Intermittent Nature of the Solar Wind Turbulence: Multifractal Analysis

A. Szczepaniak,[a] W.M. Macek,[a,b]

[a]Space Research Centre, Polish Academy of Sciences, Bartycka 18 A, 00-716 Warsaw, Poland

[b]Faculty of Mathematics and Natural Sciences. College of Sciences, Cardinal Stefan Wyszyński University, Dewajtis 5, 01-815 Warsaw, Poland

ABSTRACT

One of the features related to turbulence is the understanding of the energy transfer among different scales of the flow. Based on the Richardson's energy cascade description of several models, which take into account small-scale intermittency have been developed [1-4]. In our considerations we propose a generalization of the so-called p-model for the case when turbulence is not space filling and the energy transfer rate depends on scale. Namely, in general we assume two different scaling parameters for sizes of eddies and a probability measure parameter describing portion of energy transferred to smaller eddies. For analysis we use multifractal fromalism, which permits for an intuitive understanding of multiplicative processes [5, 6]. In particular, we analyse scaling of the velocity structure functions [7] and the energy dissipation rate [8]. Using relation of scaling exponents and dissipation rate with the multifractal formalism we obtain generalized dimensions and singularity spectra. We compare the results of the proposed model with the experimental data of the solar wind plasma parameters measured in situ by Helios 2 spacecraft in the inner heliosphere. This shows mulitfractal behaviour of the data set under study.

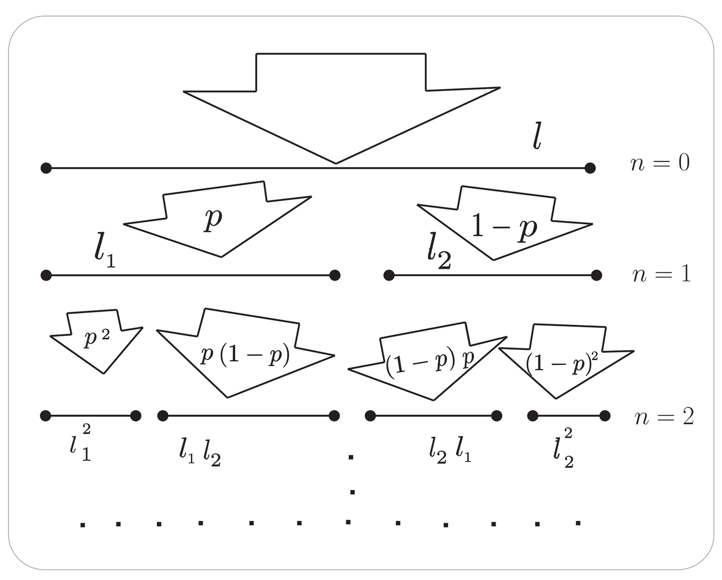
1 Theoretical Model

1.1 Generalized P-model

At each stage of construction for the proposed model we have two scaling parameters (l_1, l_2) and two different weights p and $1-p$. In order to obtain generalized dimensions and singularity spectrum for this example of multifractals we use the partition function formalism

$$\Gamma_n(l_1, l_2, p, 1-p) = \left(\frac{p^q}{l_1^{\tau(q)}} + \frac{(1-p)^q}{l_2^{\tau(q)}}\right)^n = 1 \qquad (1)$$

where n is the level of construction.



$$D_q = \frac{\tau(q)}{(q-1)}$$

$$\alpha(q) = \frac{d}{dq}[\tau(q)]$$

$$f(\alpha) = q\frac{d}{dq}[\tau(q)] - \tau(q)$$

1.2 Comparison with the p-model

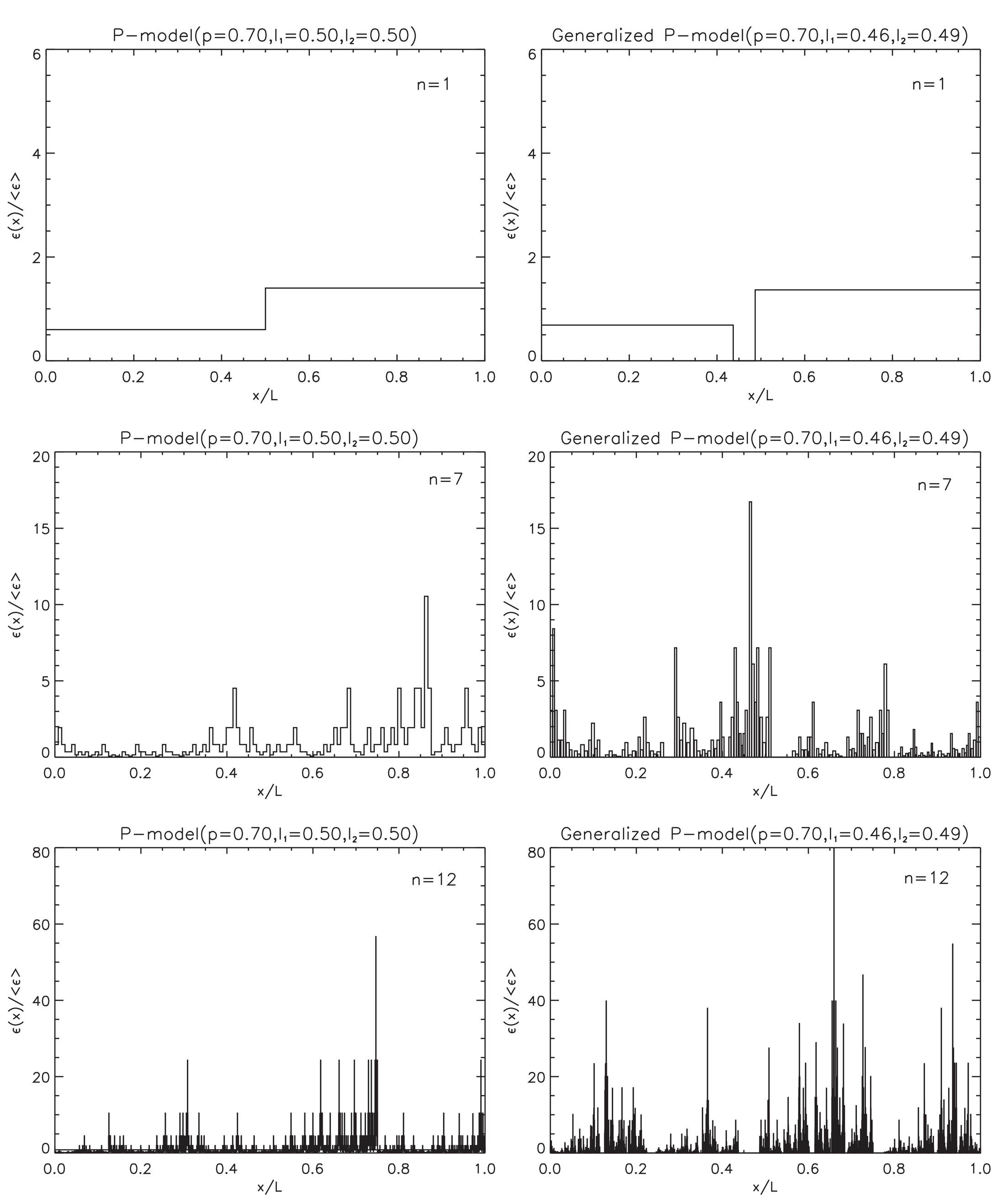


Fig.1: The multifractal measure $\varepsilon/\langle\varepsilon\rangle$ on the unit interval for p-model (left figure) and the generalized p-model (right figure). Intermittent pulses are stronger for the model with two different scaling parameters.

2 Data

	HELIOS 2 1976	
	~ 0.3 AU	~ 0.97 AU
Slow Solar Wind	099:00:00:29 - 099:23:34:27	026:00:00:33 - 026:23:58:45
	100:00:09:57 - 100:23:59:41	027:00:00:05 - 027:23:59:57
ddd:hh:mm:ss	101:00:00:21 - 101:23:59:47	028:00:00:39 - 028:23:59:59
	102:00:00:27 - 102:23:59:31	029:00:00:41 - 029:23:59:23
Fast Solar Wind	105:00:00:31 - 105:23:59:25	021:00:00:09 - 021:23:59:41
	106:00:00:45 - 106:23:59:35	022:00:00:17 - 022:23:59:51
ddd:hh:mm:ss	107:00:00:17 - 107:23:59:49	023:00:00:31 - 023:23:59:35
	108:00:00:29 - 108:23:59:55	024:00:00:07 - 024:23:59:43

3 Methods of Analysis

3.1 Structure functions scaling

$$S_u^q(l) = \langle |u(x+l) - u(l)|^q \rangle \sim l^{\xi(q)} \qquad (\eta << l << L) \qquad (2)$$

$S_u^q(l)$ - qth order structure function, $q > 0$
$u(x)$ - velocity component paraller to the l
$\xi(q)$ - scaling exponent in the inertial range

3.2 Multifractal dissipation

$$\varepsilon_l \sim \frac{S_u^3(l)}{l} \qquad \mu_i = \frac{\varepsilon_l}{\langle \epsilon_L \rangle} \qquad (3)$$

$$\sum_i \mu_i^q \sim l^{(q-1)D_q} \sim l^{d(q-1)+\xi(3q)-q\xi(3)} \qquad (4)$$

ε_l - energy dissipation rate
μ_i - probability measure
d - dimension of the space

$$D_q = d + \frac{\xi(3q) - q\xi(3)}{q-1} \qquad (5)$$

4 Results

4.1 Non-uniform energy distribution

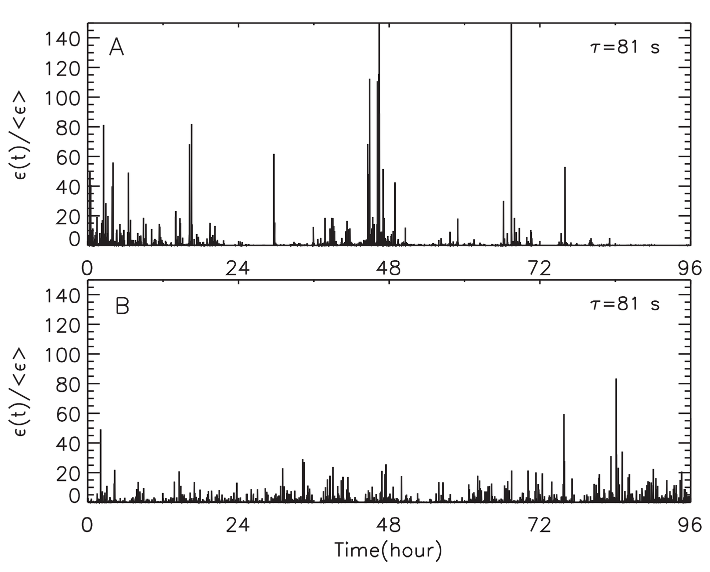


Fig.2: The intermittent nature of the dissipation rate in the slow (Fig. 2A, days 99-102) and fast (Fig. 2B, days 26-29) solar wind at 0.3 AU from the Sun. Note that the slow solar wind is more intermittent.

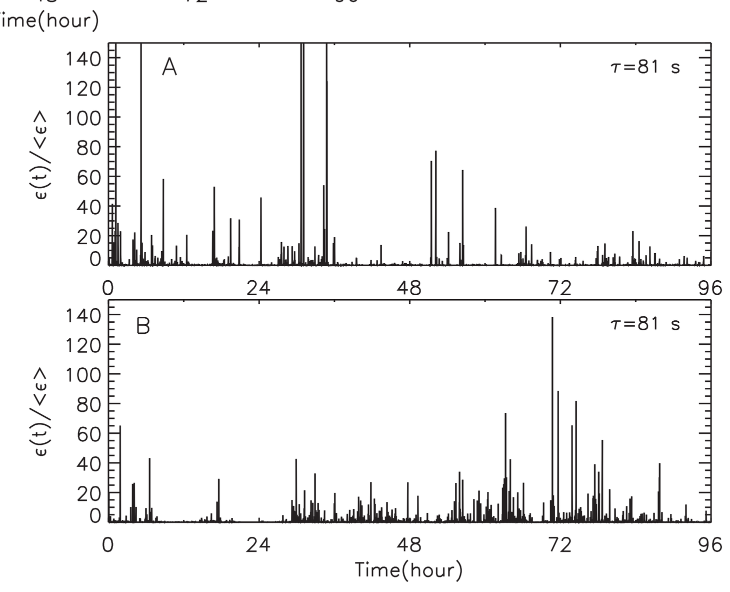


Fig.3: The intermittent nature of the dissipation rate in the slow (Fig. 3A, days 105-108) and fast (Fig. 3B, days 21-24) solar wind at distance of 0.97 AU. Note that the pulses are larger for slow solar wind than for the fast solar wind.

4.2 $\xi(q)$ as nonlinear function of q

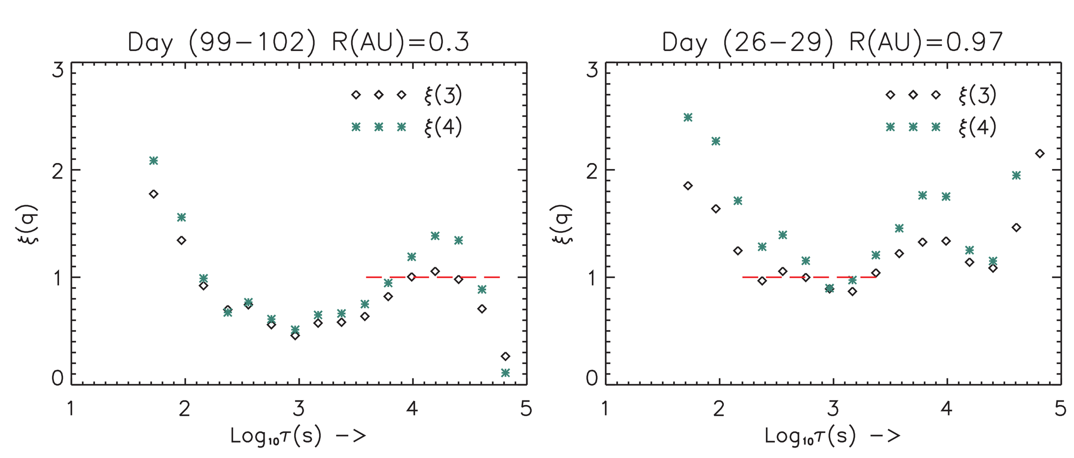


Fig.4: Measurements of $\xi(3)$ and $\xi(4)$ as a reliable indicators of the presence of an inertial range (Carbone, 1994).

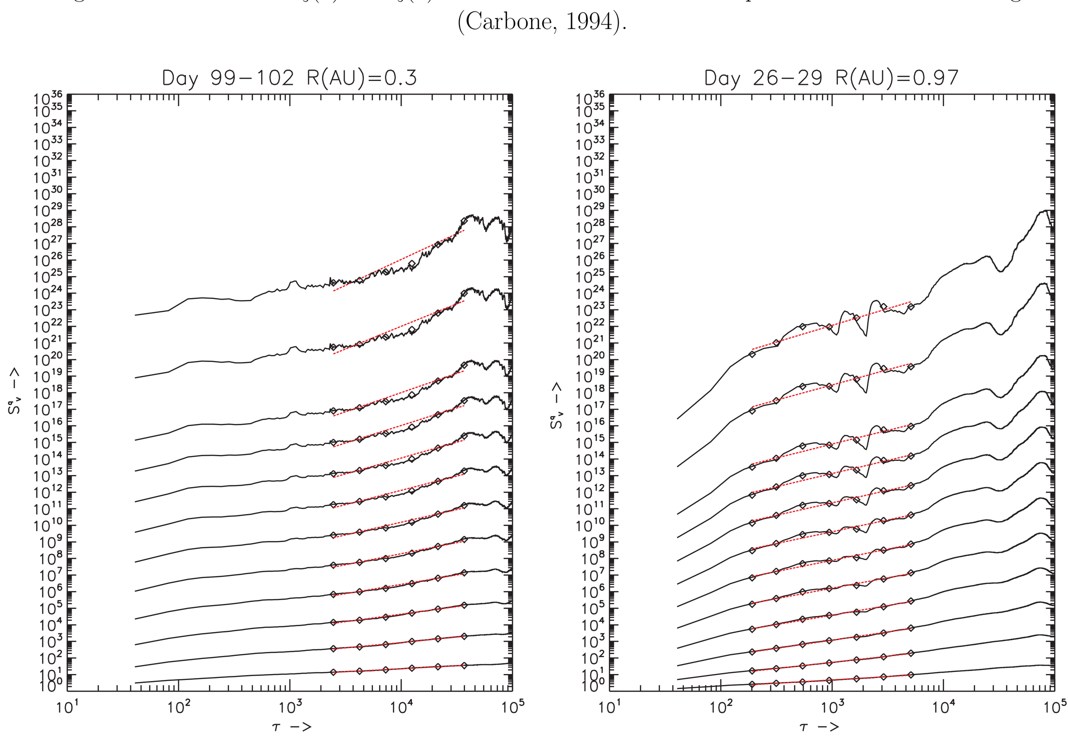


Fig.5: Structure functions of the radial velocity component for the slow solar wind. Dashed lines indicate the inertial range.

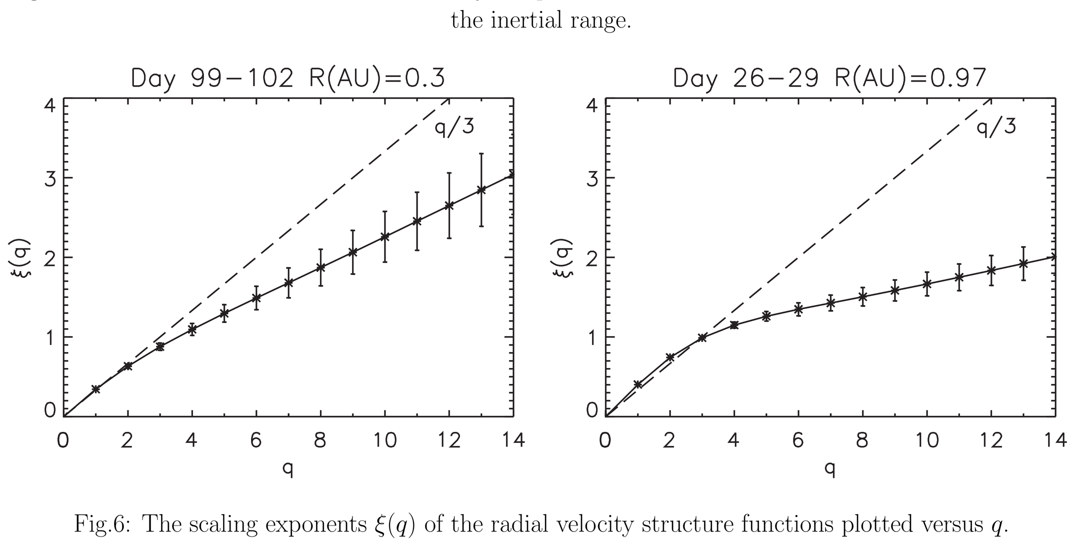


Fig.6: The scaling exponents $\xi(q)$ of the radial velocity structure functions plotted versus q.

4.3 The D_q as a function of q

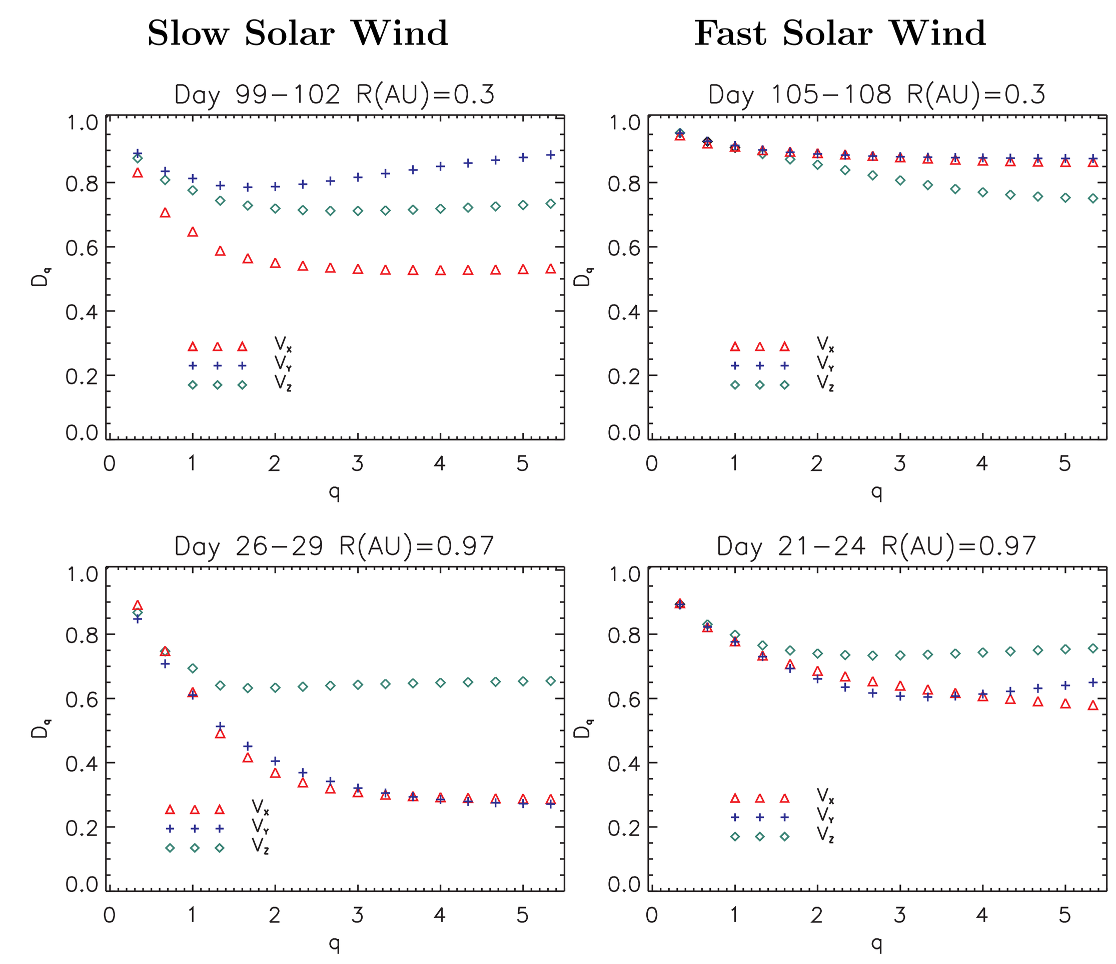


Fig.7: The generalized dimensions D_q as a function of q. The values of D_q are calculated for the velocity components of the slow (left figure) and fast (right figure) solar wind for two heliocentric distances: 0.3 AU and 0.97 AU.

4.4 Comparison with the generalized p-model

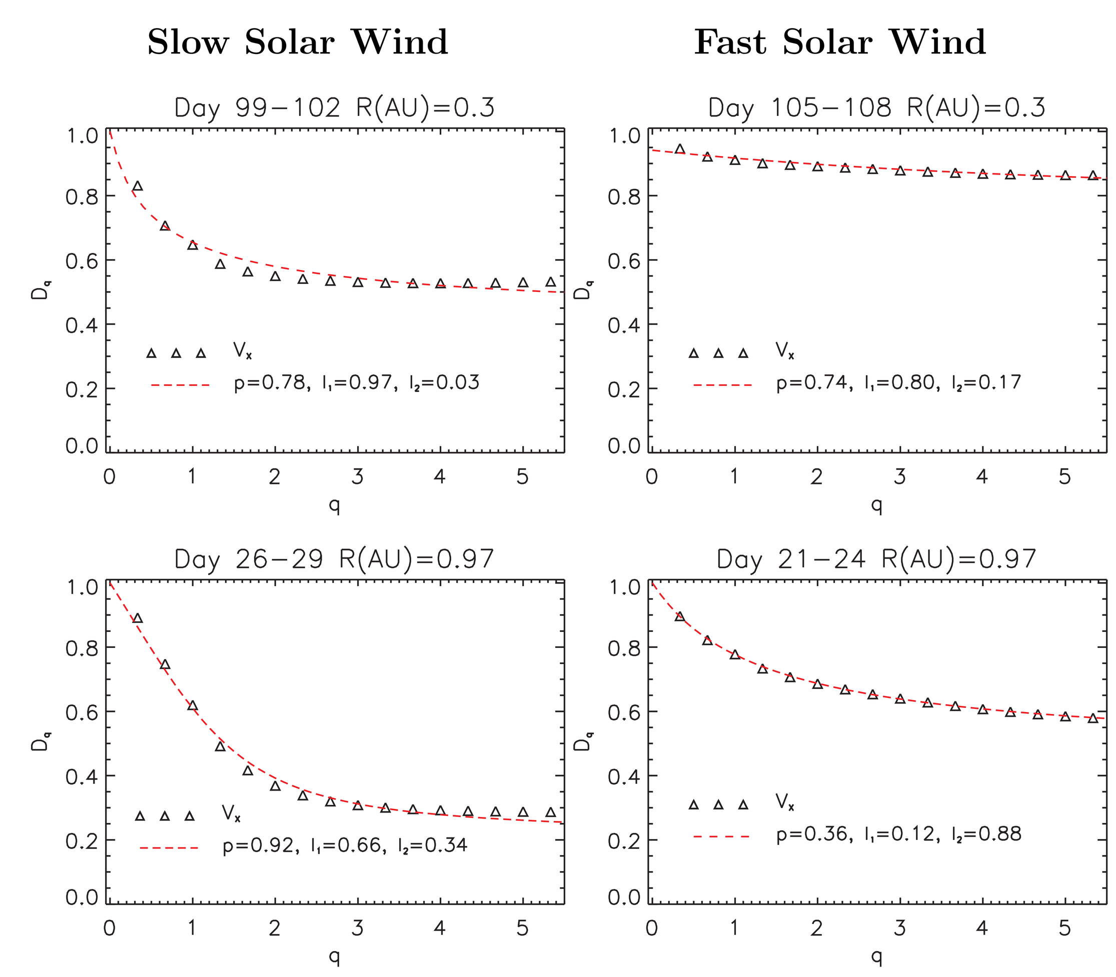


Fig.8: The generalized dimensions D_q as a function of q. The values of D_q are calculated numerically for the generalized p-model (dashed) and fitted to the solar wind (triangles).

5 Conclusions

- **We have studied departure from Kolmogorov scaling indicating multifractal (intermittent) behaviour of the solar wind in the inner heliosphere.**
- **Analysis shows that slow solar wind velocity fluctuations are more intermittent and more anistoropic than for the fast solar wind.**
- **As the heliocentric distrance increases the solar wind becomes more multifractal.**
- **Generalized dimensions for solar wind are consistent with the generalized p-model with different scaling parameters for sizes of eddies.**
- **We propose the generalized p-model set for intermittent dissipation energy cascade in the solar wind turbulence.**

References

[1] Kolmogorov, A.N., *A refinement of previous hypothesis concerning the local structure of turbulence in viscous incompressible fluid at high Reynolds number*, J. Fluid Mech., **13**, 82 (1962).

[2] Frisch, U., Sulem, P-L., Nelkin, M., *A simple dynamical model of inetrmittent fully developed turbulence*, J. Fluid Mech., **87**, 719 (1978).

[3] Benzi, R., Paladin, G., Vulpiani, A., Parisi, G., *On the multifractal nature of fully developed turbulence and chaotic systems*, J. Phys. A, **17**, 3521 (1984).

[4] Meneveau,C., Sreenivasan, K.R., *Simple multifractal cascade model for fully developed turbulence*, Phys. Rev. Lett. **59**, 1424 (1987).

[5] Meneveau, C., Sreenivasan, K.R., *The multifractal nature of turbulent energy dissipation*, J. Fluid Mech., **224**, 429 (1991).

[6] W.M. Macek, *Modeling multifractality of the solar wind*, Space Sci. Rev. **122**, 329 (2006).

[7] Anselmet, F., Gagne, Y., Hopfinger, J., Antonia, R.A., *High-order velocity structure functions in turbulent shear flows*, J. Fluid Mech., **140**, 63 (1984).

[8] Marsch, E., Tu, C.-Y., Rosenbauer, H., *Multifractal scaling of the kinetic energy flux in solar wind turbulence*, Ann. Geophys., **14**, 259 (1996).

Electronic form: http://www.cbk.waw.pl/~wawrzasz/ania