POLAR MOTION PREDICTION BY DIFFERENT METHODS IN POLAR COORDINATES SYSTEM

Kosek Wiesław
Space Research Centre, Polish Academy of Sciences
Warsaw, Poland

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Data sets

- EOPC01 (1846.0 - 2000.0), sampling interval = 0.05 years
  http://hpiers.obspm.fr/eop-pc/

- EOPC04 (1962.0 - 2002.6), sampling interval = 1 day
  http://hpiers.obspm.fr/eop-pc/

- NEOS (1976.0 - 2002.6), sampling interval = 1 day
The current method of polar motion prediction carried out in the IERS Sub-Bureau for Rapid Service and Prediction is a least-squares extrapolation of the polar motion data. The least-squares model consists of a Chandler circle, annual and semiannual ellipses, and a bias. The model is fit to the one year of polar motion data.
Mean error of polar motion prediction computed by the least-squares method from 1984.0 to 2002.7
The absolute values of the difference between polar motion data and its least squares prediction at different starting prediction epochs for different lengths of data going into the model of extrapolation.
Amplitude and phase variations of the Chandler and annual oscillations computed by the least-squares method in 3 year time intervals and Niño data

![Graph showing amplitude and phase variations of the Chandler and annual oscillations with Niño data.](image)
The FTBPF time-frequency spectra of pole coordinate data
Transformation of pole coordinate data into polar coordinates system

\[ R_t = \sqrt{(x_t - x_t^m)^2 + (y_t - y_t^m)^2}, \quad t = 1, 2, \ldots, n \]

\[ L_t = \sqrt{(x_t - x_{t-1})^2 + (y_t - y_{t-1})^2}, \quad t = 2, 3, \ldots, n \]

\( R_t, L_t \) - radius and angular distance
The transformation from the polar into the Cartesian coordinate system

\[ x_{n+1}, y_{n+1} \]

\[ x_n^m, y_n^m \]

mean pole

\[
\begin{bmatrix}
  x_{n+1} \\
  y_{n+1}
\end{bmatrix} = \begin{bmatrix}
  x_n \\
  y_n
\end{bmatrix} \cot \alpha + \begin{bmatrix}
  -y_n \\
  x_n
\end{bmatrix} + \begin{bmatrix}
  x_n^m \\
  y_n^m
\end{bmatrix} \cot \beta + \begin{bmatrix}
  y_n^m \\
  -x_n^m
\end{bmatrix}
\]

\[
cot \alpha = \left( R_{n+1}^2 + R_n^2 - L_{n+1}^2 \right) / 4P, \quad cot \beta = \left( L_{n+1}^2 + R_n^2 - R_{n+1}^2 \right) / 4P,
\]

\[
P = \sqrt{p(p - R_{n+1})(p - R_n)(p - L_{n+1})}, \quad p = \frac{R_{n+1} + R_n + L_{n+1}}{2}
\]
Autocovariance prediction of complex-valued time series

\[ Z_1, Z_2, \ldots, Z_n \quad Z_{n+1} = ? \]

\[ \hat{c}_{zz}^{(n)}(k) = \frac{1}{n} \sum_{t=1}^{n-k} z_t \overline{z}_{t+k}, \quad \text{for} \quad k = 0,1,\ldots, n-1 \]

\[ \hat{c}_{zz}^{(n+1)}(k) = \frac{1}{n+1} \sum_{t=1}^{n-k} z_t \overline{z}_{t+k}, \quad \text{for} \quad k = 0,1,\ldots, n \]

\[ R(z_{n+1}) = \sum_{k=1}^{n-k} \left| \hat{c}_{zz}^{(n)}(k) - \hat{c}_{zz}^{(n+1)}(k) \right|^2 = \min, \]

\[ z_{n+1} = x_{n+1} + i \cdot y_{n+1} \quad \text{- is the first prediction point} \]
Autocovariance prediction cont.

\[
\frac{\partial R(z_{n+1})}{\partial x_{n+1}} = 0, \quad \frac{\partial R(z_{n+1})}{\partial y_{n+1}} = 0
\]

\[
\begin{pmatrix}
    x_{n+1} \\
    y_{n+1}
\end{pmatrix} = \sum_{k=1}^{n-k} \left( \hat{a}(k) \begin{pmatrix} x_{n-k+1} \\ y_{n-k+1} \end{pmatrix} \pm \hat{b}(k) \begin{pmatrix} y_{n-k+1} \\ x_{n-k+1} \end{pmatrix} \right)
\]

\[
\hat{a}(k) = \hat{c}_{xx}^{(n)}(k) + \hat{c}_{yy}^{(n)}(k), \quad \hat{b}(k) = \hat{c}_{yx}^{(n)}(k) - \hat{c}_{xy}^{(n)}(k),
\]

\[
\sum_{k=1}^{n-k} \left( x_{n-k+1}^2 + y_{n-k+1}^2 \right)
\]
Autocovariance prediction of real-valued time series

\[ x_{n+1} = \frac{\sum_{k=1}^{n-k} \hat{c}_{xx}^{(n)}(k) x_{n-k+1}}{\sum_{k=1}^{n-k} x_{n-k+1}^2} \]

\[ \hat{c}_{xx}^{(n)}(k) = \frac{1}{n} \sum_{t=1}^{n-k} x_t x_{t+k}, \quad \text{for} \quad k = 0, 1, \ldots, n - 1 \]
5-year autocovariance predictions of the model pole coordinate data, the radius and angular distance and the transformed pole coordinates data
Autocovariance prediction errors of the model data as a function of data time span and prediction length.

Mean autocovariance prediction errors of the model data with a time span of 60 years.
Mean pole coordinates computed by the Ormsby, Butterworth and boxcar low pass filters

6-year mean
Ormsby LPF (18 years)
Butterworth LPF (7 years)
1-year least-squares predictions of the mean pole computed by the Ormsby LPF, green dots denote the IERS mean pole
The radius and angular distance of polar motion
Time-frequency FTBPF spectra of the radius and angular distance.

The beat period of the Chandler and annual oscillations computed from their least-square phases (solid line) and from the maxima and minima of the R, L time series (dots).

\[
\frac{2\pi}{T} t + (\varphi + \Delta\varphi) = \frac{2\pi}{(T + \Delta T)} + \varphi = const,
\]

\[
\frac{1}{T_{beat}} = \frac{1}{T_{An} + \Delta T_{An}} - \frac{1}{T_{Ch} + \Delta T_{Ch}}
\]
One-year autocovariance and least-squares predictions of the radius and angular distance

Autocovariance prediction

LS prediction

years

arcsec


0.00 0.10 0.20 0.30

R

L

R

L
One-year predictions of the pole coordinate data computed from the autocovariance predictions of the radius and angular distance.
The absolute values of the difference between pole coordinate data and their predictions at different starting prediction epochs.
The mean prediction errors of x and y pole coordinate data of the autocovariance and IERS predictions in the period of 1984.0 – 2002.6
CONCLUSIONS

- Polar motion prediction error depends on starting prediction epochs due to short period irregular variations and irregular phase and amplitude variations of the annual oscillation.
- The phase of the annual oscillation had maximum values before the biggest in the previous century El Niño events in 1982/83 and 1997/98. The observed increase of the annual oscillation phase indicates that another El Niño is expected in the end of this year.
- The beat period of the Chandler and annual oscillations is variable mainly due to variable annual oscillation period (phase).
- The mean prediction error of the autocovariance prediction for a few days in the future is of the same order as the mean prediction error of the current polar motion forecast curried out by the IERS Sub Bureau for Rapid Service and Prediction.
- Transform from Cartesian to polar coordinate system helps to solve the frequency resolution problem because the frequencies are transformed into their beat frequencies.
- Prediction accuracy in polar coordinate system depends on prediction error of the mean pole, radius and angular distance.
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