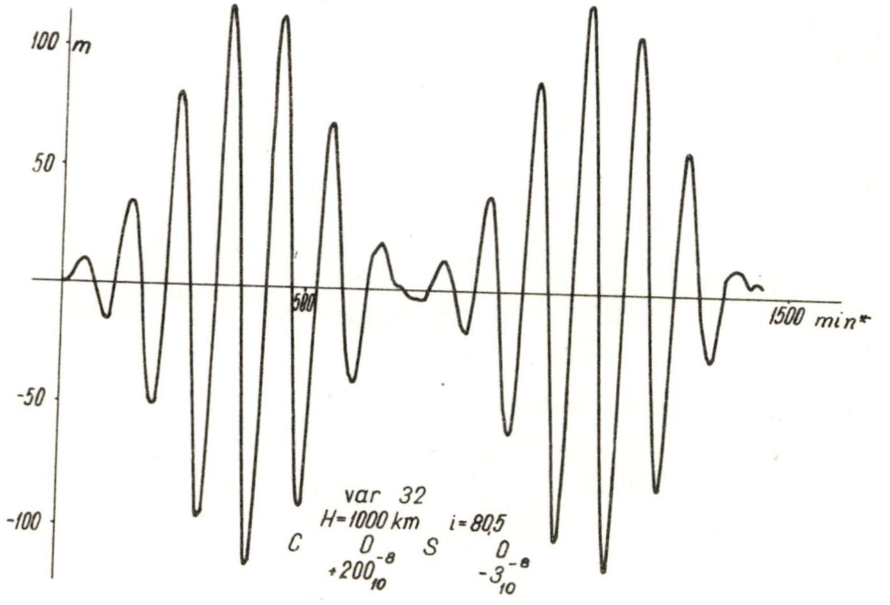

100 m
50
-50
-100
500
1500 min*
var 32
H=1000 km i=80,5
C 0 +200 10^-8
S 0 -3 10^-8

Fig. 12

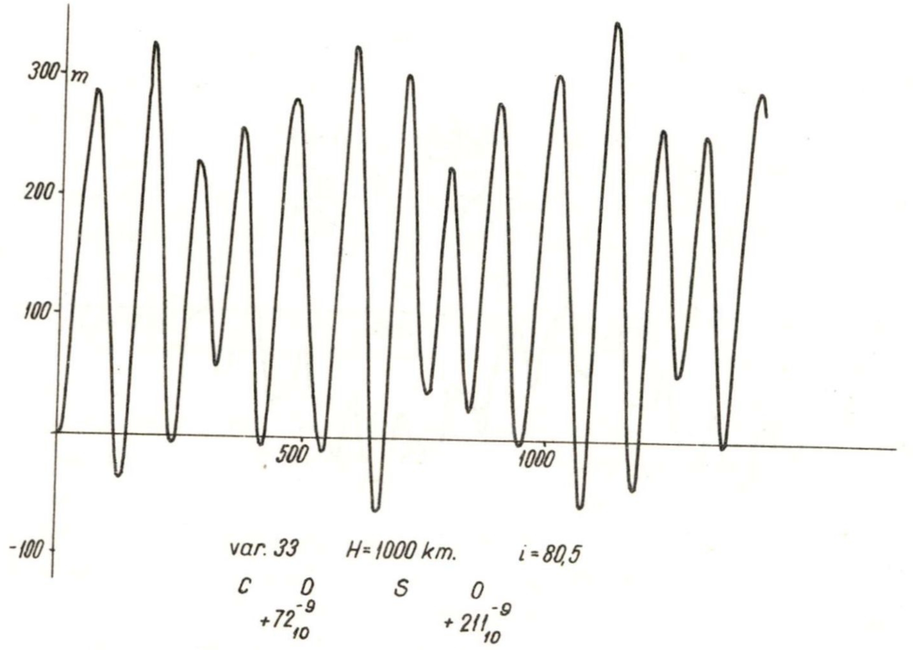
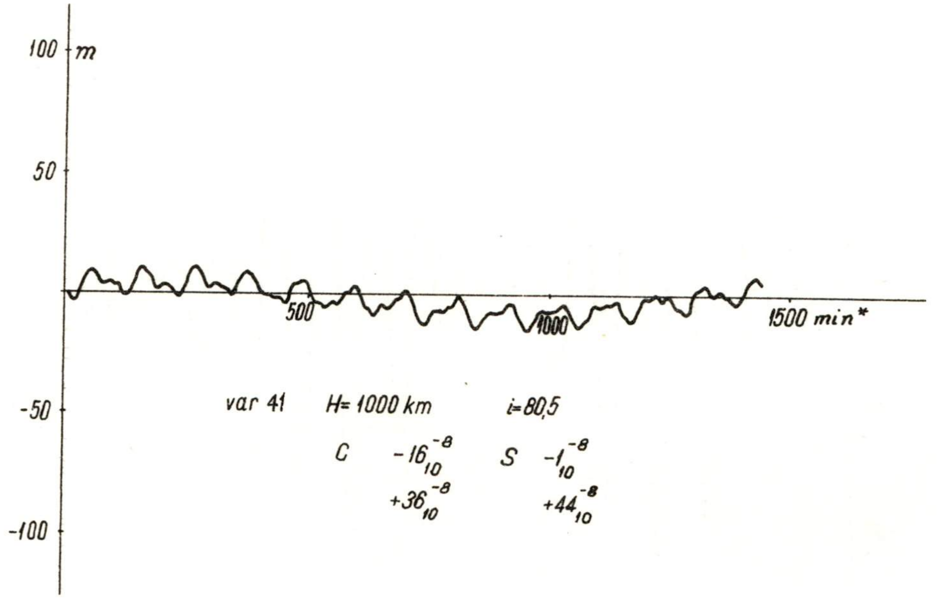
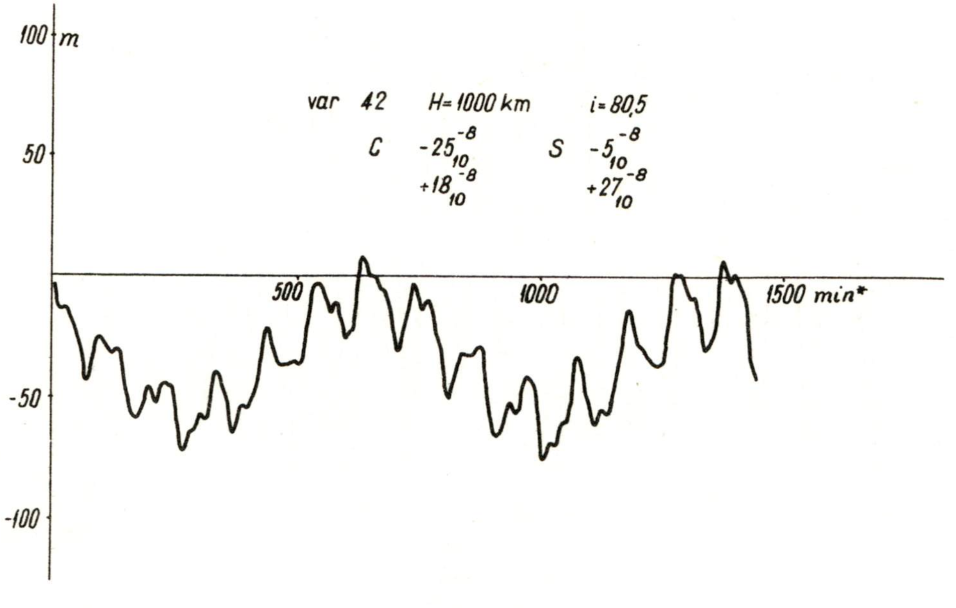

300 m
200
100
-100
500
1000
var. 33 H=1000 km. i=80,5
C 0 +72 10^-9
S 0 +211 10^-9

Fig. 13



100 m
50
-50
-100
500
1000
1500 min*
var 41 H= 1000 km i=80,5
C -16 10-8 S -1 10-8
+36 10-8 +44 10-8

Fig. 14



100 m
50
-50
-100
var 42 H=1000 km i=80,5
C -25 10-8 S -5 10-8
+18 10-8 +27 10-8
500
1000
1500 min*

Fig. 15

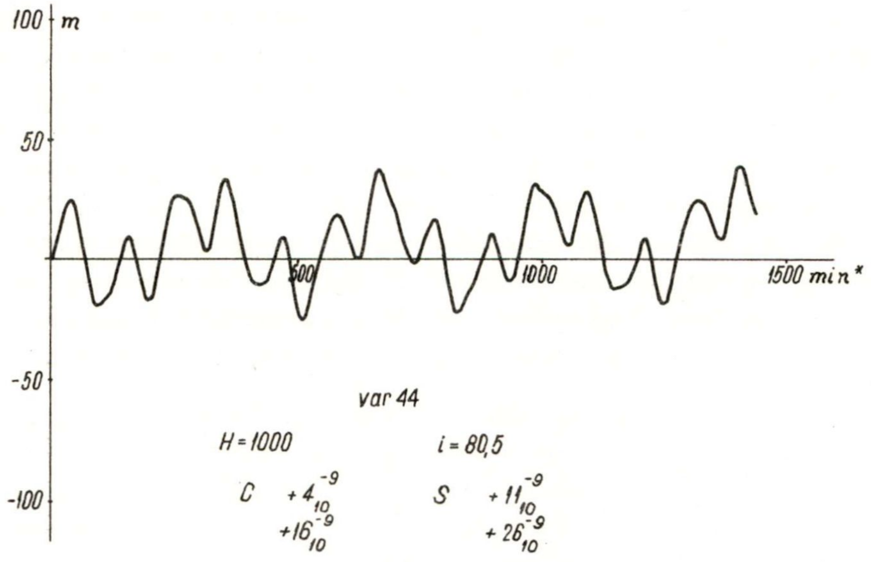


Fig. 16

9) Variant 32 - effect of the terms $C_{32} = +200 \cdot 10^{-8}$ and $S_{32} = -3 \cdot 10^{-8}$, Fig. 12.

10) Variant 33 - effect of the terms $C_{33} = +72 \cdot 10^{-9}$ and $S_{33} = +211 \cdot 10^{-9}$ Fig. 13.

11) Variant 41 - effect of the errors of terms C_{41} and S_{41} amounting to $\Delta C_{41} = 52 \cdot 10^{-8}$, $\Delta S_{41} = 45 \cdot 10^{-8}$ Fig. 14

12) Variant 42 - effect of the errors of terms C_{42} and S_{42} amounting to $\Delta C_{42} = 43 \cdot 10^{-8}$, $\Delta S_{42} = 32 \cdot 10^{-8}$ Fig. 15

13) Variant 44 - effect of the errors of terms C_{44} and S_{44} amounting to $\Delta C_{44} = 12 \cdot 10^{-9}$, $\Delta S_{44} = 15 \cdot 10^{-9}$, Fig. 16.

The integration outcomes are presented in the form of diagrams.

The following conclusions can be induced: the accuracy known for the coefficients of zonal harmonics allows for the radius-vector to be computed with an accuracy of 10^{-6}. For this purpose, the terms up to C_{40} ought to be taken into account, the accuracies known for them being adequate. As regards the perturbations produced by harmonics C_{22}, S_{22} they exceed 100 m, simultaneously, however, the accuracy of coefficients will produce an error in the determination of the lenght of radius-vector, amounting to 70 m, this giving an accuracy 10^{-5}.

A certain surprise for the author were the absolute quantities of perturbations of third degree. After having made the evaluations pertaining to the harmonics 22, we could suppose that the next terms would give smaller effects and that their omission would not bring about a drop in accuracy in computing the radius-vector below 10^{-5}. Yet the further computations have showed that in the case of term 33, their effect exceed even 300 meters for the same orbit. So, this terms can not be ommited, anyway. The influence of the errors of fourth degree harmonics can still bring about inaccuracy of the order of 70 m.

In order to examine the influence of the orbital inclination, the variant 32 was calculated for four values: $i = 80\overset{\circ}{.}5$, $63\overset{\circ}{.}4$, 30°, 0°. It appeared that only the amplitude of perturbations is undergoing variations, the characteristic shape of the curve is hardly changing: even for $i = 0$ it remains as it was. The greatest perturbations occur with the mean inclinations of the orbit.

Moreover, computations were made, changing the height of the orbit. It was found that the quantity of the perturbation diminishes rather slowly with the growth of a. With the change of H_p from 1000 km to 3500 km the effect of the terms 22 decreases by nearly 1/5, of the terms of the third degree - by nearly 1/4, and of the fourth degree - by about 1/2. This means that even with orbits of the Midas 4 or Geos type - if we want to acquire the lenght of the radius-vector to an accuracy of 10^{-6} - the effect of terms of the fourth degree must be taken into account.

As indicated hereinbefore, simultaneously with the evaluation of perturbations in the radius-vector - the programme showed changes in the position of satellite along the trajectory (along-track variations) and in the direction perpendicular to it (across-track variations). It follows from the computations that the radial changes are, as a rule, smaller than the tangential changes, but slightly greater (though not always) than transversal ones.

CHAPTER VII

OTHER PERTURBATIONS

SECTION 1. ATMOSPHERIC DRAG

The atmospheric drag can be fairly called the major ennemy of geodetic satellites. It is for this reason that we have already at the beginning adopted assumptions tending to minimize the perturbations caused by the resistance of the medium. And so, we have adopted an orbit approximate to the circular one, at an altitude of more than 1000 km over the Earth's surface, and a satellite of a small area-to-mass ratio (the so-called heavy satellite of the type of Anna 1 B or Alouette). It might be said, in addition, that in our case only the short-period and diurnal perturbations seem to be dangerous because they do not find reflection in variations of the mean elements which can be determined from observations. We shall now try to estimate the order of magnitude of those perturbations and to find out whether they can be regarded as negligibly small.

Let us briefly recall the mechanism of the arising of perturbations produced by the atmospheric drag. Suppose that the atmosphere remains motionless with regard to the orbital plane whose eccentricity $\neq 0$. Moving in a medium offering resistance, the satellite consumes a part of its kinetic energy and loses by the same it's velocity. Since the greatest retardation occurs in the vicinity of the perigee, it is there that the largest velocity variations take place, this producing a change in the shape and in the dimensions of the orbit; it should be added that the distance to the perigee will show markedly slower variations than the distance to the apogee. Thus we shall have secular variations of two elements of the semi-axis a,

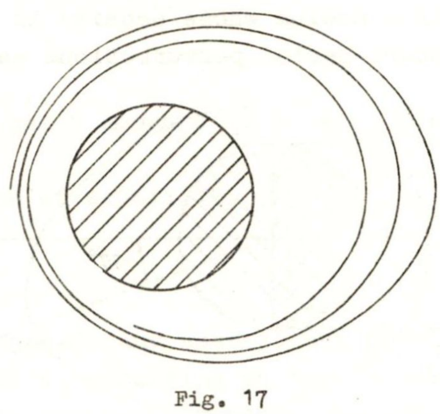


Fig. 17

and in consequence of the period P and the mean motion n, and of the eccentricity e. Still, in addition to these secular variations leading to the circular shape taken by the orbit and to the lowering of the altitude at which the satellite is moving until it collapses, short-period variations may be expected to occur, for at each point of the retardation force has a different magnitude which can be represented as a function of anomaly. It is only in the case of an orbit being circular and the atmosphere having a spherical structure that the periodical terms in perturbations of elements could be avoided. Unfortunately, the atmosphere does not represent an ideal spheroidal structure. As demonstrated by explorations based on observations of artificial satellites, the atmosphere has rather an ovoid shape produced by the difference in temperatures of the daytime and the nighttime side of the Earth globe. The axis of the bulge forms an angle of near 30^{0} in right ascension with the Earth-to-Sun direction and is orientated eastward from it. Moreover, owing to the rotation about the Earth's axis, a flattening of the atmosphere can be noticed. All that causes that even a satellite being in circular orbit, is moving in a medium whose density is a function of the place and so, short-period perturbations may occur.

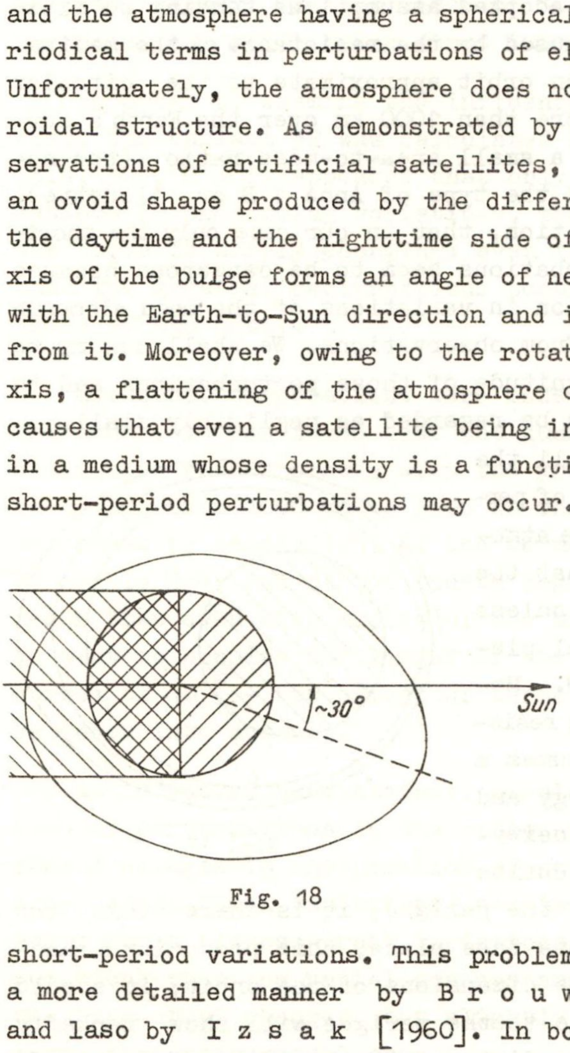


Fig. 18

If a great number of studies is devoted to the problem concerning long-period and secular variations, the theory of their computation, the determination, the atmospheric parameters from observations, rather a small number of authors have dealt with the short-period variations. This problem has been approached in a more detailed manner by B r o u w e r and H o r i [1961] and laso by I z s a k [1960]. In both cases though concrete, but rather complicated, models of the atmosphere have been adopted, this causing that after the integration of differential perturbation equations, very complicated formulae had been obtained. For our purposes, the formulae given by B a t r a k o v and P r o s k u r i n [1959] shall be used

$$\left.\begin{aligned}
\frac{da}{dt} &= -\frac{2\alpha\varrho n a^2}{(1-e^2)^{3/2}}(1+2e\cos v+e^2)^{3/2}\\
\frac{de}{dt} &= -\frac{2\alpha\varrho n a}{(1-e^2)^{1/2}}(1+2e\cos v+e^2)^{1/2}(e+\cos v)\\
\frac{d\Omega}{dt} &= \frac{di}{dt} = 0\\
e\frac{d\omega}{dt} &= -\frac{2\alpha\varrho n a}{(1-e^2)^{1/2}}(1+2e\cos v+e^2)^{1/2}\sin v\\
\frac{d\varepsilon}{dt} &= \frac{2\alpha\varrho n a e}{(1-e^2)^{1/2}}(1+2e\cos v+e^2)^{1/2}\left(\frac{(1-e^2)^{1/2}}{1+e\cos v}+\right.\\
&\quad\left.-\frac{1}{1+\sqrt{1-e^2}}\right)\sin v,
\end{aligned}\right\}\quad(7.1)$$

where

$\alpha = \frac{1}{2} C_x \frac{S}{m}$

ϱ = density of atmosphere,

C_x = aerodynamic coefficient which amounts to 2 for this type of motion

S = area of the cross-section of satellite,

m = mass of satellite,

ε = longitude in orbit.

The equations (7.1) do not contain any assumptions pertaining to the law of variability of ϱ, according to the place. The only assumption introduced is the neglecting of the rotary motion of the atmosphere; owing to this, $d\Omega = di = 0$. In our case, where the question is of examining the order of magnitude rather than an accurate determination of perturbations - and when small quantities are to be expected - we may allow ourselves for-going simplifications.

Let us assume a circular orbit in the equatorial plane: $e = 0$, $i = 0$, $a = 7378$ km. Thus

$$v = M = n \cdot t \tag{7.2}$$

and the formulae (7.1) will have the form

$$\left.\begin{aligned} \delta a &= -2\alpha n a^2 \cdot \int \varrho dt \\ \delta e &= -2 n a \cdot \int \cos M \varrho dt \\ \delta \varepsilon &= 0 . \end{aligned}\right\} \tag{7.3}$$

$\delta\omega$ for $e = 0$ takes indeterminate values, so we have not to take it into consideration.

Further, let the satellite be of the shape and of the dimensions of Alouette

$$S = 0{,}735 \text{ m}^2, \quad m = 145 \text{ kg} .$$

Therefore

$$\alpha = 0{,}0051 \text{ m}^2/\text{kg} = 0{,}051 \text{ cm}^2/\text{g}$$

$$n^2a^2 = \mu/a = 54{,}0259 \text{ km}^2/\text{sek}^2$$

$$na = 7{,}35023 \text{ km/sek} = 7{,}35023 \cdot 10^5 \text{ cm/sek}$$

$$n = 0{,}0009962 \text{ sek}^{-1}$$

$$na^2 = 54230 \text{ km}^2/\text{sek} = 5{,}4230 \cdot 10^{14} \text{ cm}^2/\text{sek}$$

$$2\alpha n a^2 = 0{,}553 \cdot 10^{14} \text{ cm}^4/\text{g} \cdot \text{sek}$$

$$2\alpha n a = 0{,}750 \cdot 10^5 \text{ cm}^3/\text{g} \cdot \text{sek} .$$

In order to compute the value of the integral $\int \varrho \, dt$, we are going to use the model of atmosphere given by M a r t i n and others [1961]. It is a model based on observations of artificial satellite, which takes into account the daytime bulge, upon assumption of the mean level of solar activity. It is presented in the form of a table containing ϱ min and ϱ max for the given altitude, and of a formula for calculating ϱ at an arbitrary place. This model is certainly not an ideal one, yet possesses the quality of being relatively simple and convenient in application. Besides, it agrees quite well with other, more recent and more complicated models.

The model presented by Martin and others gives for the altitude of 1000 km

$$\rho_{max} = 6{,}72 \times 10^{-17}\ g/cm^3 \qquad \rho_{min} = 2{,}64 \times 10^{-18}\ g/cm^3$$

Another model formulated by P a e t z o l d and Z s c h r ö n e r [1961] gives respectively

$$\rho_{max} = 9{,}701 \times 10^{-17}\ g/cm^3 \qquad \rho_{min} = 4{,}313 \times 10^{-18}\ g/cm^3$$

The Nicolet-Jacchia [J a c c h i a 1964] model considering many parameters, gives

$$\rho_{max} = 1{,}97 \cdot 10^{-17} \qquad \rho_{min} = 5{,}86 \cdot 10^{-18}$$

assuming that the coefficient of solar activity has in the 10,8 - centimeterband the following average value

$$F_{10,8} = 200 .$$

So it can be seen that the model of Martin and others may be accepted without fear of making a mistake of an order of magnitude. Martin gives the following formulae for the computation of ρ

$$\log \rho(h,\Theta) = \log \rho_{min}(h) + q(\Theta) \cdot \left[\log \rho_{max}(h) - \log \rho_{min}(h)\right], \qquad (7.4)$$

where

h - altitude above the Earth's surface

Θ - real solar time.

The values of $q(\Theta)$ are presented in Table 4 col. 3.

Substituting the respective values in the equation (7.4), we obtain the values of $\rho(1000,\Theta)$, as shown in Table 4, col.6.

$\Delta\alpha$ - stands for the difference of the right ascension between the Sun and the given point in the space; $\log \rho_{max} = 0{,}827 - 17$, $\log \rho_{min} = 0{,}421 - 18$, $\log \rho_{max} - \log \rho_{min} = 1{,}406$.

Table 4

$\Delta\alpha$	θ	$q(\theta)$	$q(\theta)\cdot(\log\rho_{max} - \log\rho_{min})$	$\lg\rho$	$\rho(g/cm^3)$
1	2	3	4	5	6
180°	0^h	0.183	0.258	$\overline{18}.678$	$4.76\ 10^{-18}$
210	2	0.106	0.149	$\overline{18}.570$	$3.72\ 10^{-18}$
240	4	0.030	0.042	$\overline{18}.463$	$2.90\ 10^{-18}$
260	5^h20^m	0.000	0.000	$\overline{18}.421$	$2.64\ 10^{-18}$
270	6	0.020	0.028	$\overline{18}.449$	$2.81\ 10^{-18}$
300	8	0.334	0.470	$\overline{18}.891$	$7.78\ 10^{-18}$
330	10	0.728	1.024	$\overline{17}.445$	$2.79\ 10^{-17}$
0	12	0.942	1.324	$\overline{17}.755$	$5.69\ 10^{-17}$
30	14	1.000	1.406	$\overline{17}.827$	$6.72\ 10^{-17}$
60	16	0.935	1.315	$\overline{17}.736$	$5.44\ 10^{-17}$
90	18	0.724	1.018	$\overline{17}.439$	$2.75\ 10^{-17}$
120	20	0.463	0.651	$\overline{17}.072$	$1.18\ 10^{-17}$
150	22	0.276	0.388	$\overline{18}.809$	$6.44\ 10^{-18}$
180	24	0.183	0.257	$\overline{18}.678$	$4.76\ 10^{-18}$

The integration will be made in the way of approximation, by the summation of products of the density ρ multiplied by the corresponding time interval. We shall observe the motion of the satellite starting with the point lying opposite to the Sun, on the other side of the Earth: ($\Delta\alpha = 180°$).

As seen from the Table 5, the explored perturbations are very small. The secular perturbations of the semi-major axis amount to a few centimeters during one revolution, the periodic variations being fully negligible. The same is to be said in respect of the eccentricity, where the variations appear at the ninth digit after the point. This seems to entitle us to state that in the present subject matter, with so selected orbital and aerodynamic conditions, the short-period and diurnal perturbations produced by the atmospheric drag are practically of no significance.

Table 5

$\Delta\alpha$	Δt sek	$\varrho \cdot \Delta t$ $g \cdot cm^{-3} \cdot sek$	Δa cm	cos M	$\varrho \cdot \Delta t \cdot \cos M$	Δe
1	2	3	4	5	6	7
180°	263	1.25 10^{-15}	-0.07	1.000	1.25 10^{-15}	- 1 10^{-10}
210	526	1.96 10^{-15}	-0.18	0.866	1.70 10^{-15}	- 2 10^{-10}
240	438	1.27 10^{-15}	-0.25	0.500	0.64 10^{-15}	- 3 10^{-10}
260	263	0.69 10^{-15}	-0.29	0.174	0.12 10^{-15}	- 3 10^{-10}
270	350	0.98 10^{-15}	-0.34	0.000	0.00	- 3 10^{-10}
300	526	4.09 10^{-15}	-0.57	-0.500	-2.04 10^{-15}	- 1 10^{-10}
330	525	14.66 10^{-15}	-1.38	-0.866	-12.70 10^{-15}	+ 8 10^{-10}
0	526	29.90 10^{-15}	-3.03	-1.000	-29.90 10^{-15}	+ 31 10^{-10}
30	525	35.31 10^{-15}	-4.98	-0.866	-30.58 10^{-15}	+ 54 10^{-10}
60	526	28.59 10^{-15}	-6.56	-0.500	-14.30 10^{-15}	+ 64 10^{-10}
90	525	14.45 10^{-15}	-7.36	0.000	0.00	+ 64 10^{-10}
120	526	6.20 10^{-15}	-7.71	0.500	3.10 10^{-15}	+ 62 10^{-10}
150	525	3.38 10^{-15}	-7.89	0.866	2.93 10^{-15}	+ 60 10^{-10}
180	263	1.25 10^{-15}	-7.96	1.000	1.25 10^{-15}	+ 59 10^{-10}

SECTION 2. EFFECT OF THE SOLAR AND LUNAR ATTRACTION

The influence of the Sun and the Moon on the motion of the artificial satellite are differring each from other only as a matter of quantity, while qualitatively they are the same .Let us first consider the influence of the Moon. We shall make here use of the formulae for the perturbation function of the exterior body. known from the celestial mechanics

$$R = \mu'\left(\frac{1}{|\bar{r} - \bar{r}'|} - \frac{\bar{r} \cdot \bar{r}'}{r'^3}\right), \tag{7.5}$$

where

μ' - the mass of the disturbing body multiplied by the gravitational constant,

$\bar{r}$ - the radius-vector of satellite,

$\bar{r}'$ - the radius-vector of the disturbing body.

Expanding $\frac{1}{|\bar{r} - \bar{r}'|}$ in series of Legendre's functions and remembering that

$$\bar{r} \cdot \bar{r}' = r \cdot r' \cos\psi \ ,$$

we obtain

$$R = \frac{\mu'}{r'}\left[1 + \frac{r}{r'}\cos\psi + \frac{1}{2}\left(\frac{r}{r'}\right)^2(3\cos^2\psi - 1) - \frac{r\,r'}{r'^2}\cos\psi\right] =$$

$$= \frac{\mu'}{r'}\left[1 + \frac{1}{2}\left(\frac{r}{r'}\right)^2(3\cos 2\psi - 1)\right], \qquad (7.6)$$

with an accuracy to the terms of the second order.

In the formula (7.6) only the second term will produce short-period perturbations, because of being the function of the angle ψ. Let us make an evaluation of the magnitude of this term.

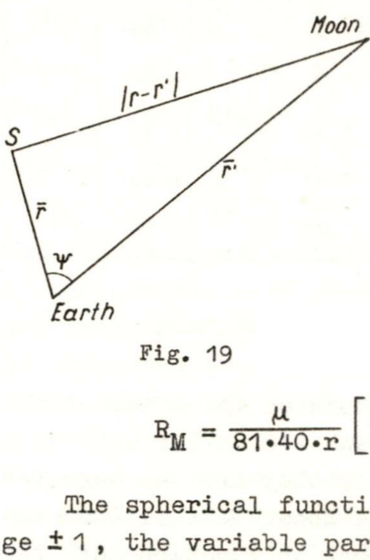


Fig. 19

In the case of the Moon μ' amounts to 1/81 of the terrestrial μ, $r' = 380\ 000$ km, r = will be adopted for 9500 km (the altitude of the satellite orbit being approximately 3000 km).

The ratio $\frac{r}{r'}$ will be 1/40.

Therefore

$$R_M = \frac{\mu}{81 \cdot 40 \cdot r}\left[1 + \frac{1}{2}\left(\frac{1}{40}\right)^2 (3\cos 2\psi - 1)\right].$$

The spherical function P_{20} taking the values in the range ± 1, the variable part of the formula (6) will assume values in the range

$$\frac{\mu}{r} \cdot 2 \cdot 10^{-7} \ .$$

For the Sun, $\mu' = 332\ 000\ \mu$, and $r' = 15\ 000\ r$.

Thus it will be

$$R_M = \frac{332}{15}\frac{\mu}{r}\left[1 + \frac{1}{2}\left(\frac{1}{1500}\right)^2 (3\cos 2\psi - 1)\right]$$

the varying part of the formula being embodied in the range

$$\frac{\mu}{r} \cdot 22 \cdot \frac{1}{225.10^6} = \frac{\mu}{r}\, 1.10^{-7} .$$

Both of these magnitudes are comparable with the effect of the error found for the harmonic C_{20}. As remembered from the Chapter VI, this error, whose value was assumed to be 5.10^{-7}, gave us perturbations of the order of one meter. Since we have to do in this case with a still smaller quantity, it can be concluded that the solar and lunar effects need not be taken into account.

SECTION 3. EFFECT OF THE SOLAR RADIATION PRESSURE

It would not be necessary at all to consider the influence of the light pressure if the satellite moving in orbit were to travel all the time in Sun rays. The force exerted by the light pressure would then be constant and could produce only secular variations in the orbit. Yet since mostly a part of the orbit lies in the area of the Earth's shadow, that factor ceases to act, and the quantity of the additional acceleration becomes dependent upon the position of satellite in orbit. Let us try to evaluate the quantities of the short-period perturbations produced by this situation. For this purpose, we shall use the method applied by W y a t t [1963] and P o l i a k h o v a [1963], where a comparision of the effect of the solar radiation pressure and the atmospheric effect is being made.

The acceleration caused by the light pressure is expressed by the formula

$$F_L = p \cdot \frac{S}{m} , \qquad (7.7)$$

where by

p - denotes the solar light pressure at the distance of the astronomical unit, exerted on a perfectly reflecting body, S - the area of the acting cross-section of satellite,

m - the mass.

The acceleration produced by the atmospheric drag can be expressed as

$$F_A = -\frac{1}{2}Cx \cdot \varrho \cdot v^2 \cdot \frac{S}{m}, \qquad (7.8)$$

where

Cx - aerodynamic coefficient,

ϱ - atmospheric density

v - velocity of satellite.

The ratio of quantities (7.7) and (7.8) will be independent of the characteristics of satellite

$$\left|\frac{F_L}{F_A}\right| = +\frac{2\,p}{Cx\,\varrho v^2}. \qquad (7.9)$$

Substituting now

$$p = 0{,}9 \cdot 10^{-4}\ g \cdot cm^{-1}\ sek^{-2}$$

$$Cx = 2$$

$$\varrho = 76.10^{-17}\ g \cdot cm^{-3}$$

$$v = 7{,}3 \cdot 10^5\ cm \cdot sek^{-1}$$

$$\left|\frac{F_L}{F_A}\right| = +\frac{1{,}8 \cdot 10^{-4}}{714{,}0 \cdot 10^{-7}} = +2{,}5\ .$$

Thus, the effect of the radiation pressure at an altitude of 1000 km is some 2,5 times stronger than the atmospheric effect. Yet in Sec. 7.1 we have demonstrated that in the region of the maximal density the atmospheric effect may give perturbations of the order of a few centimeters. So, even a tenfold stronger effect will give disturbances smaller than one meter, this being below the limit of accuracy adopted by us.

As a matter of fact, the authors of the afore-mentioned studies as well as others, like K o z a i [1963 d], for example, do not deal with the question of short-period perturbations, considering only secular terms which also depend on whether or not the satellite passes through the shadow of the Earth.

SECTION 4. OTHER POSSIBLE SOURCES OF DISTURBANCE

We are going but mention here some other factors acting upon the motion of satellite to a markedly lesser degree than the three preceding effects. Those are:

- the effect of the Earth's magnetic field,
- the effect of the electrostatic field existing in the ionosphere,
- the effect of the radiation reflected from the Earth,
- the collisions with micrometeorites,
- the relativity effect.

This list could be complemented, according to the law of interdependence of phenomena in the Nature. Yet, all those factors produce but minimal effects, and their influence on the results of any observations is hardly perceptible.

C H A P T E R VIII

APPLICATION OF THE RADIUS-VECTOR THEORY

This Chapter presents some possible applications of the theory of the artificial satellite radius-vector.They are given successively, starting with the most particular solutions imposing the fulfillment of certain specified orbital and observational conditions and ending with the quite general procedures allowing to utilize arbitrary orbits (selected, evidently, within the framework of restrictions assumed by the basic principles of the theory) as well as observations carried out in the standar way. This sequence is also in accordance with the chronology of formation of the particular concepts.

The first one was the project of observation of the satellite ALOUETTE, presented during the Conference of Observers of Artificial Satellites in Moscow [Z i e l i ń s k i 1963].

We are going to discuss it hereinafter as the project of observations of a circum-polar satellite for the determination of the radius of the Earth's parallel. The other concepts have sprung up as the particular problems had arisen - such as the tetrahedron method in triangulation - and as the work on the subject-matter was progressing. There exist probably also some other possibilities of utilizing the radius-vector of the artificial satellite, susceptible to lead to interesting solutions.

SECTION 1. DETERMINATION OF THE EARTH'S PARALLEL RADIUS

The objective of our program is to determine the Earth's parallel radii of the observing site for the investigation of the shape and the dimensions of the Earth. If we observe the satellite S from the point P twice during two consecutive passages through the great circle of the topocentric equator, we obtain a situation as in Fig. 15: the arc represents the parallel of the observing site, or more precisely, the fragment of circle circumscribed by the parallel radius of the observing site.

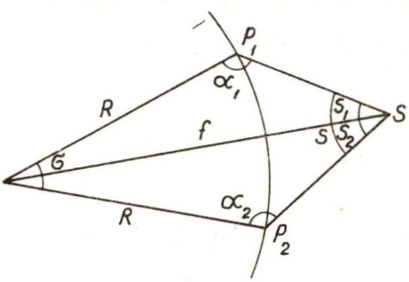


Fig. 20

After a full revolution, the satellite will be almost at the same position as before (except for the translatory motion of the Earth). During this time, the Earth will perform a revolution at an angle of σ equal to the draconic period P, this angle to be defined from the observations. The angles OP_1S and SP_2O can be determined from observations too. Assuming that we have the quantity of f, we may without difficulty solve the tetragon OP_1SP_2 and calculate the value of R (after having introduced the correction for the displacement of the point S, produced by the motion of the orbital nodes).

From ΔOP_1S

$$\frac{f}{\sin \alpha_1} = \frac{R}{\sin s_1} . \tag{8.1}$$

From ΔOP_2S

$$\frac{f}{\sin\alpha_2} = \frac{R}{\sin s_2} \tag{8.1}$$

$$\frac{\sin\alpha_2}{\sin\alpha_1} = \frac{\sin s_2}{\sin s_1}$$

yet as

$$s_1 + s_2 = s$$

thence

$$\frac{\sin\alpha_2}{\sin\alpha_1} = \frac{\sin(s - s_1)}{\sin s_1} = \sin s \cdot \mathrm{ctg}\, s_1 - \cos s \ ,$$

where from

$$\mathrm{ctg}\, s_1 = \frac{\sin\alpha_2}{\sin\alpha_1 \cdot \sin s} + \mathrm{ctg}\, s \ . \tag{8.2}$$

Having s_1, we compute

$$R = \frac{f \cdot \sin s_1}{\sin\alpha_1} = \frac{f \cdot \sin s_2}{\sin\alpha_2} \ . \tag{8.3}$$

The element f is found using the formulae

$$f = r \cdot \cos \varphi_s \tag{8.4}$$

$$\sin\varphi_s = \sin u \cdot \sin i \ , \tag{8.5}$$

where

r - radius-vector

u - argument of latitude

i - inclination.

The condition of two consecutive transits may be extended to a greater number of passages during which the satellite is observed for the second time, provided that we are able to secure the required accuracy in computing the correction $\Delta\Omega$ necessary for rectifying the position of the point S during the second observation, and to safeguard the stability of the distance f, within the limits of tolerance. We assume, as we have done before, that the long-period variations are known from observations with an adequate accuracy. As regards the short-period variations, those of them which are the function of zonal harmonics of the Earth's gravity potential will be equal to zero, for the argument of latitude is the same during the first and the second observation. Only the effects of tesseral harmonics of the diurnal period are remaining. The long-period variations are minimal in the case of ALOUETTE. They are characterized by the fact that from the launching moment, that is, from the end of September 1962 till mid May 1963 the semi-major axis a did not undergo changes within the bounds of the determination accuracy: $10^{-5} \cdot R_e$ (R_e - radius of the Earth) [NASA 1963]. Till the 18th May 1963, a = 1,15892 $\cdot R_o$; later on - till the end of August 1963, a = 1,15891 $\cdot R_o$.

T a b e l a 6

No of rev.	Influence of C_{22}	Influence of S_{22}	Total perturbation	$\left(\frac{\sum_1^{12}}{12}\right) \cdot$ no of rev.=secular perturbation	Total - secular = Diurnal perturbation
0	0	0	0	0	0
1	0	0	0	-1	$+1^{\circ}.10^{-5}$
2	$+1^{\circ}.10^{-5}$	$-1^{\circ}.10^{-5}$	0	-2	+2
3	$+1.10^{-5}$	-2	-1	-3	+2
4	$+2.10^{-5}$	-3	-1	-4	+3
5	$+3.10^{-5}$	-6	-3	-5	+2
6	$+5.10^{-5}$	-8	-3	-6	+3
7	$+7.10^{-5}$	-11	-4	-6	+2
8	$+9.10^{-5}$	-14	-5	-7	+2
9	$+12.10^{-5}$	-18	-6	-8	+2
10	$+15.10^{-5}$	-22	-7	-9	+2
11	$+18.10^{-5}$	-26	-8	-10	+2
12	$+21.10^{-5}$	-32	-11	-11	0

On this basis and in accord with the considerations of the Chapter VII, we are arriving at the conclusions that during the interval of a dozen or so of revolutions it is not necessary to take the atmospheric drag into account.

As to the influence of the nodal motion, we know that it is little for satellites with a great inclination of the orbit. In the case of ALOUETTE, it amounts to approximately $-0\overset{\circ}{.}985/1^{d}$. As it follows from the computations of Kotchina, corrected in such a way that the values of C_{22} and S_{22} be compatible with those adopted in the present work, the diurnal perturbations of the node of a polar satellite are: (Tab. 6).

The picture of diurnal perturbations shows a deviation from the mean variation proportional to time, by a magnitude of the order of $3^{\circ} \cdot 10^{-5}$, or approximately $0\overset{''}{.}1$. This corresponds to the observation accuracy $0\overset{''}{.}6$; hence the conclusion that for the computation of the correction for the alteration in the node, the application of the secular variation will do.

The influence of the inclination will be imperceptible because of the stability of that element and because of utilizing here the sine which undergoes little changes in the vicinity of 90°.

As seen from the formulae (8.4) and (8.5), an important role is played by the argument of latitude u

$$d\varphi_s = \frac{\sin i \cos u \, d u}{\cos\varphi_s}, \qquad (8.6)$$

$$d f = - r \cdot \sin\varphi_s \, d \varphi_s = - r \operatorname{tg} \varphi_s \sin i \cdot \cos u \, du . \qquad (8.7)$$

Substituting $i = 90^{\circ}$, $r = 7378$ km, and assuming that $df \leqslant 15$ m, we shall obtain the following performance of the accuracy required in defining u:

Two milliseconds may be regarded as the upper limit of accuracy to be achieved in defining the moment of observation and in determining T_{Ω}. For observational reasons - as shown in the text to follow - it will be possible to make observations at geographical latitudes $< 60^{\circ}$, this corresponding to $u < 48^{\circ}$ for an orbit of 1000 km over the Earth. So we may accept that the maximal accuracy to be achieved in the determination of u is $\approx 0''.7$, this corresponding to $0^s.003$ of $(t - T_{\Omega})$. Only in the immediate neighbourhood of the equator those requirements are losing their importance.

Table 7

u	|du|	dt
0	∞	
1^o	28"	140 ms
10^o	$2''.8$	14
20^o	$1''.5$	8
30^o	$1''.0$	5
40^o	$0''.8$	4
50^o	$0''.7$	3
60^o	$0''.6$	3
70^o	$0''.5$	2
80^o	$0''.5$	2
90^o	$0''.5$	2

Let us consider now the mode of observation and the influence of observational errors. The observation should furnish - as computational elements - the angles α_1, α_2, and σ in the plane of the parallel. The angle $(180 - \alpha)$ will be the difference between the topocentric right ascension of the satellite at the moment of its transit through the equator on the celestial sphere and the local sidereal time corresponding to this moment. The angle σ will be simply the difference between the moments of both pasages through the equator on the celestial sphere in sidereal time. Thus the observation will consist of the time recording and the measurement of the right ascension of the satellite when its declination is equal to 0.

Let differentiate the formulae (8.2) and (8.3) with regard to α_1 and α_2, considering that $d\alpha_1 = d\alpha_2 = d\alpha$

$$\frac{-\,d\,s_1}{\sin^2 s_1} = \frac{\sin\alpha_1 \cdot \cos\alpha_2 - \sin\alpha_2 \cos\alpha_1}{\sin^2\alpha_1 \sin s}\, d\alpha$$

$$d\,s_1 = \frac{\sin(\alpha_1 - \alpha_2)\,\sin^2 s_1}{\sin^2\alpha_1 \sin s} \cdot d\alpha \tag{8.9}$$

$$dR = \frac{f(\sin\alpha_1 \cdot \cos s_1 d\,s_1 - \sin s_1 \cos\alpha_1\, d\alpha)}{\sin^2\alpha_1} \tag{8.10}$$

$$dR = \frac{-f(\sin(\alpha_1 - \alpha_2)\sin^2 s_1 + \sin s_1 \cos\alpha_1 \sin^2\alpha_1 \sin s)\, d\alpha}{\sin^4\alpha_1 \cdot \sin s} \tag{8.11}$$

Adopting, for simplification, the symmetric observation $\alpha_1 = \alpha_2 = \alpha$, we obtain

$$|dR| = \left| f \cdot \frac{\sin s_1 \cos\alpha_1}{\sin^2\alpha_1} \cdot d\alpha \right|. \tag{8.12}$$

For such an observation, with an orbit of H = 1000 km and the geographical latitude of the observing site $\varphi \cong 52^{\circ}$, we have

$\alpha_1 = \alpha_2 \cong 130^{\circ}$
$S_1 \cong 29^{\circ}$
$f \cong 5500$ km
$d\alpha$ will stand for 2". Therefore

$$|dR| = 20 \text{ m} .$$

The assumption that the measurement error of angles α is equal to ± 2" seems somewhat optimistic. In addition to the very measurement accuracy of the right ascension, there is also the question of the determination accuracy of the local sidereal time, which is conditionned on the accuracy known to be for the g e o c e n t r i c geographical longitude of the observing site. For the accuracy of this element to be below 2", it means, in order to avoid the error of deflection of the vertical, we must have geodetic coordinate data in one of the newer systems (for instance: Hayford, Krosowski), in which the error of their orientation is supposed not to exceed 2".

Those coordinates need, moreover, to be reduced to the position of the instantaneous pole. The case where only astronomically determined coordinates are known will be discussed separately.

The accuracy achieved in the measurement of the angle σ is considerably higher. If we measure the time with an accuracy of $\Delta t = \pm 0^s.002$, then $\Delta\sigma = \pm 0''.03$; so we are not going to analyze here this influence.

Assuming that mf = 15 m and m α = 2", we obtain

$$m_R = 25 \text{ m} .$$

As said before, this estimate may be regarded as being too optimistic, but our observation will prove to be of avail even when m_R is much greater. In this connection we may recall that the discrepancies between the dimensions of different ellipsoids are still of the order of 100 ÷ 200 m; the same is with the position of the center of the Earth's mass. And so, if a single observation is able to supply an information of the same class as the complicated and time-consuming measurement of large areas made with the help of geodetic methods, the accuracy achieved in this case may be considered interesting.

Let us now dwell upon the question of the visibility of the satellite. It is not very often that we have the possibility of observing two consecutive transist; yet, as said before, this fact reveals no greater significance because of the stability of the selected orbit. Instead, it is more important that the satellite be observed by many other stations in order to be able to best determine the necessary orbital elements. There are, unfortunately, periods during which the visibility conditions allow only for a very short arc of the satellite to be observed. This occurs when the line of the orbital nodes is near the Earth-to-Sun direction. This effect is stronger with lower orbits and feebler with higher orbits. Therefore it is more advantageous to choose higher orbits. Next to ALOUETTE - MIDAS 3 and MIDAS 4 with orbits at an altitude of approximately 3500 km or other satellite, may be included into the observation program.

The visibility conditions define, also the range of geographical latitudes at which the observations of this type can be carried out. Since the observation has to be performed when $\delta = 0$, and on a fairly large arc of the equator, it is restricted by the maximal zenithal distance at which an observation is possible. The corresponding formulae for the computation are given by C i c h o w i c z [1963]. It follows thereof that for $Z_{max} = 85^\circ$ and in the range of $\pm 30^\circ$ of the hour angle, φ must be smaller than 60°.

SECTION 2. COMPUTATION OF THE SATELLITE TRIANGULATION BY THE TETRAHEDRON METHOD

If we observe the satellite from the point A at two positions S_1 and S_2 corresponding to two moments T_1 and T_2, we obtain the AS_1, AS_2, BS_1, BS_2 directions in the system of absolute orientation, it means, in such one whose Z-axis is covered by the rotation axis of the Earth, and the XY - plane - by the equatorial plane.

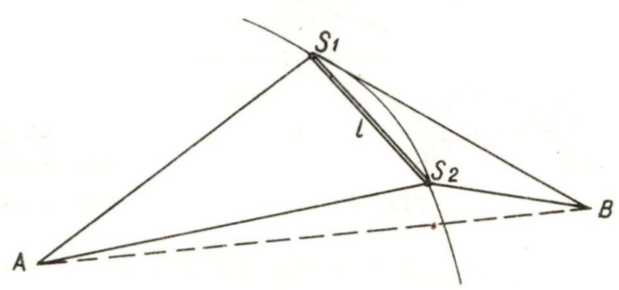


Fig. 21

If, knowing the orbital elements, we are going to calculate the length of the segment of the straight line S_1S_2, we shall be able to solve readily the tetrahedron AB S_1S_2, and to find the distance and the direction AB. Choosing arbitrarily the coordinates of the point A, we can compute the coordinates of the point A and, further on, in the same way the coordinates of all points of the network. The points of the computed network submitted afterwards to an adjustment, will obtain their coordinates in the absolutely oriented system with the length unit independent of the distance measurements performed on the Earth's surface and, hence, disencumbered of measurement errors of bases, errors of reduction from the geoid to the ellipsoid, a.s.o. Also another source of errors disappears, connected with the transposition of the length from the measured side to the unmeasured sides, because the number of "bases" exceeds markedly the number of sides.

Thanks to the participation of Poland in the International Observation Program of the Satellite ECHO 1, in 1963, a possibility was offered for the test of this concept by using the observational data collected. For this purpose, analytical formulae have been derived for the length of the base segment $S_1S_2 = l$ in the function of orbital elements, and a theoretical accuracy analysis has been made; the main idea of the experiment was to demonstrate the validity of this analysis. If it proved to be valid for ECHO 1, it ought to be good also for other satellites which fulfilling all the orbital and aerodynamic requirements, will secure much more accurate results than the balloon ECHO.

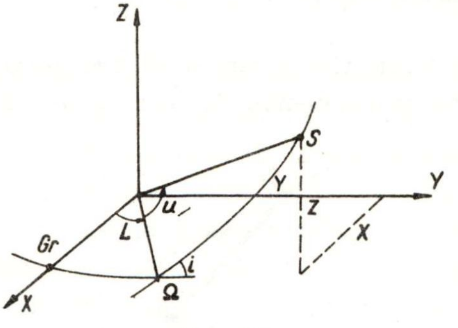


Fig. 22

As proceeds from Figure 17, the coordinates X, Y, Z may be expressed by the formulae

$$\left.\begin{aligned} X &= r\cdot\cos u\cdot\cos L - r\cdot\sin u\cos i\cdot\sin L \\ Y &= r\cdot\sin u\cdot\cos i\cdot\cos L + r\cdot\cos u\cdot\sin L \\ Z &= r\cdot\sin u\cdot\sin i\ . \end{aligned}\right\} \quad (8.13)$$

If we want to determine the distance between the points (for instance: S_1S_2), then

$$l^2 = \Delta^2 X + \Delta^2 Y + \Delta^2 Z\ , \quad (8.14)$$

where

$$\left.\begin{aligned} \Delta X &= X_2 - X_1 \\ \Delta Y &= Y_2 - Y_1 \\ \Delta Z &= Z_2 - Z_1 . \end{aligned}\right\} \quad (8.15)$$

We now assume

$$\left.\begin{aligned} &\Delta r = r_2 - r_1; \quad r = \frac{r_1 + r_2}{2}; \quad r_1 = r - \frac{\Delta r}{2}; \quad r_2 = r + \frac{\Delta r}{2} \\ &\Delta u = u_2 - u_1; \quad u = \frac{u_1 + u_2}{2} \\ &\Delta L = L_2 - L_1 \\ &i_1 = i_2 . \end{aligned}\right\} \quad (8.16)$$

Thus it will be

$$\left.\begin{aligned} X_1 &= (r - \tfrac{\Delta r}{2}) \cdot (\cos u_1 \cdot \cos L_1 - \sin u_1 \cdot \sin L_1 \cdot \cos i) \\ Y_1 &= (r - \tfrac{\Delta r}{2}) \cdot (\cos u_1 \cdot \sin L_1 + \sin u_1 \cdot \cos L_1 \cdot \cos i) \\ Z_1 &= (r - \tfrac{\Delta r}{2}) \cdot \sin u_1 \cdot \sin i \end{aligned}\right\} \quad (8.17)$$

and analogically

$$\left.\begin{aligned} X_2 &= (r + \tfrac{\Delta r}{2}) \cdot (\cos u_2 \cdot \cos L_2 - \sin u_2 \cdot \sin L_2 \cdot \cos i) \\ Y_2 &= (r + \tfrac{\Delta r}{2}) \cdot (\cos u_2 \cdot \sin L_2 + \sin u_2 \cdot \cos L_2 \cdot \cos i) \\ Z_2 &= (r + \tfrac{\Delta r}{2}) \cdot \sin u_2 \cdot \sin i \end{aligned}\right\} \quad (8.18)$$

Subtracting by members, we obtain

$$\left.\begin{aligned} X_2 - X_1 = r \cdot (&\cos u_2 \cdot \cos L_2 - \sin u_2 \cdot \sin L_2 \cdot \cos i + \\ &-\cos u_1 \cdot \cos L_1 + \sin u_1 \sin L_1 \cdot \cos i) + \\ &+\frac{\Delta r}{2} (\cos u_2 \cdot \sin L_2 - \sin u_2 \cdot \sin L_2 \cdot \cos i + \\ &+ \cos u_1 \cdot \sin L_1 - \sin u_1 \cdot \sin L_1 \cdot \cos i) \end{aligned}\right| \quad (8.19)$$

$$Y_2 - Y_1 = r\cdot(\cos u_2\cdot\sin L_2 + \sin u_2\cdot\cos L_2\cdot\cos i +$$
$$- \cos u_1\cdot\sin L_1 - \sin u_1\cdot\cos L_1\cdot\cos i) +$$
$$+\frac{\Delta r}{2}(\cos u_2\cdot\sin L_2 + \sin u_2\cdot\cos L_2\cdot\cos i +$$
$$+ \cos u_1\cdot\sin L_1 + \sin u_1\cdot\cos L_1\cdot\cos i) \tag{8.19}$$
$$Z_2 - Z_1 = r\cdot\sin i\cdot(\sin u_2 - \sin u_1) +$$
$$+\frac{\Delta r}{2}\sin i\,(\sin u_2 + \sin u_1)\,.$$

Because

$$L_2 = L_1 + \Delta L$$

we can write down, using the series expansion

$$\sin L_2 = \sin(L_1 + \Delta L) = \sin L_1 + \Delta L\cdot\cos L_1 +$$
$$-\frac{\Delta^2 L\cdot\sin L_1}{2} - \frac{\Delta^3 L\cdot\cos L_1}{6} + \ldots$$
$$\cos L_2 = \cos(L_1 + \Delta L) = \cos L_1 - \Delta L\cdot\sin L_1 + \tag{8.20}$$
$$-\frac{\Delta^2 L\cdot\cos L_1}{2} + \frac{\Delta^3 L\cdot\sin L_1}{6} + \ldots$$

If $\Delta t = 2$ min, then $\Delta L \cong 30'$, and $\Delta^3 L = 6{,}6\cdot10^{-7}$, so we can confine ourselves to the terms of the order of $\Delta^2 L$.

Substituting the respective elements in Equation (8.19), we obtain for Y-s

$$Y_2 - Y_1 = r\cdot\Big[\cos u_2\cdot\Big(\sin L_1 + \Delta L\cdot\cos L_1 - \frac{\Delta^2 L\cdot\sin L_1}{2}\Big) +$$
$$+ \sin u_2\cdot\cos i\cdot\Big(\cos L_1 - \Delta L\cdot\sin L_1 - \frac{\Delta^2 L\cdot\cos L_1}{2}\Big) +$$
$$- \cos u_1\cdot\sin L_1 - \sin u_1\cdot\cos L_1\cos i\Big] + \tag{8.21}$$

$$+\frac{\Delta r}{2}\Big[\cos u_2 \cdot (\sin L_1 + \Delta L \cdot \cos L_1) +$$

$$+ \sin u_2 \cdot \cos i \cdot (\cos L_1 - \Delta L \cdot \sin L_1 + \qquad (8.21)$$

$$+ \cos u_1 \cdot \sin L_1 + \sin u_1 \cdot \cos L_1 \cdot \cos i\Big].$$

Let change the direction of the X-axis so that L_1 bo equal to 0, and we obtain

$$Y_2 - Y_1 = r \cdot \Big[\cos i \cdot (\sin u_2 - \sin u_1) + \cos u_2 \cdot \Delta L + \qquad (8.22)$$

$$- \sin u_2 \cos i \cdot \frac{\Delta^2 L}{2}\Big] + \frac{\Delta r}{2}\Big[\cos i \cdot (\sin u_2 + \sin u_1) + \cos u_2 \Delta L\Big]$$

substituting afterwards (8.16) yields

$$Y_2 - Y_1 = r \cdot (2\sin\frac{\Delta u}{2} \cdot \cos u \cdot \cos i + \Delta L \cdot \cos u_2 - \frac{\Delta^2 L}{2} \sin u_2 \cdot \cos i) +$$

$$+ \frac{\Delta r}{2}(2\cos\frac{\Delta u}{2} \cdot \sin u \cdot \cos i + \Delta L \cdot \cos u_2) . \qquad (8.23)$$

Analogically, we have for the remaining coordinates

$$X_2 - X_1 = -r \cdot (2\sin\frac{\Delta u}{2} \cdot \sin u + \Delta L \cdot \sin u_2 \cdot \cos i + \frac{\Delta^2 L}{2} \cdot \cos u_2) +$$

$$+ \frac{\Delta r}{2}(2\cos\frac{\Delta u}{2} \cdot \cos u - \Delta L \cdot \sin u_2 \cdot \cos i) \qquad (8.23)$$

$$Z_2 - Z_1 = r \cdot 2 \cdot \sin\frac{\Delta u}{2} \cdot \cos u \cdot \sin i + \Delta r \cdot 2 \cdot \cos\frac{\Delta u}{2} \cdot \sin u \cdot \sin i .$$

Raising the equations (8.23) to the square and summing gives

$$l^2 = r^2 \cdot \Big[4 \cdot \sin^2\frac{\Delta u}{2} + 2 \cdot \sin\Delta u \cdot \cos i \cdot \Delta L + \Delta^2 L(\cos^2 u_2 + \sin^2 u_2 \cdot \cos^2 i) +$$

$$+ 2 \cdot \sin\frac{\Delta u}{2} \cdot \Delta^2 L \cdot (\sin u \cdot \cos u_2 - \cos u \cdot \sin u_2 \cdot \cos^2 i) +$$

$$+ \frac{\Delta^4 L}{4}(\cos^2 u_2 + \sin^2 u_2 \cdot \cos^2 i)\Big] + r \cdot \Delta r \cdot \Big[\Delta L \cdot \sin\Delta u \cdot \cos i +$$

$$+ \Delta^2 L(\cos^2 u_2 + \sin^2 u_2 \cdot \cos^2 i) - \cos\frac{\Delta u}{2} \cdot \Delta^2 L \cdot (\sin u_2 \cdot \cos^2 i + \qquad (8.24)$$

$$+ \cos u \cdot \cos i_2)] + \Delta^2 r \left[\cos^2 \frac{\Delta u}{2} + 2 \cdot \sin u \cdot \cos i \cdot \Delta L + \right.$$

$$\left. + \Delta^2 L \cdot (\cos^2 u_2 + \sin^2 u_2 \cdot \cos^2 i) \right. . \qquad (8.24)$$

To make it somewhat simpler, let transform the fourth expression of the first brackets, by substituting

$$\left.\begin{aligned} \sin u &= \sin(u_2 - \frac{\Delta u}{2}) = \sin u_2 - \frac{\Delta u}{2} \cdot \cos u_2 + \dots \\ \cos u &= \cos(u_2 - \frac{\Delta u}{2}) = \cos u_2 + \frac{\Delta u}{2} \cdot \sin u_2 + \dots \end{aligned}\right\} \qquad (8.25)$$

and

$$\sin \frac{\Delta u}{2} \cdot \frac{\Delta u}{2} \cong \frac{\Delta^2 u}{4} . \qquad (8.26)$$

In the third term of the second brackets we can also replace u by u_2, and designate

$$\cos^2 u_2 + \sin^2 u_2 \cdot \cos^2 i = W^2 \qquad (8.27)$$

and we obtain the final formula:

$$l^2 = r^2 \cdot \left[4\sin^2 \frac{\Delta u}{2} + 2\sin\Delta u \cdot \cos i \cdot \Delta L + \right.$$

$$\left. + \Delta^2 L \cdot (W^2 + \sin \frac{\Delta u}{2} \cdot \sin 2u_2 \cdot \sin^2 i) - \frac{\Delta^2 u \cdot \Delta^2 L}{2} \cdot W^2 + \frac{\Delta^4 L}{4} \cdot W^2 \right] +$$

$$+ r \cdot \Delta r \cdot \left[\Delta L \cdot \sin \Delta u \cdot \cos i + \frac{\Delta^2 L}{2}(W^2 - W^2 \cdot \cos \frac{\Delta u}{2})\right] + \qquad (8.28)$$

$$+ \Delta^2 r \left[\cos^2 \frac{\Delta u}{2} + \frac{\Delta L}{2} \cdot \sin \Delta u \cdot \cos i + \frac{\Delta^2 L}{4} W^2\right] .$$

It may be regarded that $\pm$ 0^s,002 is the maximal accuracy with which we can define the observation moment in the time scale, and $\pm$ 0^s,001 is the accuracy which seems to be attainable in measuring the short-time interval (of the order of minutes) between the observations. Therefore, the highest serviceable accuracy in computing the segment l appears to be $\pm$7m, that is, the distance covered by the satellite during the time of 0^s,001.

In the formula (8.28), the parameter whose accuracy has the strongest hold upon the accuracy of the whole determination performance, is represented by r. Yet we have to keep in mind that l is several times smaller than r: in the case of two-minute observation intervals - even nearly 10 times, so the accuracy needed here for the latter will not be higher than ± 70 m. Evidently, in the case of the orbit of ECHO, even such an accuracy will not be attainable. The analysis given hereinafter presents accuracies assumed to be proper for this satellite as well as the accuracy of time observations, adopted in the program of synchronous observations. It is worth adding that the time measurement error appearing in this method as the error of u and encumbering the calculus of the distance l, is present also in any other method of satellite triangulation - as the error of non-synchronism.

Let us use the known formula for estimating the mean-square error of the segment l under measurement:

$$m^2_F = \sum \left(\frac{\partial F}{\partial x_i}\right)^2 \cdot m^2_{x_i} \,, \qquad (8.29)$$

where x_i denotes the individual parameters of the function. In our case, these parameters will be

r - the geocentric distance of the satellite,
Δr - the increment in distance,
u - the argument of latitude,
Δu - the increment of argument,
ΔL - the increment in nodal longitude,
i - the inclination of orbit.

The formula for the individual partial derivatives have the following form (after discarding the terms that would give - in the product with the mean-square error of the given variable - values below one meter)

$$\frac{\partial l}{\partial r} = \frac{r}{l} \cdot (4\sin^2\frac{\Delta u}{2} + 2\sin\Delta u \cdot \cos i \cdot \Delta L) + \frac{\Delta r}{2l} \cdot \Delta L \cdot \sin\Delta u \cdot \cos i \qquad (8.30)$$

$$\frac{\partial l}{\partial \Delta r} = \frac{r}{2l} \cdot \Delta L \cdot \sin\Delta u \cdot \cos i + \frac{\Delta r}{l} \cdot \cos^2\frac{\Delta u}{2} \qquad (8.31)$$

$$\frac{\partial l}{\partial u} = \frac{r^2\Delta^2 L}{2l}(-\sin 2u + 2\cos 2u \cdot \sin\frac{\Delta u}{2}) \cdot \sin^2 i \qquad (8.32)$$

$$\frac{\partial l}{\partial \Delta u} = \frac{r^2}{l} \cdot (\sin\Delta u + \cos\Delta u \cos i \cdot \Delta L) + \tag{8.33}$$
$$+ \frac{r \cdot \Delta r}{2l} \cdot \Delta L \cdot \cos u \cdot \cos i - \frac{\Delta^2 r}{4l} \cdot \sin \Delta u$$

$$\frac{\partial l}{\partial \Delta L} = \frac{r^2}{l} \cdot \sin \Delta u \cdot \cos i + \frac{r \cdot \Delta r}{2l} \cdot \sin \Delta u \cdot \cos i \tag{8.34}$$

$$\frac{\partial l}{\partial i} = - \frac{r^2 \cdot \sin \Delta u \cdot \Delta L \cdot \sin i}{l} + \frac{r \cdot \Delta r}{2l} \Delta L \cdot \sin \Delta u \cdot \sin i . \tag{8.35}$$

As the basis for the estimation of errors to be attributed to individual parameters, the Tables of Mean Elements have been adopted - published by the Smithsonian Astrophysical Observatory [I z s a k, 1964].

In conformity with the data contained in those Tables, the following values have been accepted

$$m_\omega = \pm 0^\circ{,}5$$
$$m_{\Delta\omega/d} = 0^\circ{,}1$$
$$m_\Omega = 0^\circ{,}01$$
$$m_{\Delta\Omega/1\,\mathrm{rev}} = 0^\circ{,}001$$
$$m_i = 0^\circ{,}01$$
$$m_{Mo} = 0^\circ{,}2$$
$$m_e = 0{,}0001$$
$$m_n = 0{,}00005 \text{ rev/d}$$
$$m_a = 1 \text{ km}.$$

For the analysis, the following approximate formulae had been used, defining the parameters appearing in the formula (8.28)

$$r = a \cdot (1 - e \cdot \cos E) \tag{8.36}$$

$$\Delta r = 2 \cdot a \cdot e \cdot \sin E \cdot \sin \frac{\Delta E}{2} , \tag{8.37}$$

$u = \omega + v$, where $\operatorname{tg} \frac{v}{2} = \sqrt{\frac{1 + e}{1 - e}} \cdot \operatorname{tg} \frac{E}{2}$, and

$$E = M + e \cdot \sin E, \quad \text{and} \quad M = M_o + n \cdot (t - t_o), \tag{8.38}$$

$\Delta u = \Delta\omega + \Delta v$ where

$$\mathrm{tg}\frac{\Delta v}{2} = \sqrt{\frac{1+e}{1-e}}\cdot\left[\mathrm{tg}\frac{\Delta E}{2} + \frac{\Delta E}{8}(E_1\cdot E_2 - v_1\cdot v_2)\right] \qquad (8.39)$$

and $\Delta E = \Delta M + 2\cdot e\cdot\cos E\cdot\sin\frac{\Delta E}{2}$ and $\Delta M = n\,\Delta t$

$$\Delta L = (1 - \frac{\Delta\Omega/1\ \mathrm{rev}}{P})\cdot\Delta t\,. \qquad (8.40)$$

Applying to the formulae (8.36 ÷ 8.40) the formula (8.29) and assuming that $\Delta t = 120$ sec and $m_t = 0^s\!.01$, $a = 7500$ km, $e = 0.1$, $E = 90^o$, the following values have been obtained

$$m_r = \pm\, 1{,}2 \text{ km}$$

$$m_{\Delta r} = \pm\, 205 \text{ m}$$

$$m_u = \pm\, 0^o\!,2$$

$$m_{\Delta u} = \pm\, 3''$$

$$m_{\Delta L} = \pm\, 0''\!,1\,.$$

Substituting in the formulae (8.31 ÷ 8.36) the numerical values ($r = 7500$ km, $\Delta r = 78{,}5$ km, $l = 785$ km, $t = 120$ s, $\Delta L = 30'$, $\Delta u = 6^o$, $u = 60^o$, $i = 50^o$) and multiplying them by the corresponding mean-square errors, we obtain

$$\frac{\partial l}{\partial r}\cdot m_r = 139 \text{ m}$$

$$\frac{\partial l}{\partial \Delta r}\cdot m_{\Delta r} = 11 \text{ m}$$

$$\frac{\partial i}{\partial u}\cdot m_u = 6 \text{ m}$$

$$\frac{\partial l}{\partial \Delta u}\cdot m_{\Delta u} = 120 \text{ m}$$

$$\frac{\partial l}{\partial \Delta L}\cdot m_{\Delta L} = 24 \text{ m}$$

$$\frac{\partial l}{\partial i}\cdot m_i = 7 \text{ m}$$

and ultimately

$$m_1 = 186 \text{ m} .$$

The data of the synchronous observations campaign of 1963 contained results from eight observing stations

Uzhghorod	Zvenigorod
Nikolaev	Riga
Poznań	Rotsdam
Bucharest	Praha.

The examination of the whole documentation revealed that only a small number of observations represent pairs susceptible to be used for the formation of suitable tetrahedrons

Uzh	Riga	9
Nik	Riga	5
Poz	Riga	8
Buc	Riga	3
Uzh	Nik	3
Nik	Poz	3
Poz	Buc	2
Poz	Uzh	2
Uzh	Pr	1 .

It was decided to reduce the observations of the group of the first three pairs. We presumed that the pair Poznań-Riga would have worse results, owing to the disadvantageous position of those stations with regard to the orbit, for the computation accuracy of the chord AB depends strongly upon the shape of the tetrahedron. The most advantageous case is when the segment 1 lies in the plane perpendicular to AB. As the plane turns, the tetrahedron becomes more and more oblate until - in the extremal case - it changes into a plane, and then the problem becomes irresolvable. The direction of the chord Poznań - Riga was just close to the direction of the motion of the satellite. During the course of computations, three more observing pairs (one Poz-Riga and two Uzh-Riga) had been eliminated, their geometric configuration being very unfavorable.

The first stage of work was reduced to the computation, on the basis of orbital elements, of the distance of the segment

1. These elements were adopted in accordance with the publications of the Smithsonian Astrophysical Observatory [I z s a k 1964 b], presenting the following values: ω_0 - argument of latitude of the perigee, Ω_0 - right ascension of the ascending node, i_0 - inclination of the orbit, e_0 - eccentricity, M_0 - mean anomaly in epoch, n_0 - mean motion, n' - change in motion. These elements are given in diurnal intervals for the epoch: beginning of the 24 hours. The mode of computation of those elements is described by G a p o s h k i n [1964]; in conformity with the definition given in Chapter II, they are mean elements. For the computation of the coordinates of the satellite, the following formulae are being used:

1. Representation of mean elements in terms of the epoch: the moment of observation

$$\left.\begin{aligned} \omega &= \omega_0 + \Delta\omega \cdot \Delta t \\ \Omega_s &= \Omega_0 + \Delta\Omega \cdot \Delta t \\ i_s &= i_0 + \Delta_i \cdot \Delta t \\ e &= e_0 + \Delta e \cdot \Delta t \\ n &= n_0 + n' \cdot \Delta t \\ M &= M_0 + n \cdot \Delta t \end{aligned}\right\} . \tag{8.41}$$

2. Computation of the eccentric and true anomaly and of the argument of latitude

$$E = M + e \sin E \tag{8.42}$$

$$\sin v = \sqrt{\frac{1 - e^2}{1 - e \cdot \cos E}} \cdot \sin E \tag{8.43}$$

$$\cos v = \frac{\cos E - e}{1 - e \cos E} \tag{8.44}$$

$$u_s = \omega + v \, . \tag{8.45}$$

3. Introduction of perturbation corrections

$$\left.\begin{aligned} \Omega &= \Omega_s + \delta\Omega \\ \sin i &= \sin i_s + \delta_i \cos i_s \\ \cos i &= \cos i_s - \delta_i \sin i_s \\ \sin u &= \sin u_s + \delta u \cdot \cos u_s \\ \cos u &= \cos u_s - \delta u \cdot \sin u_s \\ r &= a\,(1 - e \cos E) + \delta r \end{aligned}\right\} \quad (8.46)$$

where a is being found from the formulae

$$a = \left(\frac{k}{n^2}\right)^{1/3}\left[1 + \frac{1}{3}\,\frac{J_2}{p^2}\sqrt{1 - e^2}\,\left(-1 + \frac{3}{2}\sin^2 i\right)\right] \quad (8.47)$$

$$p = \left(\frac{k}{n^2}\right)^{1/3}(1 - e^2) \quad (8.48)$$

$$k = 0.7537172 \times 10^5\ \mathrm{rev}^2 \cdot \mathrm{mgm}^3\,\mathrm{d}^{-2}\,; \quad J = 0.0660546\ \mathrm{mgm}^2\,. \quad (8.49)$$

The perturbation corrections taking into account the effect of the second spherical harmonic, are computed after the formulae

$$\begin{aligned} \delta u = \frac{J_2}{p^2}\Bigg[\frac{1}{2}\Bigg\{&\left(-1 + \frac{7}{6}\sin^2 i\right)\cdot\sin(2\omega + 2v) + \\ &+ e\left[\left(-1 + \frac{5}{3}\sin^2 i\right)\cdot\sin(2\omega + v) + \frac{1}{3}\left(-1 + \sin^2 i\right)\cdot\sin(2\omega + 3v)\right]\Bigg\} + \\ &- \Bigg\{\frac{1}{3}\left(-1 + \frac{3}{2}\sin^2 i\right)\left[\left(1 - \sqrt{1-e^2}\right)\sin v \cos v + \frac{e^3 \sin v}{(1 + \sqrt{1-e^2})^2}\right] + \\ &+ (v - M + e \sin v)\left(-2 + \frac{5}{2}\sin^2 i\right)\Bigg\}\Bigg] \end{aligned} \quad (8.50)$$

$$\begin{aligned} \delta_r = \frac{1}{3}\,\frac{J_2}{p}\Bigg[&\left(-1 + \frac{3}{2}\sin^2 i\right)\left(1 - \frac{1 - e\cos E}{\sqrt{1-e^2}} + \frac{e\cos v}{1 + \sqrt{1-e^2}}\right) + \\ &+ \frac{1}{2}\cos(2\omega + 2v)\sin^2 i\Bigg] \end{aligned}$$

$$\delta_{\Omega} = \frac{J_2}{p^2} \cos i \left[-(v - M + e \sin v) + \frac{1}{2} \left\{ \sin(2\omega + 2v) + \right. \right.$$

$$\left. \left. + e \left[\sin(2\omega + v) + \frac{1}{3} \sin (2\omega + 3v) \right\} \right] \right]$$

$$\delta_i = \frac{1}{2} \frac{J_2}{p^2} \cdot \sin i \cos i \left[\cos(2\omega + 2v) + \right. \qquad (8.50)$$

$$\left. + e \left\{ \cos (2\omega + v) + \frac{1}{3} \cos (2\omega + 3v) \right\} \right] .$$

4. Formulae for the evaluation of the coordinates of the satellite

$$X = r (\cos u \cos \lambda - \sin u \sin \lambda \cos i)$$

$$Y = r (\cos u \sin \lambda + \sin u \cos \lambda \cos i) \qquad (8.51)$$

$$Z = r (\sin u \sin i) .$$

5. After having found the coordinates of the two points S_1 and S_2, we compute the base segment

$$l = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2} . \qquad (8.52)$$

For the performance of those computations, a program termed ECHO (vide Appendix no 4) has been established in the ALGOL language. The Table 9 (Appendix 5) contains the data, the Table 10 (Appendix 6) - the results of this program.

In addition to these computations, a series of calculuses has been carried out by means of the same program, with the view of examining the accuracies. The diagram in Fig. 23 presents the change in the length of the base segment Δl, varying with u, assuming that $\Delta e = \pm 0{,}00007$, according to the estimate given for the orbital data. The Figurc 24 presents the effect of the error of the mean motion, assuming that $\Delta n = 0.000009$ rev/d, this being also consistent with the evaluation effectuated by the authors of the Tables of Elements. In the Table 8 we have, on the other hand, the differences of results for the same moments, computed from two sets of elements for different epochs.

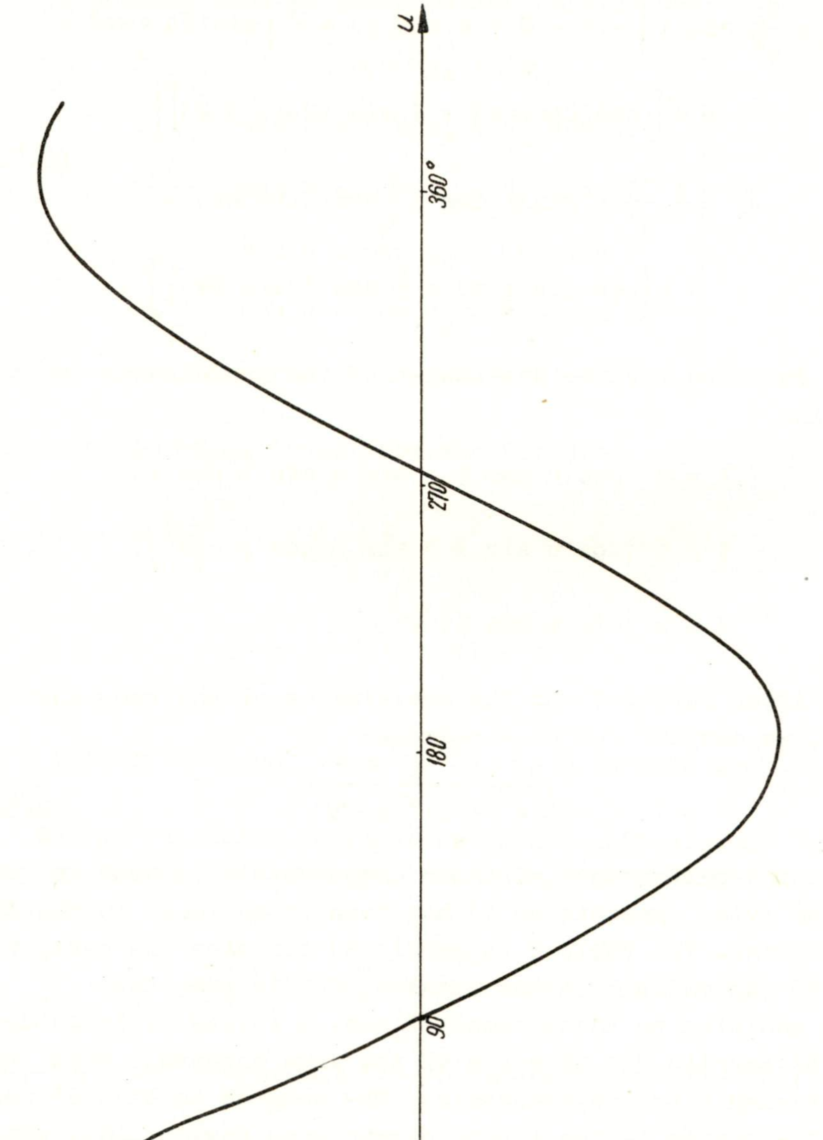

u
360°
270
180
90
0
ΔL
+60m
+50m
+40m
+30m
+20m
+10m
-10m
-20m
-30m
-40m
-50m
-60m

Fig. 23

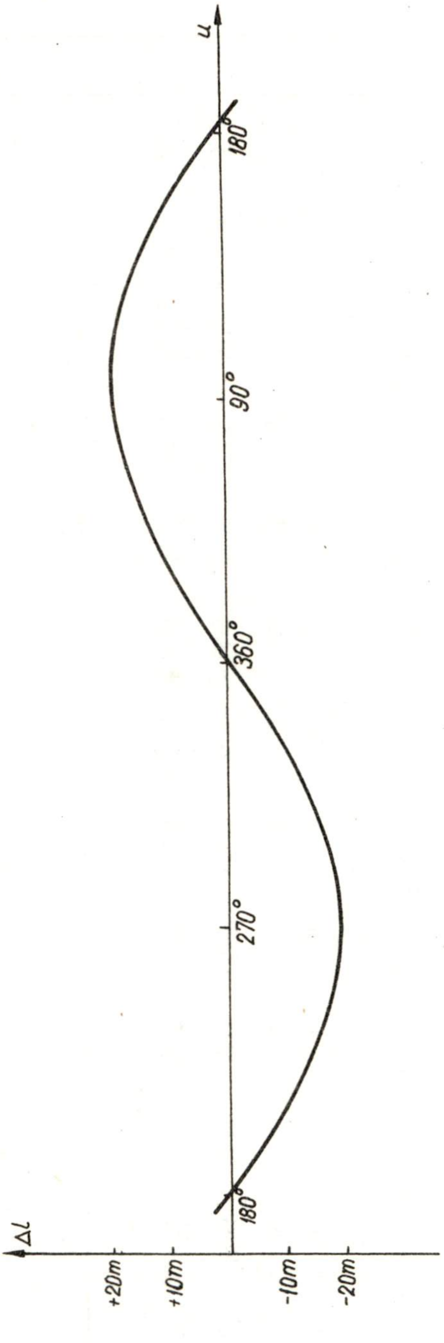


Fig. 24

Table 8

T_1	T_2	$\Delta l = l_2 - l_1$
$0^h\ 13^m\ 55^s$ 15 52	$-\ 23^h\ 43^m\ 05^s$ 44 08	+ 80 m
2 09 07 11 04	- 21 50 53 48 56	79
4 04 19 06 16	- 19 55 41 53 44	77
5 59 31 01 28	- 18 00 29 58 32	73
7 54 43 56 40	- 16 05 17 03 20	72
9 49 55 51 52	- 14 10 05 08 08	68
11 45 07 47 04	- 12 14 53 12 56	67
13 40 19 42 16	- 10 19 41 17 44	64
15 35 31 37 28	- 8 24 29 22 32	61
17 30 43 32 40	- 6 29 17 27 20	58
19 25 55 27 52	- 4 34 05 32 08	57
20 21 07 23 04	- 2 38 53 36 56	53
23 16 19 18 16	- 0 43 41 41 46	52

The data for the individual epochs were as follows

I. $\omega = 274^\circ.32$ $d\omega = 3^\circ.33$

$\Omega = 202^\circ.906$ $d\Omega = -3^\circ.297$

$i = 47^\circ.260$ $d_i = 0.002$

$e = 0.04490$ $d_e = 0.00051$

$M_o = 0.32532$ rev $n = 12.498198$ rev/d

$n' = 0.000420$ rev/d^2

II. $\omega = 277^\circ.65$ $d\omega = 3.33$

$\Omega = 199.609$ $d\Omega = -3.297$

$i = 47.262$ $di = 0.002$

$e = 0.04541$ $de = 0.00051$

$M_o = 0.82430$

$n = 12.498620$ rev/d

$n' = 0.000440$ rev/d^2.

A comparision of this kind represents a good basis for the estimation of accuracies, because the two sets of elements are to a considerable extent independent each from other. They are computed separately, on the ground of observations carried out in periods overlapping one another; thus a half of observations will be the same, the other half being different. So we may presume that the digits in the third column of the Table 8 are fairly well corresponding to the true accuracy.

The second stage of work consisted in the solution of the tetrahedron, in conformity with the known stereometric formulae. For this purpose, a program called TETRAHEDRON 2 has been established (vide Appendix 7). The data pertaining to this program are contained in the Table 11 (Appendix No 8), the results in the table 12.

As expected, the worst results are being noted with regard to the direction Riga-Poznań. Instead, for the two remaining directions the results obtained are quite satisfactory. Except for one distinctly different outcome, the mean-square error for the Riga-Uzhghorod pair is of a length of ± 67 m, and for the Riga-Nikolaev pair: ± 56 m. This is even better, indeed, than envisaged by the theoretical analysis which was rather circumspect. If compared to the results derived by Veiss, which being based on tens of thousands of observations had given

Table 12

Results of the Program TETRAHEDRON 2

Date 1963	ΔX	ΔY	ΔZ	A-B
		Riga - Poznań		
VI.2.	+548.707	-288.688	-292.790	685.762
VI.4	+548.122	-290.055	-293.260	685.982
VI.6.	+551.866	-287.922	-291.627	687.388
VI.6.	+546.277	-293.059	-294.973	686.522
VI.13.	+548.229	-289.066	-292.964	685.523
VI.13.	+549.017	-289.112	-292.739	686.077
VI.13.	+549.030	-289.125	-292.753	686.099
			l =	686.180±235 m_o=±622
		Riga - Uzhghorod		
VI.3.	+723.572	+180.591	-559.293	932.190
VI.4.	+724.251	+180.596	-559.014	932.551
VI.15.	+723.398	+181.098	-558.740	931.822
VI.17.	+723.458	+180.829	-558.774	931.836
VI.17.	+723.535	+180.881	-558.914	931.990
VI.17.	+723.595	+180.900	-559.025	932.107
VI.17.	+723.675	+180.943	-559.020	932.174
			l =	932.096± 94 m_o = ±250
		without VI.4.	l =	932.020± 67 m_o= ±164
		Riga - Nikolaev		
VI.4.	+514.844	+886.937	-683.295	1232.320
VI.4.	+515.021	+887.159	-683.185	1232.490
VI.5.	+514.521	+887.551	-682.855	1232.380
VI.9.	+514.756	+887.084	-683.078	1232.270
VI.9.	+515.045	+887.274	-683.175	1232.580
			l =	1232.408± 56 m_o= ±126

mean-square errors within the limits of 7-30 m, our results may be regarded as encouraging. There exists still the possibility of their further improvement by computing the orbital elements with the help of a special method and by taking into consideration smaller perturbations. The author hopes to be in a position to continue the work on this problem. For testing the results a comparision was made with geodetic coordinates which were available for three stations in the Soviet Union: Riga, Uzhghorod and Nikolaev, referred to the Hayfort ellipsoid, with the initial point at Potsdam.

Although, the altitudes above the sea level had not been precisely specified, nor the distances between the geoid and the ellipsoid a fairly good agreement has been obtained. From the above data the lengths of chords have been computed:

Riga - Uzhgorod 931.931 km

Riga - Nicolaev 1232.381 km .

W.Pachelski had made computations of tetrahedrons with the help of his own independent program, obtaining the same results.

SECTION 3. CONNECTION OF THE TRIANGULATION NETWORK WITH THE CENTER OF THE EARTH'S MASS

The method described in Sect. 8.2, enables to determine the coordinates of all points in a system with strictly defined directions but with an arbitrarily chosen origin.

Having evaluated the coordinates of points A and B, we can also compute the coordinates of the two remaining vertexes of the tetrahedron - S_1 and S_2. Since we know the length of the radius-vector at this point, we may put dawn

$$\left.\begin{aligned}(X_{S1} - Xo)^2 + (Y_{S1} - Yo)^2 + (Z_{S1} - Zo)^2 = r_1^2 \\ (X_{S2} - Xo)^2 + (Y_{S2} - Yo)^2 + (Z_{S2} - Zo)^2 = r_2^2\end{aligned}\right\}, \qquad (8.53)$$

where Xo, Yo, Zo are the coordinates of the mass center.

As many such equation systems can be written as many tetrahedrons have been formed in the network, this allowing for the coordinates Xo, Yo, and Zo in the adopted system to be found. After having made a parallel shift, we obtain a coordinate system with the origin at the center of the Earth's mass.

During the last years, several projects of the world triangulation network have been presented. One of the most elegant and - at the same time - one of the most efficient projects is the Zhongolovich project [1965], which was presented by the author during the scientific session of Department of Geodesy and Cartography of the Warsaw Polytechnic. This project possesses, however, some limitations consciously imposed by the author himself who considers certain difficulties to be solvable only in the future. Among those difficulties is the question of conveying the spatial network a linear scale. Zhongolovich suggests to apply here lasser-beacon for measuring one of the segments: satellite-observing station. Stating expressively that this will be a task of the future, he draws the attention to a whole series of technical problems bound with it.

The application of the method described in the present study eliminates this predicament and enables to acquire a great number of redundant data under the form of measured distances.

Another problem that the author of the world triangulation network has left irresolved is the question of relating the network to the center of the Earth's mass, although he states in the introduction that this is the very ultimate objective of any studies of this kind.

The method applying the radius-vector allows also for that part of the project to be complemented.

SECTION 4. DETERMINATION OF THE PARALLEL RADIUS BY USING THE TETRAHEDRON METHOD

In Sect. 8.1 the method of determining the parallel radius of the Earth has been discussed, in which an accurate knowledge of the geocentric distance of the observing site was being assumed. Let now give a thought to the question how we have to proceed if this condition is not satisfied.

A two fold aiming at the point S which - in reality - is moving and is considered motionless only owing to certain corrections introduced into the calculus, may be called q u a s i - s y n c h r o n o u s. The same quasi-synchronism can be applied to the observation of other selected points, for example, points remote in time from the point S, by an interval of $\pm \Delta t$. Thus, we are able to construct a tetrahedron analogous to the tetrahedron used in triangulation, whereby the place of the previous two points A and B will be taken by a single point P which - during the time separating two observations - would have changed its position in space. In the issue, we shall obtain the distance of P_1P_2 permitting to determine R immediately thereof

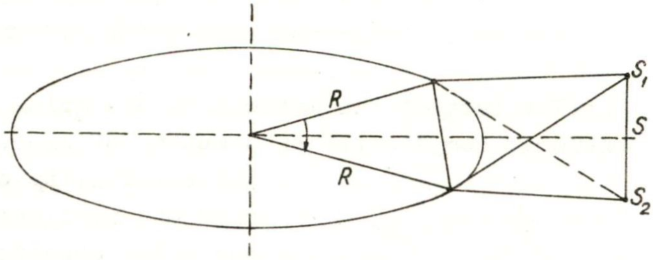


Fig. 25

$$R = \frac{P_1P_2}{2} \cdot \operatorname{cosec} \sigma/2 . \qquad (8.54)$$

Now we may venture to define an element which is perhaps even more interesting than R, namely - the parallel component of the deflection of the vertical. For this purpose, the formula (8.11) from the Sect. 8.1 can be utilized. We shall rewrite it in the form

$$\eta = \frac{\Delta R \cdot \sin^4 \alpha_1 \sin S}{f\left[\sin(\alpha_1 - \alpha_2)\sin^2 S_1 + \sin S_1 \cos\alpha_1 \sin^2\alpha_1 \sin S\right]} \qquad (8.55)$$

where

η - component of the vertical deflection

ΔR - difference between the radius determined in the manner described in Sect. 8.1 (R_I) and the radius determined in the way described here (R_{II})

$$\Delta R = R_{II} - R_I . \qquad (8.56)$$

The quasi-synchronization of points S_1 and S_2 may be achieved with the same accuracy as of the point S. Only minute variations of elements come into play here, occurring during the time of Δt 1 - 2 min; they can be easily taken care of.

SECTION 5. SYNCHRONOUS OBSERVATIONS NOT FORMING TETRAHEDRONS

The last of two methods we are going to present here, are the most complicated as a matter of analysis, but they offer the possibility of utilizing the existing observational material as well as any other observations to be made in the future with the help of the standard procedures.

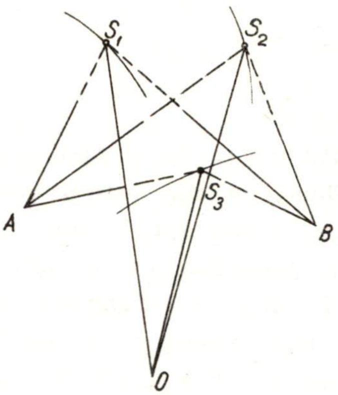


Fig. 26

Let suppose that synchronous observations of a satellite or of satellites being at points S_1, S_2, S_3 are to be carried out from two points A and B. Each of those observations will permit for the following equation system to be formulated

$$\frac{y_{si} - y_A}{x_{si} - x_A} = \mathrm{tg}\, t_{Ai} \qquad \frac{y_{si} - y_B}{x_{si} - x_B} = \mathrm{tg}\, t_{Bi}$$

$$\left.\begin{aligned} \frac{z_{si} - z_A}{\sqrt{(x_{si} - x_A)^2 + (y_{si} - y_A)^2}} &= \mathrm{tg}\, \delta_{Ai} \\ \frac{z_{si} - z_B}{\sqrt{(x_{si} - x_B) + (y_{si} - y_B)^2}} &= \mathrm{tg}\, \delta_{Bi} \end{aligned}\right\} \qquad (8.57)$$

$$x_{si}^2 + y_{si}^2 + z_{si}^2 = r_i^2$$

where

x_{si}, y_{si}, z_{si} - are the satellite coordinates at the moment of the i-th observation,

x_A, y_A, z_A, x_B, y_B, z_B - are the coordinates of the stations,

t_{Ai}, t_{Bi} - are the hour angles referred to Greenwich, observed from stations A and B,

δ_{Ai}, δ_{Bi} - are the declinations observed from A and B.

Three observations will give a system of 15 equations with 15 unknowns. The solution of a system in which three equations are of the second degree is certainly not an easy matter but, after all, not hopeless. As the coordinates of points A and B are always known with a fairly good approximation and as the coordinates of points S_1, S_2 and S_3 can be evaluated to a slightly worse approximation, the numerical method of iteration may be applied without fear of the process to prove discrepant.

Apart from the accuracy of observations and the accuracy of radii-vectors, the accuracy of such a determination will depend on the geometrical configuration. The most favorable conditions will prevail when the directions at points A and B form approximately right angles with each other or, in other words, when the points S_1, S_2 and S_3 are largely spaced. It follows thereof that the distance between the stations should not exceed the altitude of the satellite above the Earth, otherwise, it would be hardly possible to attain a favorable configuration.

SECTION 6. NONSYNCHRONOUS OBSERVATIONS

Let us suppose that at the station A the observations are made in such a way that during one flight the satellite is being observed at two or more places on the sky, at intervals of several minutes. This allows to evaluate the length of the segments l in space, analogous to the base segments appearing in tetrahedrons. We are going to consider three of such observations. Each of them permits for the following equation to be set down

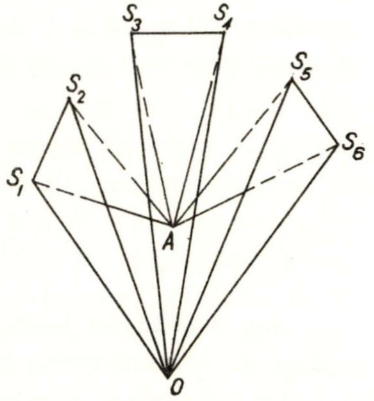


Fig. 27

$$\frac{y_1 - y}{x_1 - x} = \operatorname{tg} t_1 \qquad \frac{y_2 - y}{x_2 - x} = \operatorname{tg} t_2$$

$$\frac{z_1 - z}{\sqrt{(x_1-x)^2 + (y_1-y)^2}} = \operatorname{tg}\delta_1, \frac{z_2 - z}{\sqrt{(x_2-x)^2 + (y_2-y)^2}} = \operatorname{tg}\delta_2$$

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 = l_{12}^2 \qquad (8.58)$$

$$x_1^2 + y_1^2 + z_1^2 = r_1^2$$

$$x_2^2 + y_2^2 + z_2^2 = r_2^2$$

where

$x_1, y_1, z_1, x_2, y_2, z_2$ - are the coordinates of the points S_1 and S_2,

x, y, z - are the coordinates of the observing site,

$t_1, t_2, \delta_1, \delta_2$ - are the observed hour angles and declinations respectively.

The three observations give a system of 21 equations with 21 unknowns.

The matter of solving this system stands like in the preceding case. Owing to the fact that we know the approximate coordinates of all points, we are in a position to solve this system fully uniquely with the aid of the iteration method, although some more real solutions exist here.

As it always is, the accuracy depends strongly upon the configuration. So, for the point A to be properly determined a good location of observed points S_1 is needed.

CHAPTER IX

FINAL CONCLUSIONS

The considerations presented in this study and the outcomes of computations made in connection with the work, lead to the following conclusions:

1^o. If we have at our disposal a set of observations carried out with an accuracy to ± 2" and ± $0^s.002$, well distributed in space and time, it is possible to compute the orbital elements, especially the semi-major axis and the eccentricity, with an accuracy better than 10^{-6}. Applying therewith an appropriate observation program, we can disburden completely the semi-major axis and partially the eccentricity of possition errors of observing stations (Chapter IV).

2^o. The present state of knowledge on the Earth's gravity field permits to compute the radius-vector accurate to 10^{-5} - if the satellite is travelling on an orbit of H_p equal to 1000 km and i being equal to 80^o. It is the lowest orbit within the adopted limitations H = 1000 - 3000 km - having been purposely chosen for our calculations as the least favorable. With higher orbits the effect of harmonics of higher orders is rapidly dropping because of the high powers r appearing in the denominator (Chapter VI).

3^o. Perturbation influences of atmospheric provenance produced by the attraction of other celestial bodies or generated by other causes - are small and need not be taken into consideration (Chapter VII).

Let reflect on the question in what sense those results offer possibilities of utilizing the theory of radius-vector. Although the accuracy to 10^{-5} may be not very attractive for problems being resolved on the Earth's surface such for example, as the connection of continents, but becomes rather interesting with the problem of referring surface points to the center of the Earth's mass. The studies existing so far in this domain did not pass the mentioned accuracy limit, neither. However, the investigations on the Earth's gravity field are making a rapid progress. The accuracy known for the C_{33} and S_{33} harmonics responsible for the largest perturbations, is continually improving and will, no doubt, in the nearest future attain the same order of accuracy as the zonal harmonics. This gives foundation for hope that it will be possible to achieve also with regard to this part of the theory an accuracy of the order of at least $2 \cdot 10^{-6} - 3 \cdot 10^{-6}$. Still, it seems necessary to remark that it may be hardly possible to improve here the accuracy by means of augmenting the number of observations because the errors, having their source in the accuracy of the theory, would not be of an accidental nature.

It is the most pleasant duty of the author to express his gratitude to all persons who have not spared advice or support during his work on the above problem:

- to Professor W. Opalski - for some valuable remarks and observations,
- to Dr L.Cichowicz - for his generous help and initiative having encouraged the author to take up innovative subjects,
- to Mr. W.Pachelski, M.Sc., and Mr.K.Ziółkowski, M.Sc., from the Computing Center of the Polsih Academy of Sciences - for imparting their experiences in the domain of orbital computation, numerical integration and programmation, and
- to the personnel of the Establishment for Numerical Computations of the Warsaw University, where the calculation operations had been carried out.

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A P P E N D I X 1

Table 3

The results of the investigation of the Earth´s gravity field

Author, year, satellites, the method	m	C_{2m} S_{2m}	C_{3m} S_{3m}	C_{4m} S_{4m}	C_{5m} S_{5m}	C_{6m} S_{6m}	C_{7m} S_{7m}	C_{8m} S_{8m}	C_{9m} S_{9m}
O´Keefe, Eckels Squires - 1959 Vanguard 1	0	-1082.5	+2.4	+1.7	+0.1				

d.c.Tab. 3

Author, year, satellites, the method	m	C_{2m} S_{2m}	C_{3m} S_{3m}	C_{4m} S_{4m}	C_{5m} S_{5m}	C_{6m} S_{6m}	C_{7m} S_{7m}	C_{8m} S_{8m}	C_{9m} S_{9m}
King-Hele - 1961 Sputnik 2 Vanguard 1 Explerer 7	0	-1082.79	-	+1.4					
Zhongolovich - 1960 Sputnik 2 Sputnik 3 Sputnik 3 R	0	-1083.3	+2.	+4.1					
Kozai - 1961 Explorer 7 Vanguard 3 Vanguard 1	0	-1082.21	+2.29	+2.1	+0.23				
Michielsen - 1961 Transit 1B Vanguard 1 Sputnik 4	0	-1082.7	+2.4	+1.7	+0.1	-0.7	+0.5	-0.1	-0.5
Newton, Hopfield, Kline - 1961 Transit 2A Vanguard 1 Transit 1B	0	-	+2.36	-	+0.19	-	+0.28		
Smith - 1961 Sputnik 3, Vanguard 1, Transit 1B, Tiros 1	0	-1083.15	-	+1.4	-	+0.7	-		
Shelkey - 1962	0	-1082.61	-	+1.52	-	+0.73	-		
Kozai - -1962x)	0	-1082.48	+2.562	+1.84	+0.064	-0.39	+0.470	+0.02	-0.117
Kozai - - 1964axx)	0	-1082.65	+2.53	+1.62	+0.21	-0.61	+0.32	+0.24	+0.10
Smith - 1962	0	-	+2.37	-	+0.05				
Smith - 1963	0	-	+2.44	-	+0.18	-	-0.30		
Kozai - 1964bxxx)	0	-1082.645	+2.546	+1.649	+0.210	-0.646	+0.333	+0.270	+0.053
King-Hele, Cook, Rees 1963xxxx)	0	-1082.86	-	+1.03	-	-0.72	-	-0.34	

d.c.Tab. 3

Author, year, satellites, the the method	m	C_{2m} S_{2m}	C_{3m} S_{3m}	C_{4m} S_{4m}	C_{5m} S_{5m}	C_{6m} S_{6m}	C_{7m} S_{7m}	C_{8m} S_{8m}	C_{9m} S_{9m}
Kaula 1961 satellites + gravimetry + triangulation	0	-1082.61	+2.05	+1.43	-0.08	-0.20			
	1	-	+0.14 +0.42	-0.60 -0.14					
	2	+0.48 -0.25	+0.33 -0.01	+0.10 +0.09					
	3		+0.079 +0.195	+0.031 -0.001					
	4			-0.002 +0.008					
Kaula - - 1963 1959 α 1, 1959 η, 1960 ι 2	0	-1082.49	+2.59	+1.65	+0.10	-0.36	+0.39		
	1	-	+1.91 -0.12	-0.20 +0.44					
	2	+1.19 -1.10	+0.12 +0.03	-0.01 +0.07					
	3		-0.043 +0.103	+0.030 +0.010					
	4			-0.005 +0.012					
Kaula - - 1963 1959 α 1, 1959 η, 1960 ι 2, 1961 δ 1, 1961 $\alpha\delta$ 1	0	-1082.44	+2.57	+2.01	+0.07	-0.32	+0.46		
	1	-	+1.64 +0.15	-0.31 +0.35					
	2	+1.21 -0.89	-0.01 +0.14	0.00 +0.08					
	3	-	+0.097 +0.106	+0.010 +0.025					
	4			0.000 0.004					
Uotila - 1962 gravimetry	1		+0.11 -0.68	-0.14 -0.19					
	2	+0.45 -1.45	+0.41 -0.01	+0.18 +0.10					
	3		+0.208 +0.339	+0.072 -0.038					
	4			+0.014 -0.003					

d.c.Tab. 3

Author, year, satellites, the method	m	C_{2m} S_{2m}	C_{3m} S_{3m}	C_{4m} S_{4m}	C_{5m} S_{5m}	C_{6m} S_{6m}	C_{7m} S_{7m}	C_{8m} S_{8m}	C_{9m} S_{9m}
Kozai - 1961b	1		+2.99	+0.25					
			+1.18	+0.08					
1958δ2, 1959α1, 1959η	2	+0.60							
		-2.24							
Kozai - 1963	1		+1.89	-0.28					
			+0.15	-0.44					
	2	+0.72	+0.12	-0.04					
		-0.95	-0.08	-0.05					
	3		-0.063	+0.035					
			+0.083	+0.001					
	4			+0.016					
				+0.026					
Izsak - 1963	1		+1.12	-0.288					
Vanguard 2,			+0.062	-0.321					
Vanguard 3,	2	+0.968	+0.091	+0.035					
Explorer 9,		-0.40	-0.018	+0.012					
Echo 1R,	3		+0.072	+0.021					
Midas 4			+0.124	+0.015					
	4			+0.010					
				+0.016					
Anderle Oesterwinter 1963 Anna Ib	0	-1082.466	+2.476	+1.405	+0.140				
	1			-0.68					
				-0.38					
	2	+1.84		+0.101					
		-0.99		+0.269					
	3			+0.158					
				-0.036					
Guier - 1963	1		+1.77	-0.57					
			+0.19	-0.46					
	2	+1.68	+0.29	+0.06					
		-0.64	-0.02	+0.27					
	3		+0.147	+0.080					
			+0.140	-0.003					
	4			-0.008					
				+0.007					

d.c.Tab. 3

Author, year, satellites, the method	m	C_{2m} S_{2m}	C_{3m} S_{3m}	C_{4m} S_{4m}	C_{5m} S_{5m}	C_{6m} S_{6m}	C_{7m} S_{7m}	C_{8m} S_{8m}	C_{9m} S_{9m}
Izsak - 1964 xxxxx)	1	-	+0.87 -0.27	-0.17 -0.24					
	2	+0.75 +0.61	+0.08 -0.09	-0.025 +0.052					
	3		-0.070 +0.129	+0.017 -0.005					
	4			-0.002 +0.006					

x) 1957 β, 1958 β, 1958 β2, 1958 δ1, 1958 δ2, 1959 α1, 1959 η, 1959 ι1, 1960 β2, 1960 γ1, 1960 γ2, 1960 η, 1960 η1, 1960 ι2, 1961 o2

xx) 1959 α1, 1959 η, 1960 ι2, 1961 o1, 1961 o 2, 1961 αδ1, 1962 αε, 1961 ν,

xxx) 1959 α1, 1959 η. 1960 ι2, 1960 ν, 1960 o, 1961 αδ1, 1962 αε, 1962 βμ1, 1962 βυ.

xxxx) 1961 α1, 1960 η1, 1962 o 1, 1960 ι2, 1961 δ1, 1959 α1, 1961 ν,

xxxxx) 1959 α1, 1959 α2, 1959 η, 1961 δ1, 1962 βμ1, 1961 o 1, 1961 o 2, 1961 αδ1.

APPENDIX 2

Programme RKG

<u>begin</u> <u>comment</u> This programme has to compute the influence of particular harmonics of the Earth gravity field on the motion of artificial satellite. Input data: index = subscript of the harmonic, C,S, - one-dimensional arrays of the values of harmonics Cnm, Snm, m = Legendre coefficient, t = degree of harmonic, l = total number of steps.After printing on the monitor of the word < interval > the value of the output interval (number of steps) should be introduced. A = semi-major axis in megameters, e = eccentricity, I = inclination, w = argument of perigee, W = R.A. of node in degrees. The computation is performed by the numerical integration, using Runge-Kutta-Gill method.Step of integration - one sidereal minute; <u>integer</u> index, i, j, p, n, l, h, int, t;

<u>real</u> d,s,x,y,z,r,m,PX,QX,PY,QY,PZ,QZ,cts,sts;

```
array  X[1:2], Y[1:2], Z[1:2], R[1:2], VX[1:2], VY[1:2], VZ[1:2],C[1:2],
       S[1:2], time [1:2], k[1:7], g[1:7], q[1:2,1:7], a[1:4], b [1:4],
       c[1:4];
procedure KROK;
for p: = 1,2 do
begin
    g[1]: = time[p];
    g[2]: = VX[p];
    g[3]: = VY[p];
    g[4]: = VZ[p];
    g[5]: = X[p];
    g[6]: = Y[p];
    g[7]: = Z[p];
    for j: = 1,2,3,4 do
begin
    x: = g[5];
    y: = g[6];
    z: = g[7];
    r: = sqrt(x↑2+y↑2+z↑2) ;
    s: = g[1]× 6.28318531/1440;
HERE PUT FRAGMENT OF PROGRAMME FOR PROPER HARMONICS:
    cts: = m×cos(txs);
    cts: = m×sin(txs);
    k[2]: = -5.5002277₁₀-3×(x/r↑3 + 1.62405₁₀ - 3×(-5 × x × z↑2/r↑7
            + x/r↑5) + cts × (C[p] × PX + S[p]× QX) +sts × (C[p] × QX
            - S[p]× PX));
    k[3]: = -5.5002277₁₀ - 3×(y/r↑3 + 1.62405₁₀ -3× (-5 × y × z ↑2/r ↑7
            + y/r↑5) +cts × (C[p] × PY + S[p] × QY)+sts ×(C[p] × QY - S[p] ×PY));
    k[4]: = -5.5002277₁₀-3×(z/r↑3 + 1.62405₁₀- 3×(-5×z↑3/r↑7 + 3×z/r↑5)
            + cts × (C[p]× PZ + S[p] × QZ)+ sts ×(C[p]× QZ- S[p] × PZ));
    k[5]: = g[2];
    k[6]: = g[3];
    k[7]: = g[4];
for i: = 1 step 1 until 7 do
begin
    d: =a[j] ×(k[i]- b[j] ×q[p, i]);
    g[i]: = g[i]+ h×d;
    q[p,i]: = q[p,i] + 3×d - c[j] × k[i];
end i;
```

```
for i: = 1 step 1 until 7 do
output (⊀+nddddddd₁₀ - dd⊁, g[i], outsp(5), k[i],outsp(5),q[p,i],outcr );
output (⊀+nd⊁,p,outsp(3),j,outsp(3),i,outcr);
output (⊀+nddddddd₁₀ - dd⊁, x ,
outsp(5),y,outsp(5),z, outsp(5),r,
outsp(5),s,outcr, PX, outsp(3),
QX,outsp(3),PY,outsp(3), QY,outsp(3),
PZ,outsp(3),QZ,outcr,
cts,outsp(5),sts,outcr,outcr);
end j;
time [p]: = g[1];
VX[p]: = g[2];
VY[p]: = g[3];
VZ[p]: = g[4];
X[p]: = g[5];
Y[p]: = g[6];
Z[p]: = g[7];
R[p]: = sqrt (X[p]↑2+Y[p]↑2+Z[p]↑2);
end KROK;
begin
real A,e,I,w,W,P1,P2,Q1,Q2,R1,R2,N,G;
input (index, C,S,l,m,t);
writetext (⊀⊀interval⊁⊁);
int: = typein;
writecr;
input (A,e,I,w,W);
I: = I/57.295780;
w: = w/57.29578;
W: = W/57.295780;
P1: = cos(w)×cos(W) - sin(w)×cos(I)×sin(W);
P2: = -sin(w)×cos(W) - cos(w)× cos(I)×sin (W);
Q1: = cos(w)×sin(W)+sin(w)×cos(I)×cos(W);
Q2: = -sin(w)×sin(W) + cos(w)×cos(I)×cos(W);
R1: = sin(w)×sin(I);
R2: = cos(w)×sin(I);
A: = A/6.378165;
N: = sqrt(5.5002277₁₀- 3/A↑3);
r: = Ax(1- e);
```

```
R[1]: = R[2]: = r;
X[1]: = X[2]: = r ×P1;
Y[1]: = Y[2]: = r ×Q1;
Z[1]: = Z[2]: = r ×R1;
G: = A↑2× N ×sqrt(1-e↑2)/r;
VX[1]: = VX[2]: = G × P2;
VY[1]: = VY[2]: = G × Q2;
VZ[1]: = VZ[2]: = G × R2;
time[1]: = time [2]: = 0;
a[1]: = c[1]: = c[4]: = 0.5;
a[2]: = c[2]: = 1 - sqrt (0.5);
a[3]: = c[3]: = 1 + sqrt (0.5);
a[4]: = 1/6;
b[1]: = b[4]: = 2;
b[2]: = b[3]: = 1;
for p: = 1,2 do
for i: = 1 step 1 until 7 do
q[p,i]: = 0;
h: = 1;
n: = 0;
k[1]: = 1;
outtext (outcr,⪡Variant No:⪢, output (⪡ddd⪢, index), outcr,
⪡ harmonics values:⪢, output(⪡+nddd₁₀-d⪢,C[1], outsp(3),
S[1],outcr,outsp(20),C[2],outsp(3), S[2]),outcr, outcr,
⪡Orbital elements:
a =⪢, output(⪡-nddd.dddddd⪢,Ax6.378165), outsp(5),
⪡ =⪢, output(⪡-n.dddd⪢,e), outcr,⪡ I,w,W ⪢,
output(⪡- ndd.ddd⪢, I x 57.296, outsp(3), w x 57.296, outsp(3),
W x 57.296), ⪡deg ⪢,outcr, ⪡ P= ⪢,output(⪡ nddd.ddd⪢, 6.28318/N),
⪡ sid.min.
Initial coordinates and velocities:
X,Y,Z, ⪢, output(⪡ +nddd.dddddd⪢, X[1],
outsp(5), Y[1], outsp(5), Z[1], outsp(5)),
⪡earth radii
VX, VY, VZ ⪢, output(⪡ -nddddd₁₀ - dd⪢, VX[1],
outsp(5), VY[1], outsp(5), VZ[1]), outcr,
⪡min ⑦ dr ⑧ du ⑧ dw
  0 ⑦ 0 0 ⑦ 0 0 ⑦ 0 0
```

```
⋟);
end;
for n: = n+1 while n≤1 do
begin
  KROK;
  if n/int = n: int then
  begin
    real V;
    array L,M,N, dr[1:3];
    V: = sqrt (VX[1]↑2 + VY [1]↑2 + VZ[1]↑2);
    L[2]: = VX[1]/V;
    M[2]: = VY[1]/V;
    N[2]: = VZ[1]/V;
    L[3]: = X[1]/R[1];
    M[3]: = Y[1]/R[1];
    N[3]: = Z[1]/R[1];
    L[1]: = M[2] × N[3]- M[3] × N[2];
    M[1]: = L[3] × N[2]- L[2] × N[3];
    N[1]: = L[2] × M[3]- L[3] × M[2];
    V: = sqrt(L[1]↑ 2 + M[1]↑2 + N[1]↑2);
    L[1]: = L[1]/V;
    M[1]: = M[1]/V;
    N[1]: = N[1]/V;
    for p: = 1,2,3 do
    dr[p]: =(L[p]×(X[2]-X[1])+ M[p]×(Y[2]- Y[1]) + N[p] × (Z[2]- Z[1])) ×
            × 63781650;
    output (⋞dddd⋟,n,outsp(3));
    output (⋞+nddd d⋟, dr[3], outsp(3), dr[2], outsp(3),
    dr[1], outcr);
  end:
  if kbon then
  write(⋞dddd⋟,n);
  end;
end;
```

Example of input data for programme RKG variant 32:

```
32,0,200₁₀-9,0,-30₁₀-9,
l 720, m 15, t 2,
orbit 7.392 mgm, e 0.0025, i 63.4, w 0, W 0;
```

APPENDIX 3

Fragments of the RKG programme serving for the computation of the particular harmonics

```
x30: PX: = (5/2)×(-7×x×z↑3/r↑9+3×x×z/r↑7);
     PY: = (5/2)×(-7×y×z↑3/r↑9+3×y×z/r↑7);
     PZ: = (1/2)×(-35×z↑4/r↑9+30×z↑2/r↑7-3/r↑5);
     QX: = QY: = QZ: = 0;

x40: PX: = (1/8)×(-315×x×z↑4/r↑11+210×x×z↑2/r↑9-15×x/r↑7);
     PY: = (1/8)×(-315×y×z↑4/r↑11+210×y×z↑2/r↑9-15×y/4↑7);
     PZ: = (5/8)×(-63×z↑5/r↑11+70×z↑3/r↑9-15×z/r↑7);
     QX: = QY: = QZ: = 0;

x50: PX: = (21/8)×(-33×x×z↑5/r↑13+30×x×z↑3/r↑11-5×x×z/r↑9);
     PY: = (21/8)×(-33×y×z↑5/r↑13+30×y×z↑3/r↑11-5×y×z/r↑9);
     PZ: = (3/8)×(-231×z↑6/r↑13+315×z↑4/r↑11-105×z↑2/r↑9+5/r↑7);
     QX: = QY: = QZ: = 0;

X22: PX: = 2×x/r↑5 - 5×(x↑2-y↑2)xx/r↑7;
     QX: = 2×y/r↑5 - 10×x↑2×y/r↑7;
     PY: = -2×y/r↑5 - 5×y×(x↑2-y↑2)/r↑7;
     QY: = 2×x/r↑5 - 10×x×y↑2/r↑7;
     PZ: = -5×z×(x↑2-y↑2)/r↑7;
     QZ: = -10×x×y×z/r↑7;

X31: PX: = -35×x↑2×z↑2/r↑9+5×(x↑2+z↑2)/r↑7-1/r↑5;
     QX: = -35×x×y×z↑2/r↑9+5×x×y/r↑7;
     PY: = QX;
     QY: = PX;
     PZ: = 15×x×z/r↑7-35×x×z↑3/r↑9;
     QZ: = 15×y×z/r↑7-35×y×z↑3/r↑9;

X32: PX: = 2×x×z/r↑7-7×x×z×(x↑2-y↑2)/r↑9;
     QX: = 2×x×z/r↑7-14×x↑2×y×z/r↑9;
     PY: = -2×y×z/r↑7-7×y×z×(x↑2-y↑2)/r↑9;
     QY: = 3×(x↑2-y↑2)/r↑7-14×x×y↑2×z/r↑9;
     PZ: = (x↑2-y↑2)/r↑7-7×z↑2×(x↑2-y↑2)/r↑9;
     QZ: = 2×x×z/r↑7-14×x×y×z↑2/r↑9;
```

```
X33: PX: = 3×(x↑2-y↑2)/r↑7-7×x↑2×(x↑2-3×y↑2)/r↑9;
     QX: = 6×x×y/r↑7-7×x×y×(3×x↑2-y↑2)/r↑9;
     PY: = -6×x×y/r↑7-7×x×y×(x↑2-3×y↑2)/r↑9;
     QY: = 3×(x↑2-y↑2)/r↑7-7xy↑2x(3xx↑2-y↑2)/r↑9;
     PZ: = -7×x×z×(x↑2-3×y↑2)/r↑9;
     QZ: = -7×x×z×(3×x↑2-y↑2)/r↑9;

X41: PX: = -3×z/r↑7+7×z×(3×x↑2+z↑2)/r↑9-63×x↑2×z↑3/r↑11;
     QX: = 21×x×y×z/r↑9-63×x×y×z↑3/r↑11;
     PY: = QX;
     QY: = PX;
     PZ: = -3×x/r↑7+42×x×z↑2/r↑9-63×x×z↑4/r↑11;
     QZ: = -3×y/r↑7+42×y×z↑2/r↑9-63×y×z↑4/r↑11;

X42: PX: = -2×x/r↑7+7×x×(x↑2-y↑2+2×z↑2)/r↑9-63×x×z↑2×(x↑2-y↑2)/r↑11;
     QX: = -2×y/r↑7+14×y×(x↑2)/r↑9-126×x↑2×y×z↑2/r↑11;
     PY: = 2×y/r↑7+7×y×(x↑2-y↑2-2×z↑2)/r↑9-63×y×z↑2×(x↑2-y↑2)/r↑11;
     QY: = -2×x/r↑7+14×x×(y↑2+z↑2)/r↑9-126×y↑2×x×z↑2/r↑11;
     PZ: = 21×z×(x↑2-y↑2)/r↑9-63×z↑3x(x↑2-y↑2)r↑11;
     QZ: = 42×x×y×z/r↑9-126×x×y×z↑3/r↑11;

X43: PX: = 3×z×(x↑2-y↑2)/r↑9-9×z×(x↑2-3×y↑2)/r↑11;
     QX: = 6×x×y×z/r↑9-9×x×y×z×(3×x↑2-y↑2)/r↑11;
     PY: = -6×x×y×z/r↑9-9×x×y×z×(x↑2-3×y↑2)/r↑11;
     QY: = 3×z×(x↑2-y↑2)/r↑9-9×y↑2×z×(3×x↑2-y↑2)/r↑11;
     PZ: = x×(x↑2-3×y↑2)/r↑9-9×x×z↑2×(x↑2-3×y↑2)/r↑11;
     QZ: = y×(3×x↑2-y↑2)/r↑9-9×y×z↑2x(3×x↑2-y↑2)/r↑11;

X44: PX: = 4×x×(x↑2-3×y↑2)/r↑9-9×x×((x↑2-y↑2)↑2-4×x↑2×y↑2)/r↑11;
     QX: = 4×y×(3×x↑2-y↑2)/r↑9-36×x↑2×y×(x↑2-y↑2)/r↑11;
     PY: = -4×yx(3×x 2-y↑2)/r↑9-9×y×((x↑2-y↑2)↑2-4×x↑2×y↑2)/r↑11;
     QY: = 4×x×(x↑2-3×y↑2)/r↑9-36×x×y↑2×(x↑2-y↑2)/r↑11;
     PZ: = -9×z×((x↑2-y↑2)↑2-4×x↑2×y↑2)/r↑11;
     QZ: = -36×x×y×z×(x↑2-y↑2)/r↑11;
```

APPENDIX 4

Programme ECHO

begin **comment** This programme has been established for the computation of the lenght of the segment in space empraced between two points in which the satellite was at two known moments. Input data: T - one-dimensional **array** containing the differences between the moments of observations and the epoch of orbital elements. w, dw, W, dW, io, di, eo, de, MO, n, n1 - - the mean orbital elements, according to the scheme of the SAO Special Report respectively: Argument of perigee, change of perigee, r.a. of the node, change of the node, inclination, change of inclination, eccentricity, mean anomaly at the epoch, mean motion, change of the mean motion. SO - sidereal greenwich time at Oh UT;

```
integer q;
real l,wo,dw,WO,dW,io,di,eo,de,MO,n,n1,SO;
array t [1:2], X[1:2], Y[1:2], Z[1:2];
real procedure TSEK(a);
real a;
begin real d, f;
integer b,c;
  if a = 0 then TSEK: = 0 else
  begin
    f: = abs(a);
    b: = entier(f/10^4);
    c: = entier ((f-bx10^4)/100);
    d: = f-bx10^4-cx10^2;
    TSEK: = (b×3600 + c × 60 + d)×(if a < 0 then -1 else 1);
  end
end;
INPUT:
  input(T,wo,dw,WO,dW,io,di,eo,de,MO,n,n1,SO);
  SO: = TSEK(SO);
S: for q: = 1 step 1 until 2 do
  begin real TG,M,E,E1,sinv,cosv,v,u,sini,cosi,sini2,p,øu,sinu,cosu,a, r,
            ør,øi,øW,L,w,W,i,e;
T[q]: = TSEK(T[q]);
TG: = T[q] × 236.558/86400+SO+T[q];
T[q]: = T[q]/86400;
w: = (wo+dw×T[q])/57.295780;
W: = (WO+dW×T[q])/57.295780;
i: = (io+di×T[q])/57.295780;
e: = eo+de×T[q];
```

```
    M: = (MO+n×T[q]+ n1 × (T[q]↑2));
    M: = (M-entier(M)) × 6.2831853;
    E: = M;
RK: E1: = M + e×sin(E);
    E: = M+ e×sin(E1);
    if abs(E1-E) >₁₀-7 then go to RK;
    sinv: =(sqrt(1-e↑2) × sin(E))/(1-e×cos(E));
    cosv: =(cos(E)-e)/(1-e×cos(E));
    v: = if abs(cosv)≥abs(sinv) then
         arctan(sinv/cosv)+(if cosv<0 then 3.14159265 else if sinv≥0
         then 0 else 6.28318531) else arctan(-cosv/sinv) +(if  sinv≥0
         then 1.57079633 else 4.71238898);
    u: = w+v;
    sini2: = sin(i) ↑2;
    p: = ((75371.72/(n+n1 x T[q]) ↑2) ↑(1/3)) x (1-e↑2);
    øu: = (0.0660546/p↑2)x (((-1+ 7 sini2/6) × sin(2×w+2×v)+ e × ((-1 +
          5×sini2/3) × sin(2×w+ v)+ ((-1+ sini2) × sin(2×w+ 3×v))/3))/2
          -((-1 + 3 × sini 2/2) × ((1- sqrt(1-e↑2)) × sinv × cosv+(e↑3)×sinv×
          /(1+sqrt(1-e↑2))↑2))/3-(v-M+e×sinv)×(-2+5×sini2/2));
    sinu: = sin(u)+øu×cos(u);
    cosu: = cos(u)-øu×sin(u);
    a: = (p/(1-e↑2))×(1+(0.0660546/(3×p↑2))×sqrt(1-e↑2)×(-1 +3×sini 2/2));
    ør: = (0.0660546/(3×p))×((-1+3×sini2/2)×(1-(1-e×cos(E))/sqrt(1-e↑2)
          +e cosv/(1+sqrt(1-e↑2)))+cos(2×w+2×v)×sini2/2);
    r: = a×(1-e×cos(E))+ør;
øi: = ((0.0660546/p↑2)×sin(i)×cos(i)/2)×(cos(2×w+2×v)+e×(cos(2×w+v)
      +cos(2×w+3×v)/3));
sini: = sin(i)+øi×cos(i);
    cosi: = cos(i)-øi×sin(i);
    øW: = (0.0660546/p↑2)×cos(i)×(-v+M-e×sinv+(sin(2×w+2×v)+e×(sin(2×w+v)
        +sin(2×w+3×v)/3))/2)
    W: = W+øW;
    L: = W-TG× 15/206264.81;
    X[q]: = r×(cosu × cos(L)- sinu × cosi × sin(L));
    Y[q]: = r×(cosu × sin(L)+ sinu × cosi × cos(L));
    Z[q]: = r×sinu×sini;
                              output(⊰-d.dddddd⊱,T[q],outsp(3),
M,outsp(3),E,outsp(3),v,outcr,p,outsp(3),a,outsp(3),r,
```

```
    outcr,øu,outsp(3),ør,outsp(3),øi,outsp(3),øW,outcr, W,outsp(3),L,outcr,
    X[q],outsp(3), Y[q],
    outsp(3), Z[q],outcr,outcr);
end;
  l: = sqrt(abs(X[2]-[1])↑2+abs(Y[2]-Y[1])↑2+abs(Z[2]-Z[1])↑2);
  outtext(⋞l =⋟,output(⋞nd.dddddd⋟,l),outcr,outcr);
go to INPUT
end;
```

APPENDIX 5

T a b l e 9

Input data for the programme ECHO

```
VI pierwszy
T: -0 49 37
   -0 47 37
w= 264.19 dw=3.31 W=212.788 dW=-3.295 i=47.240 di=0.005
e= 0.04312 de=0.00066 M=0.83158 n=12.496514 nn=0.000680; s=16 35 01.833;

VI trzeci
T: -0 43 40
   -0 41 39
w=270.86 dw=3.32 W=206.202 dW=-3.287 i=47.256 di=0.007
e=0.04437 de=0.00070 M=0.82703 n=12.497670 nn=0.000700; s=16 42 54.937;

VI czwarty
T: -1 43 35
   -1 41 35
w= 274.32 dw=3.46 W=202.906 dW=-3.296 i=47.260 di=0.004
e= 0.04490 de=0.00053 M=0.32532 n=12.498198 nn=0.000420; s=16 46 51.490;

VI piąty
T: -2 47 37
   -2 45 36

w= 277.65 dw=3.33 W=199.609 dw=-3.297 i=47.262 di=0.002
e= 0.04541 de=0.00051 M=0.82430 n=12.498620 nn=0.000440; s= 16 50 48.044;

T: -2 45 36
   -2 43 40

w= 277.65 dw=3.33 W=199.609 dw=-3.297 i=47.262 di=0.002
e= 0.04541 de=0.00051 M=0.82430 n=12.498620 nn=0.000440;s=16 50 48.044;

T: -0 43 41
   -0 41 44
```

w= 277.65 dw=3.33 W=199.609 dw= -3.297 i=47.262 di=0.002
e= 0.04541 de=0.00051 M=0.82430 n=12.498620 nn=0.000440; s=16 50 48.044;

VI szósty
T: -1 39 36
-1 37 45

w=281.02 dw=3.37 W=196.306 dW=-3.300 i=47.261 di= -0.001
e= 0.04591 de=0.00050 M=0.32365 n=12.498905 nn=0.000200; s=16 54 44.601;

VI siódmy
T: -0 47 37
-0 45 36

w= 284.37 dw=3.35 W=193.008 dW= -3.301 i=47.260 di= -0.001
e= 0.04647 de=0.00056 M=0.82316 n=12.499152 nn=0.000320; s=16 58 41.160;

T: -0 45 36
-0 43 45

w=284.37 dw=3.35 W=193.008 dW=-3.301 i=47.260 di=-0.001
e= 0.04647 de=0.00056 M=0.82316 n=12.499152 nn=0.000320; s=16 58 41.160;

VI dziesiąty
T: -1 51 41
-1 49 36

w=294.55 dw=3.37 W=183.111 dW=-3.300 i=47.265 di=0.002
e= 0.04780 de=0.00033 M=0.32298 n=12.499926 nn=0.000222; s=17 10 30.844;

T: -1 49 36
-1 47 44

w= 294.55 dw=3.37 W=183.111 dW=-3.300 i=47.265 di=0.002
e= 0.04780 de=0.00033 M=0.32298 n=12.499926 nn=0.000222; s=17 10 30.844;

VI czternasty
T: -1 53 40
-1 51 43

w= 308.42 dw=3.49 W=169.909 dW=-3.306 i=47.268 di=0.001
e= 0.04895 de=+0.00022 M=0.32520 n=12.500913 nn=0.000270;s=17 26 17.077;

T: -1 51 43
-1 49 37

w= 308.42 dw=3.49 W=169.909 dW=-3.306 i=47.268 di=0.001
e= 0.04895 de=+0.00022 M=0.32520 n=12.500913 nn=0.000270;s=17 26 17.077;

T: -1 49 37
-1 47 44

w= 308.42 dw=3.49 W=169.909 dW=-3.306 i=47.268 di=0.001
e= 0.04895 de=+0.00022 M=0.32520 n=12.500913 nn=0.000270;s=17 26 17.077;

VI szesnasty
T: -1 43 45
-1 41 44

w= 315.52 dw=3.65 W=163.315 dW=-3.298 i=47.272 di=0.001
e= 0.04930 de=0.00017 M=0.32735 n=12.501435 nn=0.000340;s=17 34 10.181;

VI osiemnasty
T: -1 55 40
-1 53 44

w= 322.67 dw=3.71 W=156.711 dW=-3.298 i=47.274 di=0.000
e= 0.04946 de=0.00014 M=0.33037 n=12.502007 nn=0.000310;s=17 42 03.288;

T: -1 53 44
-1 51 40

w= 322.67 dw=3.71 W=156.711 dW=-3.298 i=47.274 di=0.000
e= 0.04946 de=0.00014 M=0.33037 n=12.502007 nn=0.000310;s=17 42 03.288;

T: -1 51 40
-1 49 37

w= 322.67 dw=3.71 W=156.711 dW=-3.298 i=47.274 di=0.000
e= 0.04946 de=0.00014 M=0.33037 n=12.502007 nn= 0.000310;s=17 42 03.288;

T: -1 49 37
-1 47 38

w= 322.67 dw=3.71 W=156.711 dW=-3.298 i=47.274 di=0.000
e= 0.04946 de=0.00014 M=0.33037 n=12.502007 nn=0.000310; s=17 42 03.288;

APPENDIX 6

Table 10

The results of the programme ECHO

Date 1963	Time	U T	l
V.31	$23^h10^m23^s$	- $23^h12^m23^s$	0.776545 Mgm
VI.2	23 16 20	- 23 18 21	0.777179
VI.3	22 16 25	- 23 18 25	0.772085
VI.4	21 12 23	- 21 14 24	0.785811
VI.4	21 14 24	- 21 16 20	0.750678
VI.4	23 16 19	- 23 18 16	0.750801
VI.5	22 20 24	- 22 22 15	0.711222
VI.6	23 12 23	- 23 14 24	0.778848

d.c.Tab. 10

Date 1963	Time	U T	1
VI.6	23 14 24	- 23 16 15	0.712856
VI.9	22 08 19	- 22 10 24	0.812686
VI.9	22 10 24	- 22 12 16	0.725269
VI.13	22 06 20	- 22 08 17	0.762964
VI.13	22 08 17	- 22 10 23	0.817849
VI.13	22 10 23	- 22 12 16	0.730585
VI.15	22 16 15	- 22 18 16	0.775171
VI.17	22 04 20	- 22 06 16	0.758751
VI.17	22 06 16	- 22 08 20	0.807184
VI.17	22 08 20	- 22 10 23	0.797148
VI.17	22 10 23	- 22 12 22	0.768307

APPENDIX 7

Programme TETRAHEDRON 2

```
begin comment This programme has to compute the lenght and the coordi-
nates of the vector linking the two observing stations. The meaning of
input data: year, month, day of the epoch of the Greenwich sidereal ti-
me which follows the date of observation, month and day of observation,
the first moment of simultaneous observation in UT, r.a.and declination
observed at the first station, r.a. and declination observed of the se-
cond station, the second moment of observation, right ascensions and de-
clinations as above, the lenght of the segment in space;
integer i;
real L,SO,TP,TQ,ÆAP,ÆAQ,DAP,DAQ,ÆBP,ÆBQ,DBP,DBQ,tAP,tAQ,tBP,tBQ,G,H,I,J,
     W,AP1,PP1,QP1.BP2,PP2,QP2,AP3,PP3,BP3,AP4,QP4,BP4,W1,W2,W3,W4;
array VG[1:3],VH[1:3],VI[1:3],VJ[1:3],P1[1:3],P2[1:3],P3[1:3], P4[1:3],
     VL[1:3], VW[1:3];
procedure PROSTA(V,D,t);
  array V;
  real D,t;
```

```
begin
  V[1]: = cos(D)xcos(t);
  V[2]: = cos(D)xsin(t);
  V[3]: = sin(D);
end PROSTA;
procedure PL(AA,M,N);
array AA,M,N;
begin array A[1:3];
  A[1]: = M[2]x N[3]-M[3]x N[2];
  A[2]: = M[3]x N[1]-M[1]x N[3];
  A[3]: = M[1]x N[2]-M[2]x N[1];
  for i: = 1,2,3 do
  AA[i]: = A[i]/sqrt(A[1]↑2 + A[2]↑+ A[3]↑ 2)
end PL;
real procedure KDNT(M,N);
array M,N;
begin real cos,sin;
  cos: = (M[1]x N[1]+ M[2]x N[2]+ M[3]x N[3])/sqrt((M[1]↑2+ M[2]↑2 +
          + M[3]↑2)x(N[1]↑2+N[2]↑2+N[3]↑2));
  sin: = sqrt(1-cos↑2);
  KDNT: = sin
end KDNT;
  real procedure BOK(b,sina,sinb);
real b,sina,sinb;
begin real a;
  a: = b xsina/sinb;
  BOK: = a
end BOK;
real procedure INTIME;
begin real a;
integer b,c,sig;
  a: = inone;
  sig: = sign(a);
  a: = abs(a);
  b: = entier(a): 100;
  c: = b: 100;
  INTIME: = (a-b x 40 - c x 240)/86400 x sig
```

```
end INTIME;
real procedure INANG;
begin real a;
integer b,c,sig;
  a: = inone;
  sig: = sign(a);
  a: = abs(a);
  b: = entier(a):100;
  c: = b:100;
  INANG: = (a-b'x 40-c x 2400)/206264.81 x sig
end INANG;
real procedure SGr(T);
real T;
SGr: = (SO-(1-T)x 1.00273791) x 6.2831853;

ART:
  L: = inone;
  L: = inone;
  L: = inone;
  SO: = INTIME;
  outcopy(≮<d≯); outcr;
  TP: = INTIME;
   AP: = INANG; DAP: = INANG;
   BP: = INANG; DBP: = INANG;
  TQ: = INTIME;
   AQ: = INANG; DAQ: = INANG;
   BQ: = INANG; DBQ: = INANG;
  L: = inone;
  tAP: = ÆAP-SGr(TP);
  tAQ: = ÆAQ-SGr(TQ);
  tBP: = ÆBP-SGr(TP);
  tBQ: = ÆBQ-SGr(TQ);
  PROSTA(VG,DAP,tAP);
  PROSTA(VH,DAQ,tAQ);
  PROSTA(VI,DBP,tBP);
  PROSTA(VJ,DBQ,tBQ);
  PL(P1,VH,VG);
  PL(P2,VI,VJ);
  PL(P3,VI,VG);
```

```
  PL(P4,VH,VJ);
  PL(VL,P2,P1);
  PL(VW,P3,P4);
  AP1: = KDNT(VG,VH);
  PP1: = KDNT(VL,VG);
  QP1: = KDNT(VH,VL);
  BP2: = KDNT(VI,VJ);
  PP2: = KDNT(VI,VL);
  QP2: = KDNT(VL,VJ);
  AP3: = KDNT(VG,VW);
  PP3: = KDNT(VG,VI);
  BP3: = KDNT(VW,VI);
  AP4: = KDNT(VH,VW);
  QP4: = KDNT(VH,VJ);
  BP4: = KDNT(VW,VJ);
  G: = BOK(L,QP1,AP1);
  H: = BOK(L,PP1,AP1);
  I: = BOK(L,QP2,BP2);
  J: = BOK(L,PP2,BP2);
  W1: = BOK(H,QP4,BP4);
  W2: = BOK(G,PP3,BP3);
  W3: = BOK(J,QP4,AP4);
  W4: = BOK(I,PP3,AP3);
  W: = (W1+W2+W3+W4)/4;
  output(≮-ddd.ddd≯,W,outcr);
  for i: = 1,2,3 do
  output(≮+ndd.ddd≯,W x VW[i],outsp(3));
  outcr;
outtext(≮≪katy godzinne ≯,outcr);
  output(≮-n.dddddd≯,tAP,outsp(3),tAQ,outsp(3),tBP,outsp(3),tBQ,outcr);
  outtext(≮≪P1≯,outsp(9),≮≪P2≯,outsp(9),≮≪P3≯,outsp(9),≮≪P4≯,outsp (9),
  ≮≪VL≯, outsp(9), ≮≪ VW≯,outcr);
  for i: = 1,2,3 do
  output(<+ndd.ddd>,P1[i],outsp(3),P2[i],outsp(3),P3[i],outsp(3),P4[i],
  outsp(3),VL[i], outsp(3),VW[i],outcr); outtext(≮≪W=≯,output(≮-ddd.ddd≯,
  W1,outsp(3),W2,outsp(3),W3,outsp(3),W4,outcr),≮≪wektory≯,
  outcr,outsp(3),≮≪VG≯,outsp(9),≮≪VH≯,outsp(9),≮≪VI≯,outsp(9),≮≪VJ≯ ,
  outsp(9), ≮≪ VL≯,outcr;
```

```
for i: = 1,2,3 do
output(≮+ndd.ddd≯,Gx VG[i]outsp(3), H xVH[i], outsp(3), I xVI[1],outsp (3),
Jx VJ[i],outsp(3), L x VL[i], outcr);
outtext(≮<sinusy  katow≯, outcr,outsp(3),≮<A≯, outsp(10),≮<B≯, outcr);
output(≮+n.dddddd≯, AP1,outsp(3),BP2,outcr);

go to ART
end;
```

APPENDIX 8

Table 11

Input data for the programme TETRAHEDRON 2

Rok 1963
Czas gw Gr na 6 mies 1.0 d : 16 35 01.833
data obs. 5 mies 31 d

T	alfa	delta	alfa	delta
POZNAŃ			RYGA	
23 10 23, 23 12 23, l=776.545 km	253 16 57.13, 267 55 48.08,	-7 55 43.85, -0 25 15.72,	245 35 41.50, 257 35 00.70,	-11 48 08.14, - 6 01 46.02,
1963 6 mies 3.0 d:	16 42 54.937			
6 mies 2 d 23 16 20, 23 18 21, l=777.179	301 12 02.05, 318 04 38.54,	+17 17 05.16, +18 25 17.88,	286 22 51.78, 304 25 36.94,	+10 08 28.52, +12 56 40.77,
1963 6 mies.5.0 d:	16 50 48.044			
6 mies 4 d 23 16 19, 23 18 16, l=750.801	303 55 28.56, 320 27 22.78,	+21 46 13.47, +20 42 48.33,	288 06 59.09, 306 20 25.29,	+14 20 08.57, +15 17 08.77,
1963 6 mies 7.0 d:	16 58 41.160			
6 mies 6 d 23 12 23, 23 14 24, l=778.848	264 14 02.71, 288 16 31.21,	+21 39 12.20, +25 06 02.75,	252 31 44.39, 272 17 52.72,	+10 55 29.77, +15 21 48.48,

d.c.Tab. 11

T	alfa	delta	alfa	delta
1963 6 mies 7.0 d: 6 mies 6 d 23 14 24, 23 16 15, l=712.856	 16 58 41.160 288 16 31.21, 307 46 13.99,	 +25 06 02.75, +24 16 59.69,	 272 17 52.72, 291 26 09.41,	 +15 21 48.48, +16 54 47.25,
1963 6 mies 14.0 d: 6 mies 13 d 22 06 20, 22 08 17, l=762.968	 17 26 17.077 215 20 56.61, 236 45 40.48,	 +18 22 31.45, +25 44 22.27,	 212 59 51.87, 228 39 55.00,	 + 8 12 43.45, +13 13 35.25,
1963 6 mies 14.0 d: 6 mies 13 d 22 08 17, 22 10 23, l=817.855	 17 26 17.077 236 45 40.48, 265 07 26.83,	 +25 44 22.27, +29 03 38.44,	 228 39 55.00, 250 06 29.10,	 +13 13 35.25, +17 02 51.00,
1963 6 mies 14.0 d: 6 mies 13 d 22 10 23, 22 12 16, l=730.593	 17 26 17.077 265 07 26.83, 288 09 45.70,	 +29 93 38.44, +26 37 18.29,	 250 06 29.10, 270 57 54.09,	 +17 02 51.00, +17 29 49.46,
UZHOROD			RYGA	
1963 6 mies 4.0 d: 6 mies 3 d 22 16 25, 22 18 25, l=772.088	 16 46 51.490 297 41 55.71, 314 34 00.36,	 +20 59 13.42, +22 19 29.47,	 284 10 11.68, 299 55 33.30,	 + 6 15 43.32, + 9 43 00.53,
1963 6 mies 5.0 d: 6 mies 4 d 23 16 19, 23 18 16, l=750.801	 16 50 48.044 301 45 10.37, 322 35 31.00,	 +33 57 02.52, +30 56 38.06,	 288 06 59.09, 306 20 25.29,	 +14 20 08.57, +15 17 08.77,
1963 6 mies 16.0 d: 6 mies 15 d 22 16 15, 22 18 16, l=775.185	 17 34 10.181 322 24 31.61, 330 23 12.88,	 +17 18 43.40, +10 18 24.17,	 308 57 56.77, 318 32 00.75,	 + 5 20 35.35, + 0 44 53.67,

d.c.Tab. 11

T	alfa	delta	alfa	delta
1963				
6 mies 18.0 d:	17 42 03.288			
6 mies 17 d				
22 04 20,	196 09 05.96,	+28 24 08.22,	209 50 48.71,	+12 09 47.04,
22 06 16,	215 30 47.82,	+36 16 37.50,	226 42 55.50,	+15 05 37.70,
l=758.758				
1963				
6 mies 18.0 d:	17 42 03.288			
6 mies 17 d				
22 06 16,	215 30 47.82,	+36 16 37.50,	226 42 55.50,	+15 05 37.70,
22 08 20,	246 42 51.52,	+40 30 56.18,	248 26 38.01,	+15 44 07.57,
l=807.193				
1963				
6 mies 18.0 d:	17 42 03.288			
6 mies 17 d				
22 08 20,	246 42 51.52,	+40 30 56.18,	248 26 38.01,	+15 44 07.57,
22 10 23,	278 58 57.68,	+35 21 38.53,	270 07 38.25,	+13 00 34.27,
l=797.159				
1963				
6 mies 18.0 d:	17 42 03.288			
6 mies 17 d				
22 10 23,	278 58 57.68,	+35 21 38.53,	270 07 38.25,	+13 00 34.27,
22 12 22,	300 27 23.44,	+25 34 30.65,	287 38 36.04,	+ 8 20 41.24,
l=768.319				
NIKOLAJEW			RYGA	
1963				
6 mies 5.0 d:	16 50 48.044			
6 mies 4 d				
21 12 23.	244 06 57.39,	+ 4 20 14.47,	252 34 04.22,	- 9 30 13.24,
21 14 24,	263 39 41.73,	+14 23 40.71,	266 09 53.82,	- 3 13 05.48,
l=785.811				
1963				
6 mies 5.0 d:	16 50 48.044			
6 mies 4 d				
21 14 24,	263 39 41.73,	+14 23 40.71,	266 09 53.82,	- 3 13 05.48,
21 16 20,	283 57 30.32,	+21 21 13.39,	279 48 14.44,	+ 2 07 28.04,
l=750.679				
1963				
6 mies 6.0 d:	16 54 44.601			
6 mies 5 d				
22 20 24,	329 48 01.73,	+31 24 36.76,	317 09 36.43,	+12 30 43.85,
22 22 15,	340 55 16.22,	+26 17 25.79,	327 37 29.69,	+11 08 04.17,
l=711.229				

d.c.Tab. 11

T	alfa	delta	alfa	delta
1963 6 mies 10.0 d: 6 mies 9 d 22 08 19, 22 10 24, l=812.689	 17 10 30.844 200 25 51.20, 213 26 37.08,	 +16 56 02.95, +27 58 06.74,	 222 23 59.43, 238 16 20.78,	 + 1 47 40.98, + 8 02 18.67,
1963 6 mies 10.0 d: 6 mies 9 d 22 10 24, 22 12 16, l=725.273	 17 10 30.844 213 26 37.08, 232 28 22.20,	 +27 58 06.74, +38 59 47.96,	 238 16 20.78, 255 53 37.00,	 + 8 02 18.67, +12 56 54.46,

ПРИМЕНЕНИЕ РАДИУС-ВЕКТОРА ИСКУССТВЕННОГО СПУТНИКА ЗЕМЛИ В КАЧЕСТВЕ МЕРЫ ДЛИНЫ В ГЕОДЕЗИИ

Краткое содержание

Пользуясь третьим законом Коплера можно определить полуось орбиты спутника на основании наблюдаемого периода вращения. Если эксцентриситет орбиты является известной величиной, тогда можно вычислить радиус-вектор спутника для произвольного момента даже в том случае, когда известны только приближенные элементы орбиты.

Настоящая статья касается прежде всего исследования, с какой точностью можно определить радиус-вектор спутника. При предпосылках, что эксцентриситет орбиты близок к нулю, высота перигея 1000 - 3000 км, а отношение массы спутника к его поверхности характеризуется сравнительно большой величиной, доказывается, что вычисление радиус-вектор можно произвести с точностью 10^{-5}.

Основной помехой, препятствующей получению результатов высшей точности, является слабое знакомство коэффициентов тессеральных гармоник гравитационного поля Земли. Остальными помехами возмущений являются: ошибки коэффициентов зональных гармоник, атмосферное сопротивление, давление света, притяжение луны и солнца. Все последние факторы имеют значительно меньшее значение.

В статье подан также метод наблюдений и вычислений элементов орбиты спутника. При соблюдении этого метода исключаются ошибки координат точек наблюдений.

Последняя часть статьи содержит несколько способов использования известного радиус-вктор для определения координат точек земной поверхности в геоцентрической системе координат, отнесенной к оси вращения Земли и плоскости экватора.

APPLICATION OF THE RADIUS-VECTOR OF ARTIFICIAL SATELLITE AS LENGHT MEASURE FOR GEODETIC PURPOSES

S u m m a r y

By utilizing the third Kepler's Law, it is possible to define the semi-axis of satellite orbit on the basis of the observed revolution period. If the eccentricity of the orbit is also known, then it is possible to compute the radius-vector of the satellite for an arbitrary moment, even if the remaining elements were known only approximately.

The present study is devoted, first of all, to the examination of the degree of accuracy suspectible to be achieved in the determination of the radius-vector. Assuming that the orbital eccentricity in near zero, that the altitude of perigee is $1000 \div 3000$ km and that the satellite has a small area/mass ratio, the radius-vector can be computed as shown in Chapter VI, with an accuracy of 10^{-5}. A poor knowledge of coefficients of tesseral harmonics of Earth's gravity field seems to be essential impediment in achieving a higher precision. The other sources of perturbations, such as: coefficient errors of zonal harmonics,atmospheric drag, solar radiation pressure, solar and lunar attraction may be regarded as lesser disturbance causes. Also the way of observation and computation of orbital elements, eliminating the effect of errors of the observing site coordinates is showed.

The Chapter VIII presents some modes of using the known radius-vector for determining the coordinate points on the Earth's surface in the geocentric system oriented in conformity with thedirection of the revolution axis and the equatorial plane.