Multiscale Multifractal Solar Wind Turbulence

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ABSTRACT

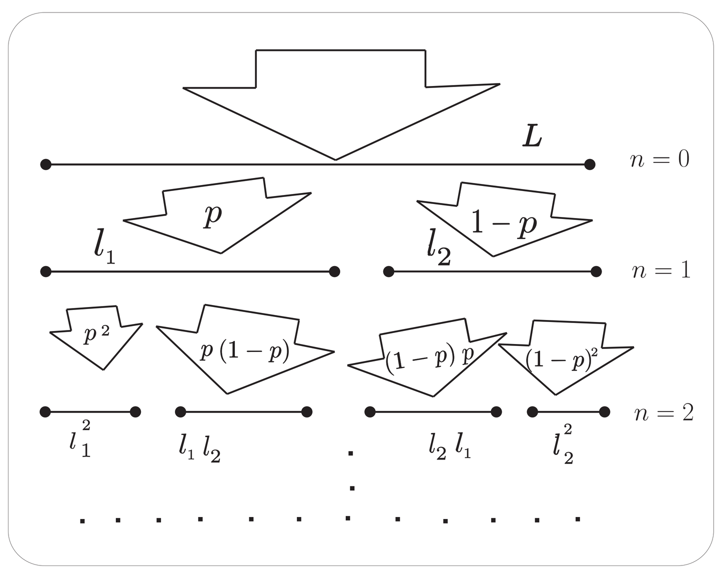
The concept of multiscale multifractality is of great importance for the heliophysics because it allows us to look at intermittent turbulence in the solar wind. Starting from Richardson's (1922) scenario of turbulence, many authors still attempts to recover the observed scaling exponents, using some simple and more advanced fractal and multifractal phenomenological models of turbulence describing distribution of the energy flux between cascading eddies at various scales [1, 2]. In particular, the multifractal spectrum has been investigated using Voyager (magnetic field) data in the outer heliosphere [3] and using Helios (plasma) data in the inner heliosphere [4]. We have also analysed the multifractal spectrum directly on the solar wind attractor and have shown that it is consistent with that for the multifractal measure of a two-scale weighted Cantor set [5]. Further, to quantify scaling of solar wind turbulence, we also consider this generalized Cantor set with two different scales describing nonuniform distribution of the kinetic energy flux between cascading eddies of various sizes. We investigate the multifractal spectra depending on two rescaling parameters and one probability measure parameter [6]. We demonstrate that the universal shape of the multifractal spectrum resulting from the multiscale nature of the cascade is often rather asymmetric. Moreover, we observe the evolution of multifractal scaling of the solar wind in the inner and outer heliosphere [7]. It is worth noting that for the model with two different scaling parameters a better agreement with the solar wind data is obtained, especially for the negative index of the generalized dimensions. Hence we hope that this somewhat more general model could be a useful tool for analysis of the intermittent turbulence in space plasmas also during Cross-Scale Mission.

1 Theoretical Model

1.1 Generalized P model

At each stage of construction of the weighted two-scale Cantor set we have two scaling parameters (l_1, l_2) and two different weights p and $1-p$. To obtain generalized dimensions and singularity spectra for this multifractal set we use the partition function at the nth level of construction

$$\Gamma_n^q(l_1, l_2, p) = \left(\frac{p^q}{l_1^{\tau(q)}} + \frac{(1-p)^q}{l_2^{\tau(q)}}\right)^n = 1 \tag{1}$$



$$D_q = \frac{\tau(q)}{(q-1)}$$

$$\alpha(q) = \frac{\mathrm{d}\,\tau(q)}{\mathrm{d}q}$$

$$f(\alpha) = q\,\alpha(q) - \tau(q)$$

Fig. 1: Two-scale weighted Cantor set model for asymmetric solar wind turbulence [5].

1.2 Comparison With the P model

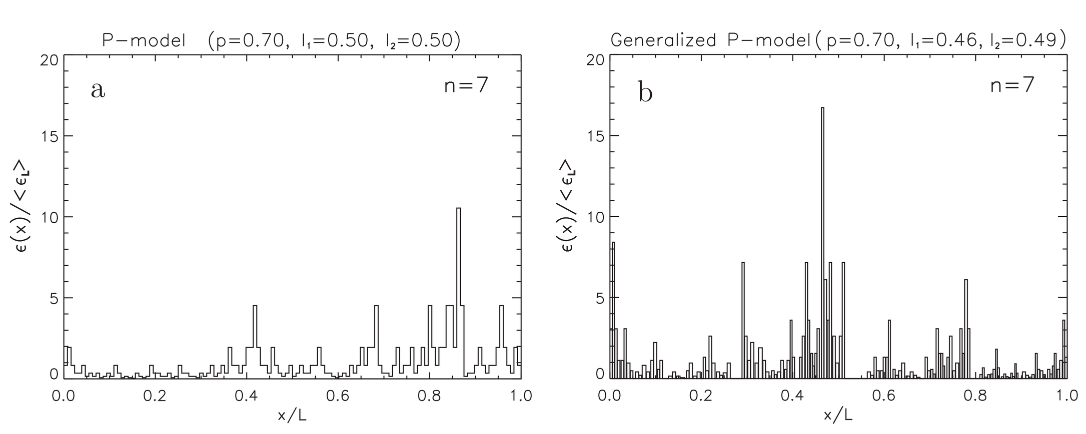


Fig. 2: The multifractal measure $\mu = \varepsilon / \langle \varepsilon_L \rangle$ on the unit interval for (a) the usual one-scale p-model and (b) the generalized two-scale cascade model. Intermittent pulses are stronger for the model with two different scaling parameters [6].

2 Methods of Data Analysis

2.1 Energy Transfer Rate

$$\varepsilon(l) \sim \frac{|v_x(x+l) - v_x(x)|^3}{l} \qquad p_i(l) = \frac{\varepsilon_i(l)}{\sum_{i=1}^{N} \varepsilon_i(l)} \tag{2}$$

2.2 The Generalized Dimensions

$$\bar{\mu}(q,l) \equiv \sqrt[q-1]{\langle (p_i)^{q-1} \rangle_{\mathrm{av}}} \qquad \bar{\mu}(q,l) \propto l^{D_q} \tag{3}$$

$$D_q = \lim_{l \to 0} \frac{\log_{10} \bar{\mu}(q,l)}{\log_{10} l} \tag{4}$$

2.3 The Singularity Spectrum

averages $\langle \ldots \rangle$ with respect to generalized pseudoprobability measures

$$\mu_i(q,l) \equiv \frac{p_i^q(l)}{\sum_{i=1}^{N} p_i^q(l)} \tag{5}$$

$$\alpha(q) = \lim_{l \to 0} \frac{\langle \log_{10} p_i(l) \rangle}{\log_{10}(l)} \qquad f(q) = \lim_{l \to 0} \frac{\langle \log_{10} \mu_i(q,l) \rangle}{\log_{10}(l)} \tag{6}$$

3 Solar Wind Data

2.5 AU (1978)
25 AU (1987-1988)
50 AU (1996-1997)

4 Results

4.1 Non-uniform Energy Distribution

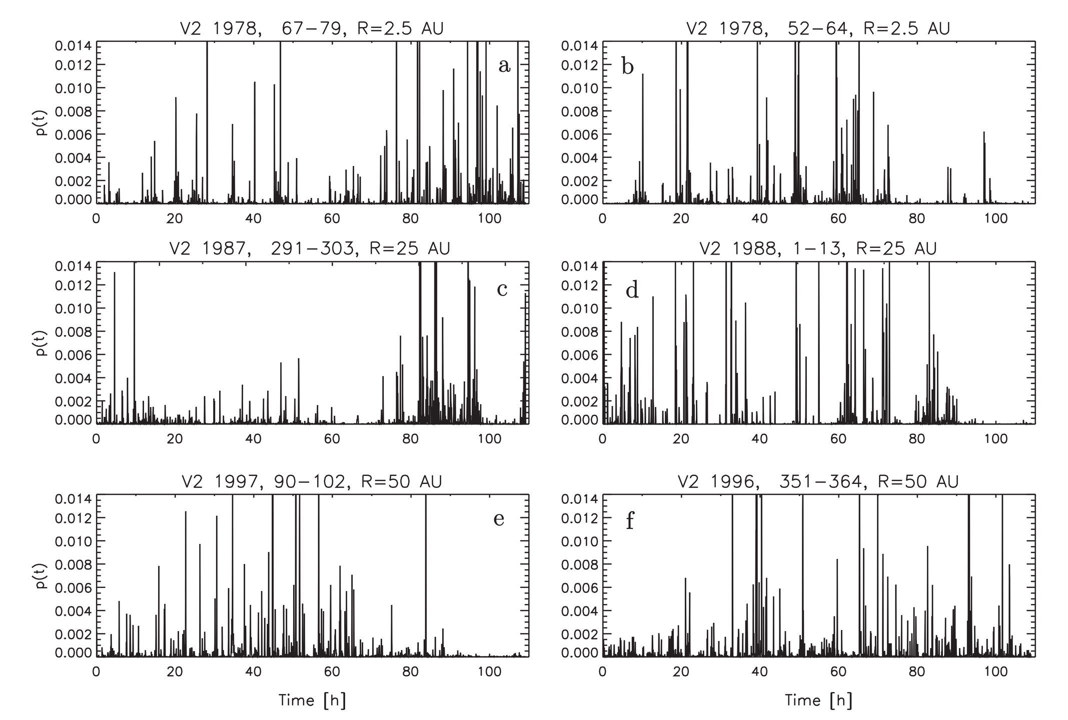


Fig. 3: The normalized transfer rate of the energy flux $p(t) = \varepsilon_i(t) \,/ \sum \varepsilon_i(t)$ obtained using data of the v_x velocity components measured by Voyager 2 during solar minimum (1978, 1987–1988, 1996–1997) at 2.5, 25, and 50 AU for the slow (a), (c), and (e) and fast (b), (d), and (f) solar wind, correspondingly [7].

4.2 Turbulence Scaling

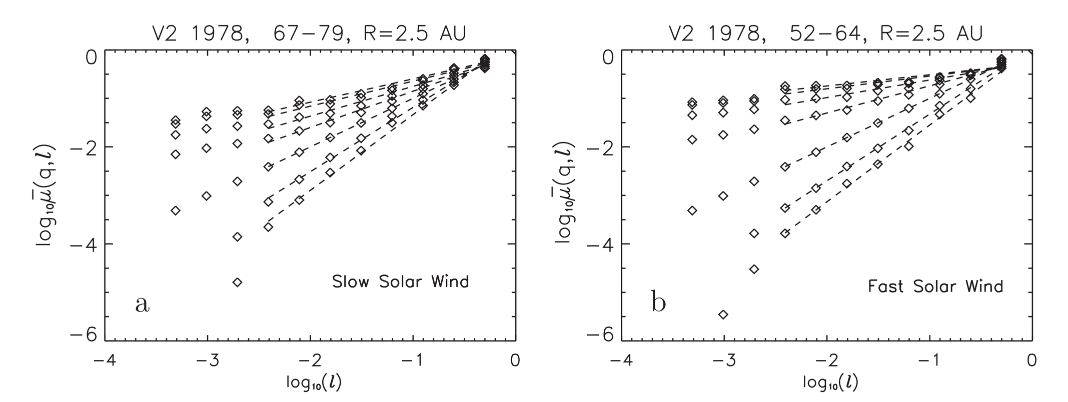


Fig. 4: Plots of the generalized average probability of cascading eddies $\log_{10} \bar{\mu}(q,l)$ versus $\log_{10} l$ for the following values of q: 6, 4, 2, 1, 0, -1, -2, Ref. [7].

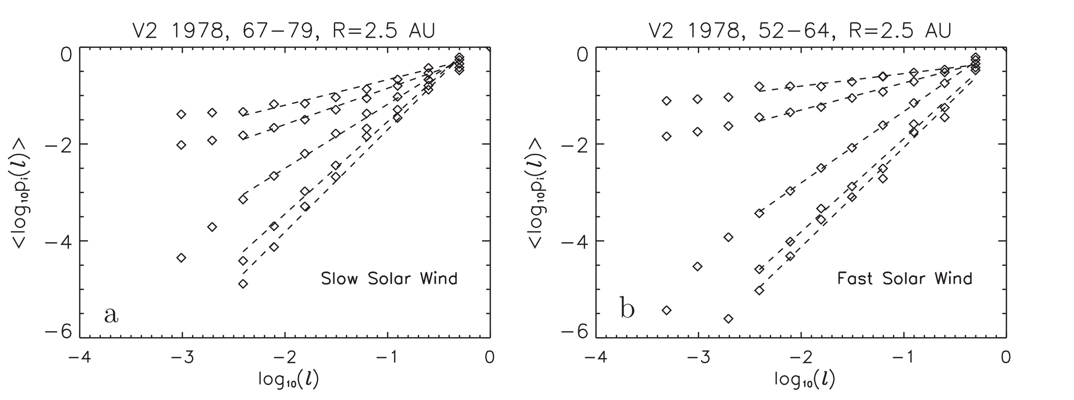


Fig. 5: Plots of the generalized average logarithmic probability of cascading eddies $\langle \log_{10} p_i(l) \rangle$ versus $\log_{10} l$ for the following values of q: 4, 2, 1, 0, -1, -2, Ref. [7].

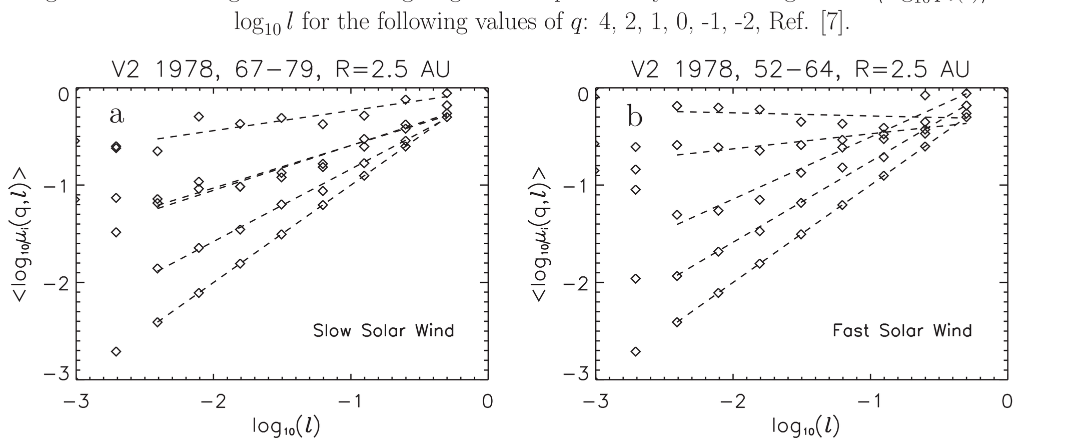


Fig. 6: Plots of the generalized average pseudoprobability of cascading eddies $\langle \log_{10} \mu_i(q,l) \rangle$ versus $\log_{10} l$ for the following values of q: 4, 2, 1, 0, -1, -2, Ref. [7].

4.3 The Generalized Dimensions

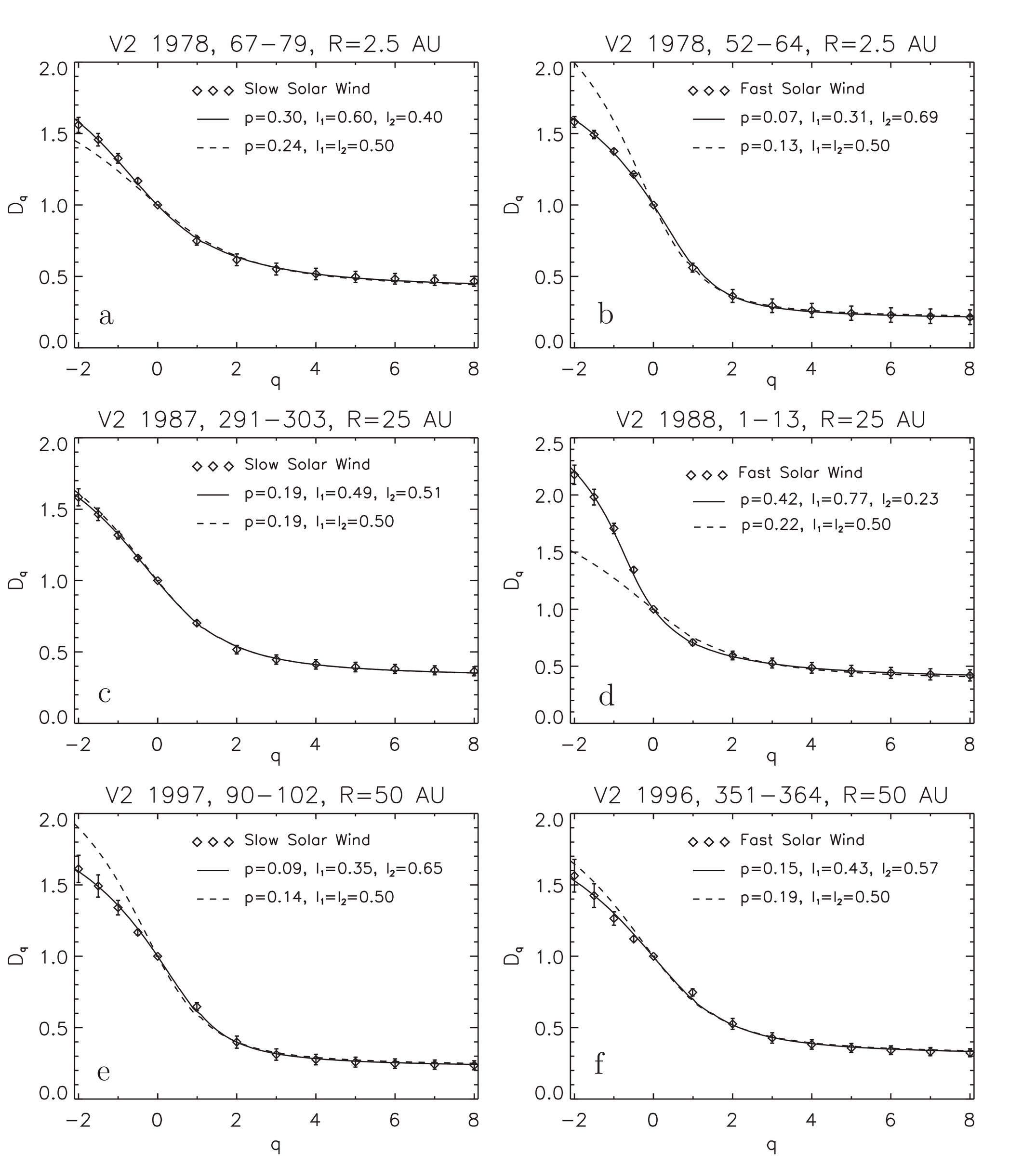


Fig.7: The generalized dimensions D_q for the one-scale p-model (dashed) and the generalized two-scale (continuous lines) model [7].

4.4 The Singularity Spectrum

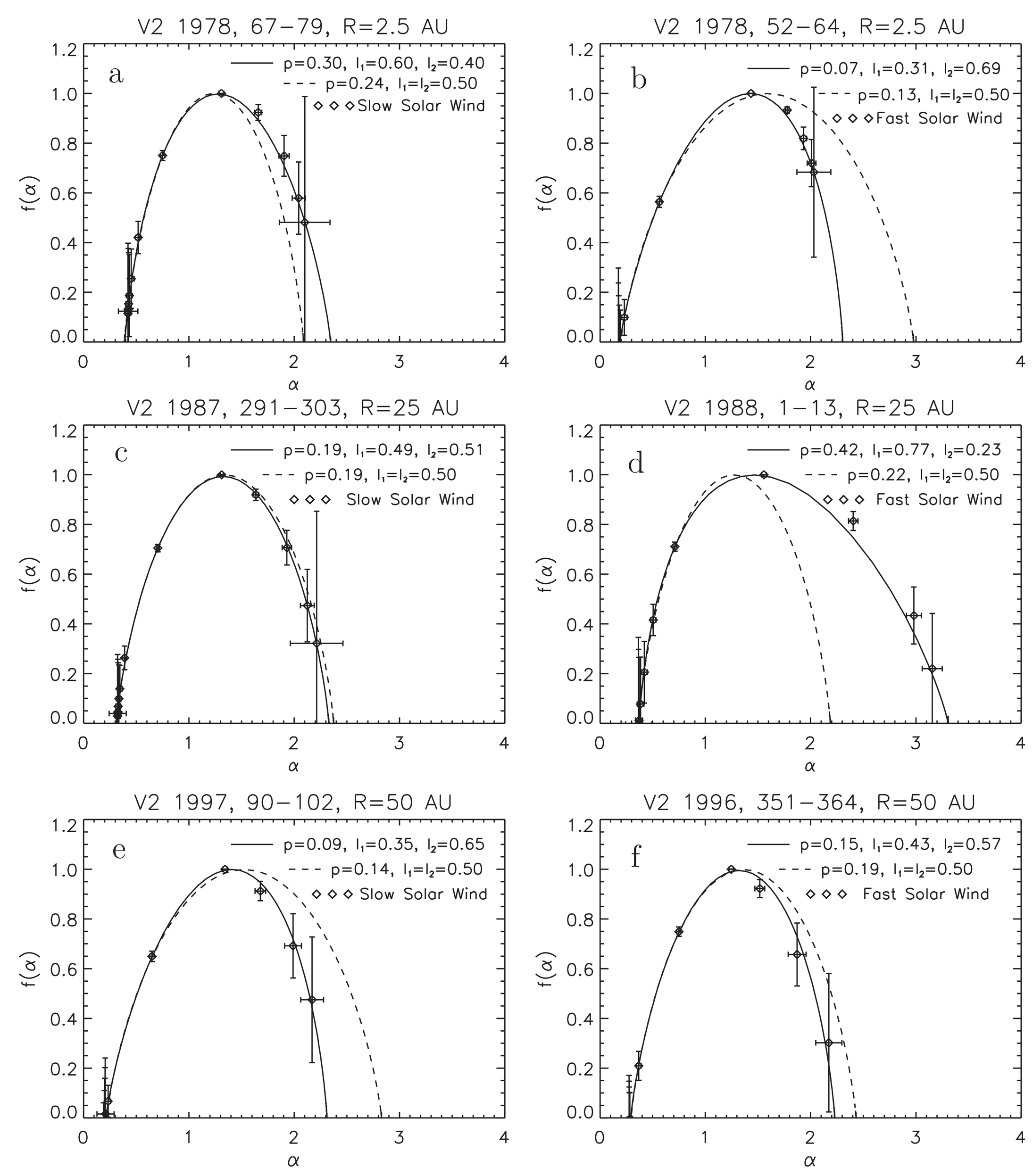


Fig.8: The singularity spectrum $f(\alpha)$ as a function of α calculated for the one-scale p-model (dashed lines) and the generalized two-scale (continuous lines) models with parameters fitted to the multifractal measure $\mu(q,l)$ using data measured by Voyager 2 during solar minimum (1978, 1987–1988, 1996–1997) at 2.5, 25, and 50 AU (diamonds) for the slow (a, c, and e) and fast (b, d, and f) solar wind, correspondingly [7].

Degree of multifractality:

$$\Delta \equiv \alpha_{\max} - \alpha_{\min} = D_{-\infty} - D_{\infty} = \left| \frac{\log(1-p)}{\log l_2} - \frac{\log(p)}{\log l_1} \right|$$

Measure of asymmetry: $A \equiv \frac{\alpha_0 - \alpha_{\min}}{\alpha_{\max} - \alpha_0}$ $f(\alpha_0) = 1$

Table 1. Degree of Multifractality Δ and Asymmetry A for Solar Wind Data in the Outer Heliosphere During Solar Minimum

Heliospheric Distance (Year)	Slow Solar Wind	Fast Solar Wind
2.5 AU (1978)	$\Delta = 1.95, A = 0.91$	$\Delta = 2.12, A = 1.54$
25 AU (1987-1988)	$\Delta = 2.02, A = 0.98$	$\Delta = 2.93, A = 0.66$
50 AU (1996-1997)	$\Delta = 2.10, A = 1.14$	$\Delta = 1.94, A = 0.95$

5 Conclusions

- **We have studied the inhomogeneous rate of the transfer of the energy flux indicating multifractal and intermittent behavior of solar wind turbulence in the heliosphere.**
- **We demonstrate that the universal shape of the multifractal spectrum resulting from the multiscale nature of the cascade is often rather asymmetric; the fast wind during solar minimum exhibits strong asymmetric scaling.**
- **The degree of multifractality and degree of asymmetry are correlated with the heliospheric distance and we observe the evolution of multifractal scaling of the solar wind in the inner and outer heliosphere.**
- **The degree of multifractality for the solar wind in the outer heliosphere is somewhat greater for fast solar wind velocity fluctuations than that for the slow solar wind.**
- **The generalized dimensions for solar wind are consistent with the generalized p model for both positive and negative q, but rather with different scaling parameters for sizes of eddies, while the usual p-model can only reproduce the spectrum for $q \geq 0$.**
- **We propose the generalized Cantor set p model for analysis of turbulence in various environments also during Cross-Scale Mission.**

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