Multifractal Intermittent Turbulence in Space Plasmas

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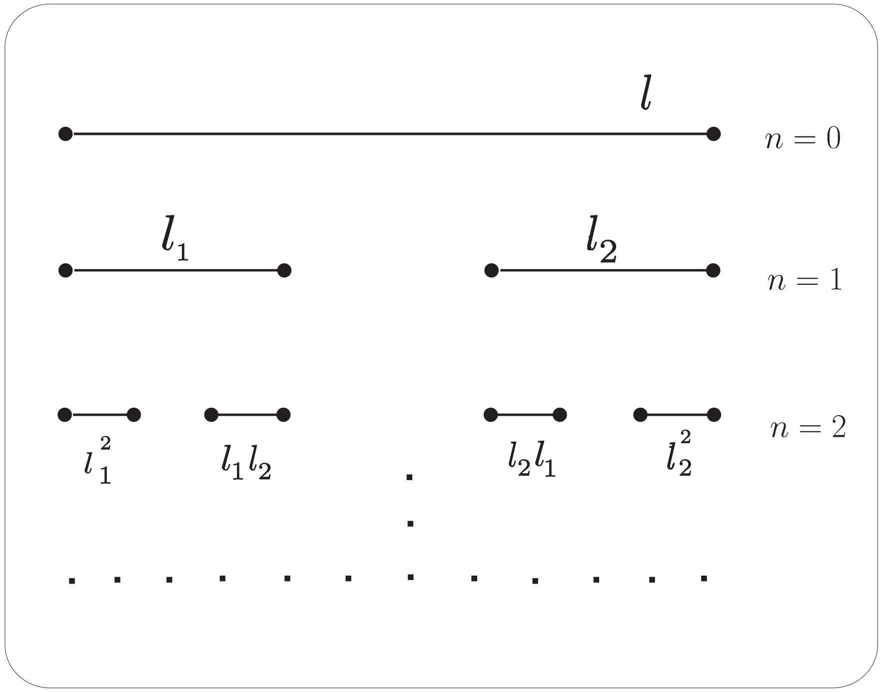
Prologue

A **fractal** is a rough or fragmented geometrical object that can be subdivided in parts, each of which is (at least approximately) a reduced-size copy of the whole. Fractals are generally *self-similar* and independent of scale (fractal dimension).

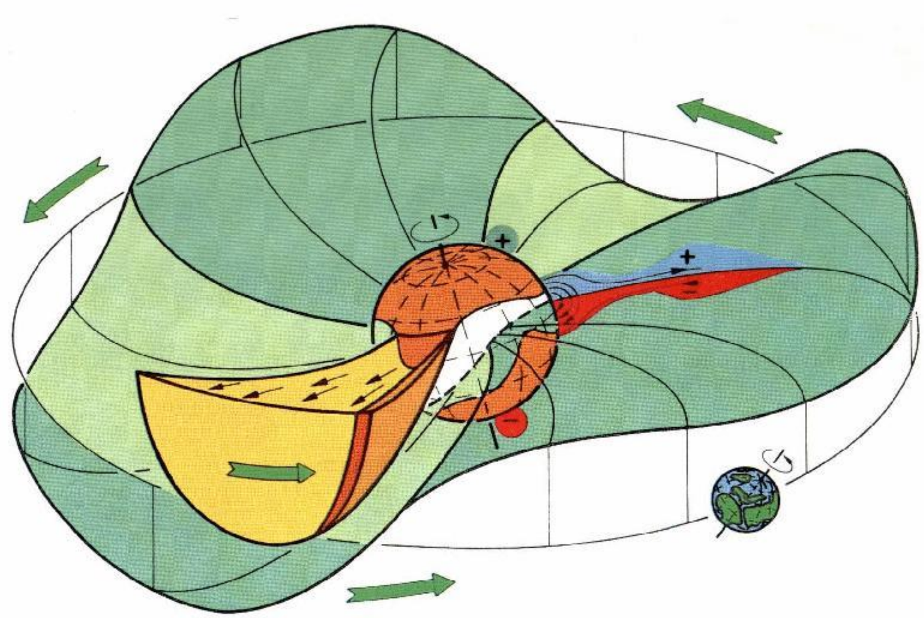
A **multifractal** is a set of intertwined fractals. Self-similarity of multifractals is point dependent (spectrum of dimensions). A deviation from a strict self-similarity is also called **intermittency**.



Two-scale **Cantor** set.

Solar Wind

In the inner heliosphere the solar wind streams are of two forms called the slow (≈ 400 km s^{-1}) and fast (≈ 700 km s^{-1}). The solar wind most likely originates from nonlinear processes in the solar corona. The fast wind associated with coronal holes is relatively uniform and stable, while the slow wind is more turbulent and quite variable.



A schematic model of the solar wind ”ballerina”: the Sun’s two hemispheres are separated by a neutral layer of a form reminiscent of a ”ballerina’s skirt”, taken from (Schwenn and Rosenbauer, 1984).

Importance of Multifractality

The concept of multiscale multifractality is of great importance for space plasmas because it allows us to look at intermittent turbulence in the solar wind (e.g., Marsch and Tu, 1997; Bruno *et al.*, 2001). Starting from Richardson's (1922) scenario of turbulence, many authors still attempts to recover the observed scaling exponents, using some simple and more advanced fractal and multifractal phenomenological models of turbulence describing distribution of the energy flux between cascading eddies at various scales.

In particular, the multifractal spectrum has been investigated using Voyager (magnetic field fluctuations) data in the outer heliosphere (e.g., Burlaga, 1991, 2001) and using Helios (plasma) data in the inner heliosphere (e.g., Marsch *et al.*, 1996; Macek and Szczepaniak, 2008). We have also analysed the multifractal spectrum directly on the solar wind attractor and have shown that it is consistent with that for the multifractal measure of a two-scale weighted Cantor set (Macek, 2007).

The multifractal scaling has also been tested using Ulysses observations (Horbury *et al.*, 1997) and with Advanced Composition Explorer (ACE) and WIND data (e.g., Hnat *et al.*, 2003; Kiyani *et al.*, 2007; Szczepaniak and Macek, 2008).

Recently, to quantify scaling of solar wind turbulence, we also consider the generalized Cantor set with two different scales describing nonuniform distribution of the kinetic energy flux between cascading eddies of various sizes. We investigate the multifractal spectra depending on two rescaling parameters and one probability measure parameter (Macek and Szczepaniak, 2008).

We demonstrate that the universal shape of the multifractal spectrum resulting from the multiscale nature of the cascade is often rather asymmetric. Moreover, we observe the evolution of multifractal scaling of the solar wind in the inner and outer heliosphere (Macek and Wawrzaszek, 2009).

It is worth noting that for the model with two different scaling parameters a better agreement with the solar wind data is obtained, especially for the negative index of the generalized dimensions. Hence we hope that this somewhat more general model could be a useful tool for analysis of the intermittent turbulence in space plasmas.

- W. M. Macek, Multifractality and intermittency in the solar wind, Nonlin. Processes Geophys. **14**, 695–700 (2007).
- W. M. Macek and A. Szczepaniak, Generalized two-scale weighted Cantor set model for solar wind turbulence, Geophys. Res. Lett., **35**, L02108 (2008).
- A. Szczepaniak and W. M. Macek (2008), Asymmetric multifractal model for solar wind intermittent turbulence, Nonlin. Processes Geophys., **15**, 615–620 (2008).
- W. M. Macek and A. Wawrzaszek (2009), Evolution of asymmetric multifractal scaling of solar wind turbulence in the outer heliosphere, J. Geophys. Res., A013795, in press.

Turbulence Cascade

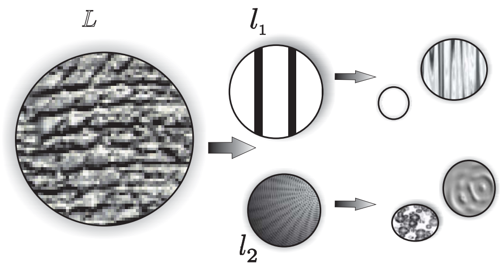


Fig. 1. Schematics of binomial multiplicative processes of cascading eddies. A large eddy of size L is divided into two smaller *not necessarily equal* pieces of size l_1 and l_2. Both pieces may have different probability measures, as indicated by the different shading. At the n-th stage we have 2^n various eddies. The processes continue until the Kolmogorov scale is reached (Meneveau and Sreenivasan, 1991; Macek *et al.*, 2009).

Methods of Data Analysis

Energy Transfer Rate and Probability Measures

In the inertial range ($\eta \ll l \ll L$)

$$\varepsilon_l \sim \frac{|u(x+l)-u(x)|^3}{l} \qquad \mu_i = \frac{\varepsilon_l}{\langle \varepsilon_L \rangle} \tag{1}$$

where $u(x)$ and $u(x+l)$ are velocity components parallel to the longitudinal direction separated from a position x by a distance l.

To each ith eddy of size l in turbulence cascade ($i = 1, \ldots, N = 2^n$) we associate a probability measure

$$p_i(l) = \frac{\varepsilon_i(l)}{\sum_{i=1}^{N} \varepsilon_i(l)} \tag{2}$$

This quantity can roughly be interpreted as a probability that the energy is transferred to an eddy of size $l = v_{\mathrm{sw}} t$.

As usual the time-lags can be interpreted as longitudinal separations, $x = v_{\mathrm{sw}} t$ (Taylor's hypothesis).

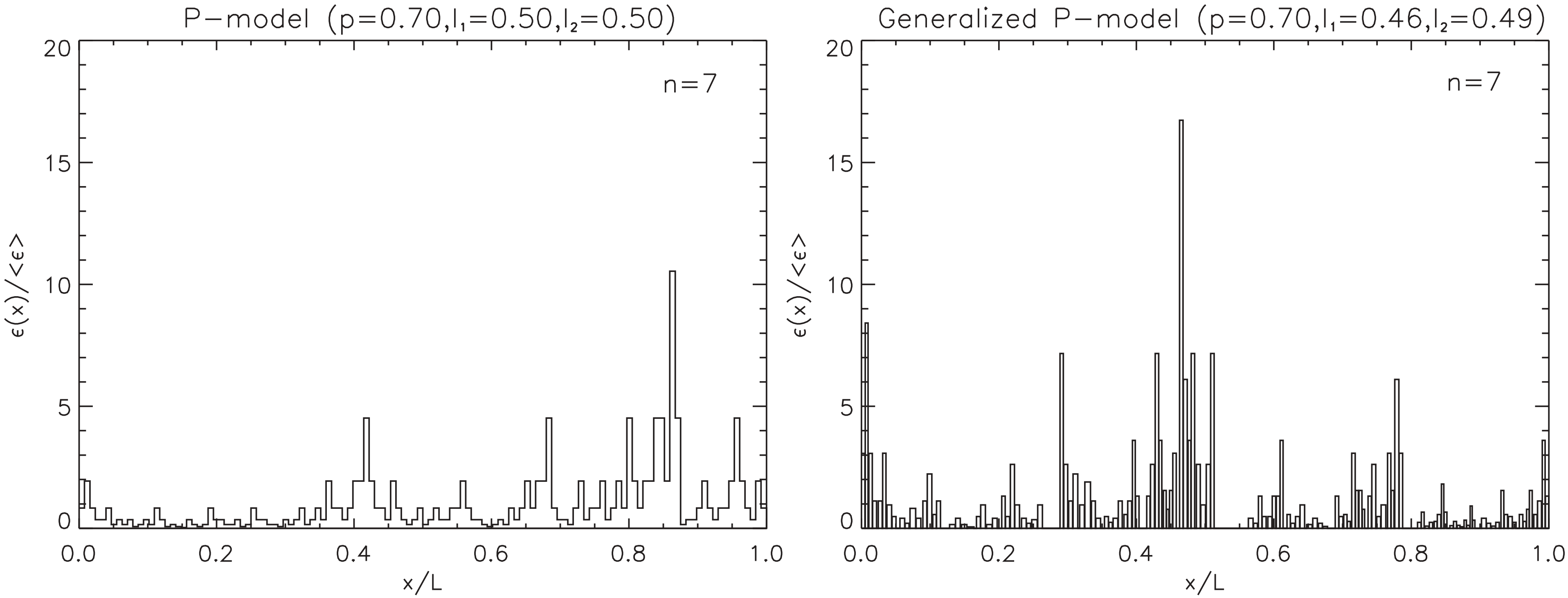


Fig. 1. The multifractal measure $\mu = \varepsilon / \langle \varepsilon_L \rangle$ on the unit interval for (a) the usual one-scale *p*-model (Meneveau and Sreenivasan, 1987) and (b) the generalized two-scale cascade model. Intermittent pulses are stronger for the model with two different scaling parameters (Macek and Szczepaniak, 2008).

Generalized Scaling Property

The generalized dimensions are important characteristics of *complex* dynamical systems; they quantify multifractality of a given system (Ott, 1993). In the case of turbulence cascade these generalized measures are related to inhomogeneity with which the energy is distributed between different eddies (Meneveau and Sreenivasan, 1991). In this way they provide information about dynamics of multiplicative process of cascading eddies. Here high positive values of $q > 1$ emphasize regions of intense energy transfer rate, while negative values of q accentuate low-transfer rate regions (cf. Chhabra *et al.* 1989).

Using ($\sum p_i{}^q \equiv \langle p_i{}^{q-1}\rangle_{\mathrm{av}}$) a generalized average probability measure of cascading eddies

$$\bar{\mu}(q,l) \equiv \sqrt[q-1]{\langle (p_i)^{q-1}\rangle_{\mathrm{av}}} \tag{3}$$

we can identify D_q as scaling of the measure with size l

$$\bar{\mu}(q,l) \propto l^{D_q} \tag{4}$$

Hence, the slopes of the logarithm of $\bar{\mu}(q,l)$ of Eq. (4) versus $\log l$ (normalized) provides

$$D_q = \lim_{l\to 0} \frac{\log \bar{\mu}(q,l)}{\log l} \tag{5}$$

Measures and Multifractality

Similarly, we define a one-parameter q family of (normalized) generalized pseudoprobability measures (Chhabra and Jensen, 1989; Chhabra *et al.*, 1989)

$$\mu_i(q,l) \equiv \frac{p_i^q(l)}{\sum_{i=1}^{N} p_i^q(l)} \tag{6}$$

Now, with an associated fractal dimension index $f_i(q,l) \equiv \log \mu_i(q,l)/\log l$ for a given q the multifractal singularity spectrum of dimensions is defined directly as the averages taken with respect to the measure $\mu(q,l)$ in Eq. (6) denoted from here on by $\langle \ldots \rangle$

$$f(q) \;\equiv\; \lim_{l\to 0} \sum_{i=1}^{N} \mu_i(q,l)\; f_i(q,l) = \lim_{l\to 0} \frac{\langle \log \mu_i(q,l) \rangle}{\log(l)} \tag{7}$$

and the corresponding average value of the singularity strength is given by (Chhabra and Jensen, 1987)

$$\alpha(q) \;\equiv\; \lim_{l\to 0} \sum_{i=1}^{N} \mu_i(q,l)\; \alpha_i(l) = \lim_{l\to 0} \frac{\langle \log p_i(l) \rangle}{\log(l)}. \tag{8}$$

Mutifractal Models for Turbulence

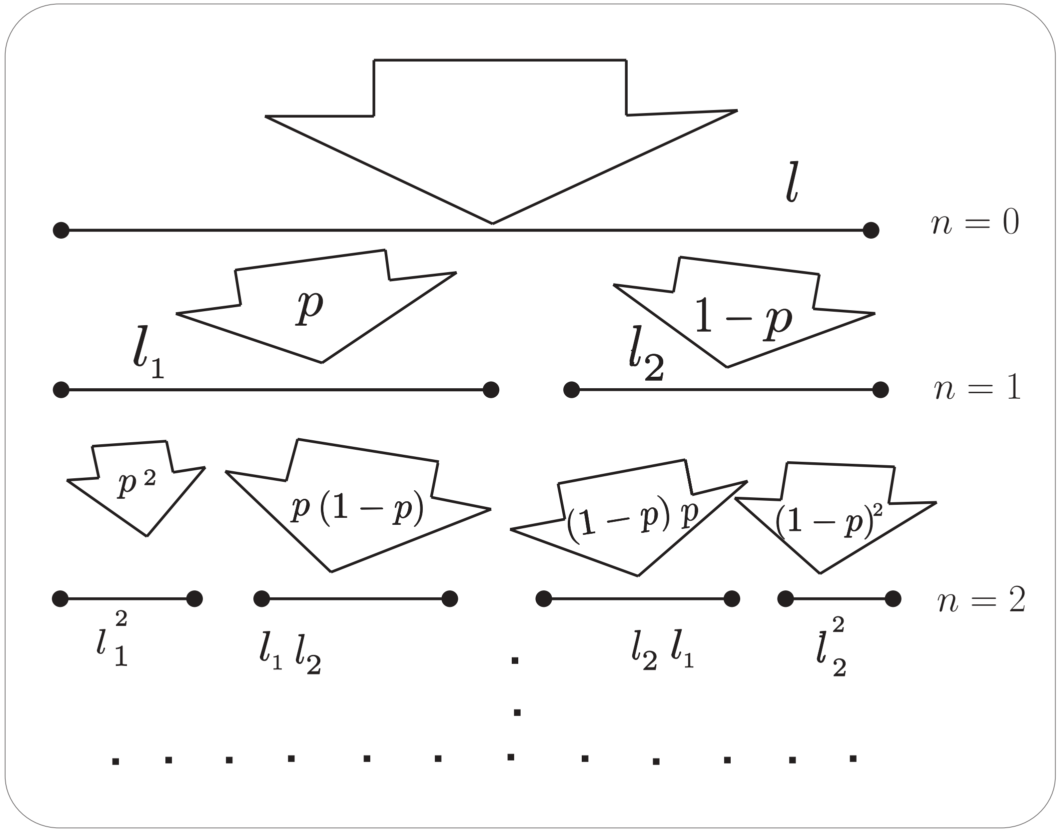


Fig. 1. Generalized two-scale Cantor set model for turbulence (Macek, 2007).

Solutions

For the generalized self-similar weighted Cantor set we have for $\tau(q) \equiv (q-1)D_q$ (Hentschel and Procaccia 1983; Halsey *et al.*, 1986)

Transcendental equation

$$\frac{p^q}{l_1^{\tau(q)}} + \frac{(1-p)^q}{l_2^{\tau(q)}} = 1 \tag{9}$$

Legendre transformation

$$\alpha(q) = \frac{\mathrm{d}\,\tau(q)}{\mathrm{d}q}, \tag{10}$$

$$f(\alpha) = q\alpha(q) - \tau(q). \tag{11}$$

Parameters:

- probability of providing energy to one eddy is p to the other eddy is $1-p$
- $l_1 + l_2 \leq 1$, two rescaling parameters for size of eddies (for space filling turbulence $l_1 + l_2 = 1$).

For $l_1 = l_2 = s$ and any q in Eq. (9) one has for the generalized dimensions

$$\tau(q) \equiv (q-1)D_q = \frac{\ln[p^q + (1-p)^q]}{\ln s}. \qquad (12)$$

Space filling turbulence ($s = 1/2$):
the multifractal cascade $p-$model for fully developed turbulence,
the generalized weighted Cantor set (Meneveau and Sreenivasan, 1987).
The usual middle one-third Cantor set (without any multifractality):
$p = 1/2$ and $s = 1/3$.

The values of parameter p are related to the usual models, which are based on the *p*-model of turbulence (e.g., Meneveau and Sreenivasan, 1987).

These values of p obtained are roughly consistent with the fitted value in the literature both for laboratory and the solar wind turbulence, which is in the range

$$0.1 \le p \le 0.3$$

(e.g., Burlaga, 1991; Carbone, 1993; Carbone and Bruno, 1996; Marsch *et al.*, 1996).

Degree of Multifractality and Asymmetry

The difference of the maximum and minimum dimension (the least dense and most dense points in the phase space) is given, e.g., by Macek (2006, 2007)

$$\Delta \equiv \alpha_{\max} - \alpha_{\min} = D_{-\infty} - D_{\infty} = \left| \frac{\log(1-p)}{\log l_2} - \frac{\log(p)}{\log l_1} \right|. \quad (13)$$

In the limit $p \to 0$ this difference rises to infinity (degree of multifractality).

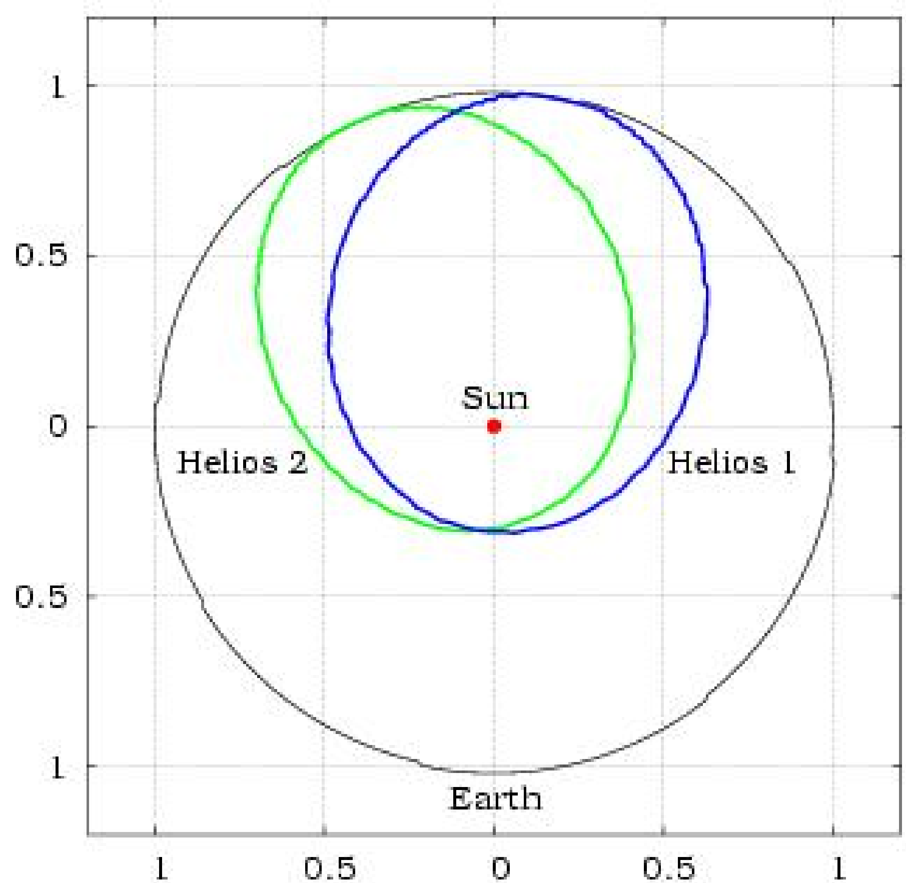
The degree of multifractality Δ is simply related to the deviation from a simple self-similarity. That is why Δ is also a measure of intermittency, which is in contrast to self-similarity (Frisch, 1995, chapter 8).

Using the value of the strength of singularity α_0 at which the singularity spectrum has its maximum $f(\alpha_0) = 1$ we define a measure of asymmetry by

$$A \equiv \frac{\alpha_0 - \alpha_{\min}}{\alpha_{\max} - \alpha_0}. \quad (14)$$

Helios Spacecraft

Orbits



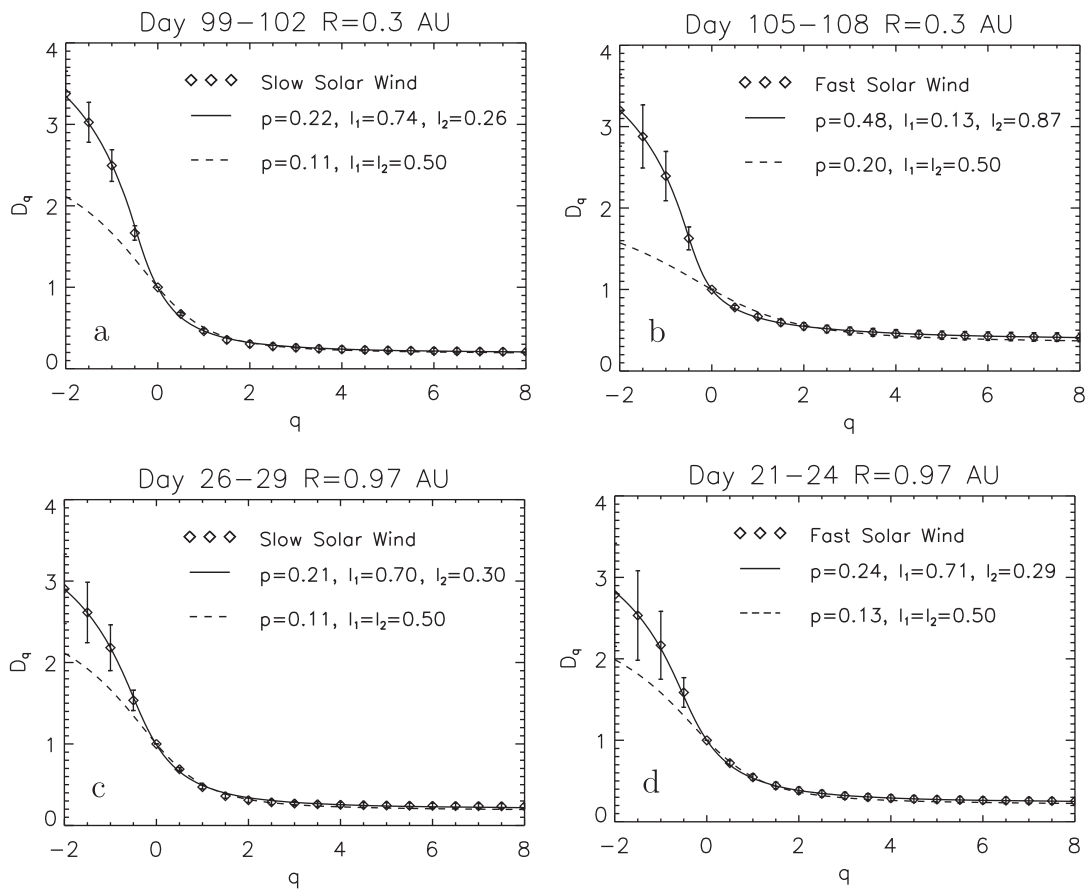


Fig. 3. The generalized dimensions D_q as a function of q. The values for one-dimensional turbulence are calculated for the generalized two-scale (continuous lines) model and the usual one-scale (dashed lines) *p*-model and fitted using the v_x radial velocity components (diamonds) for the slow (a) and fast (b) solar wind streams at distances of 0.3 AU and 0.97 AU, correspondingly (Macek and Szczepaniak, 2008).

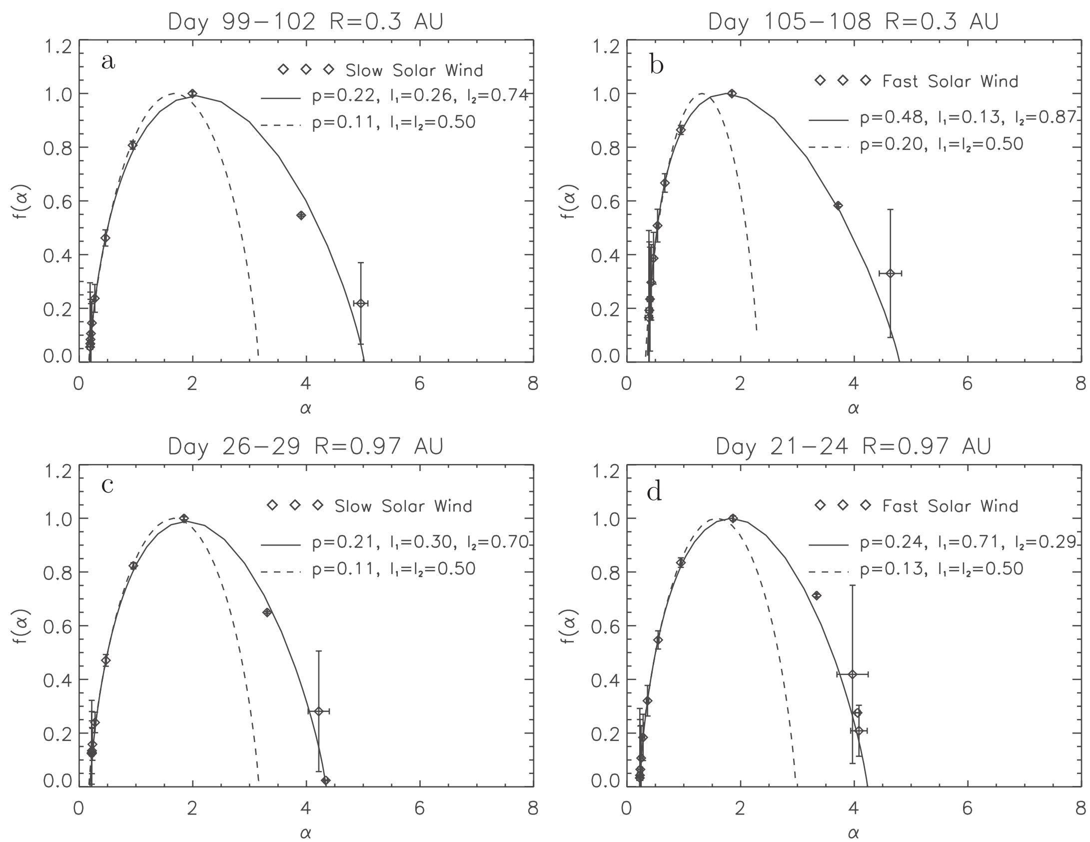


Fig. 4. The singularity spectrum $f(\alpha)$ as a function of α.. The values for one-dimensional turbulence are calculated for the generalized two-scale (continuous lines) model and the usual one-scale (dashed lines) *p*-model and fitted using the v_x radial velocity components (diamonds) for the slow (a) and fast (b) solar wind streams at distances of 0.3 AU and 0.97 AU, correspondingly (Macek and Szczepaniak, 2008).

ACE Spacecraft

2006 Days 172-176
2006 Days 354-358
2001 Days 191-195
2001 Days 273-277

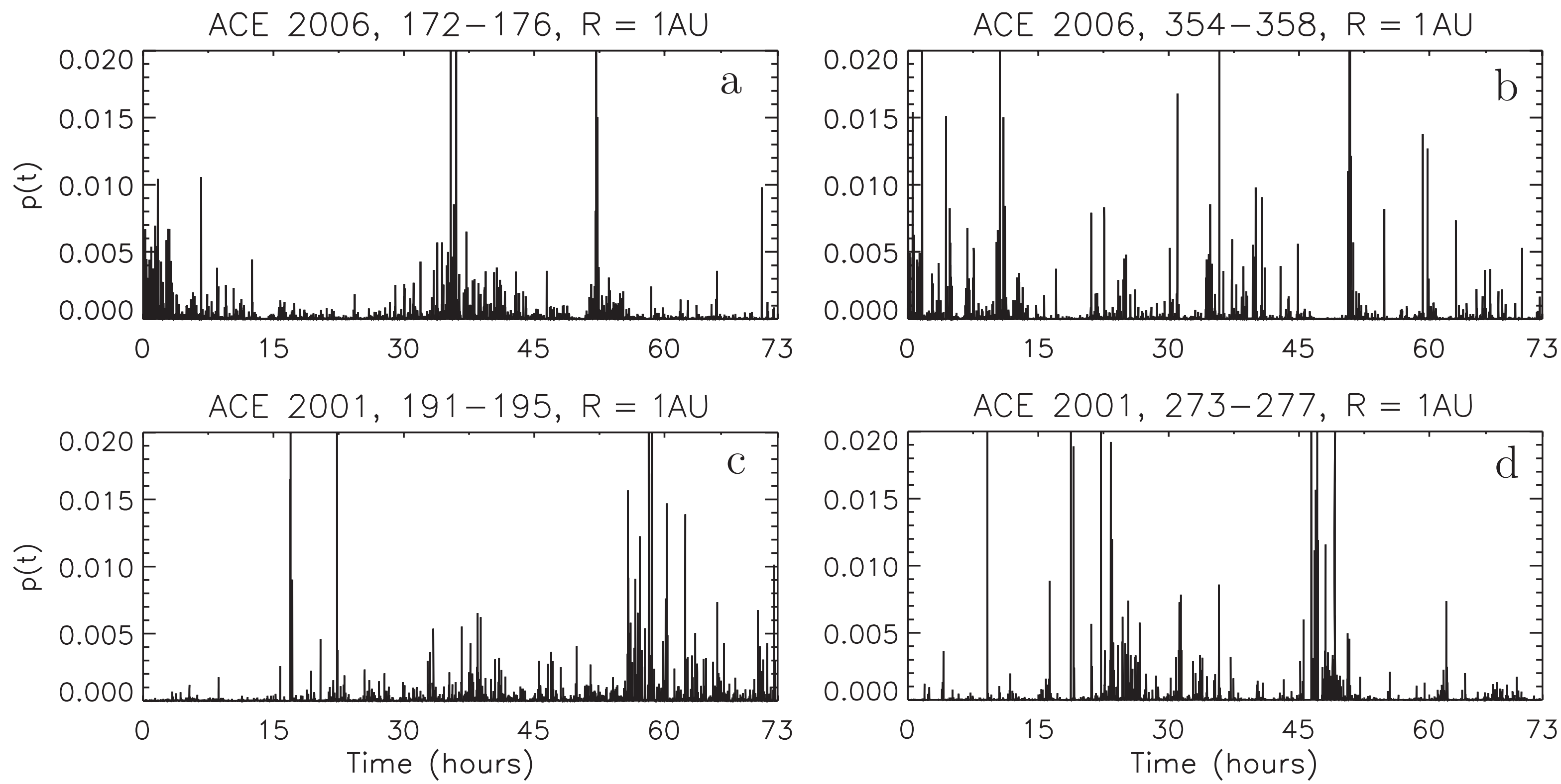


Fig. 2. The normalized transfer rate of the energy flux $p(t) = \varepsilon_i(t)\ /\ \sum \varepsilon_i(t)$ obtained using data of the $u = v_x$ velocity components measured by ACE at 1 AU for the slow (a) and (c) and fast (b) and (d) solar wind during solar minimum (2006) and maximum (2001), correspondingly (Macek *et al.*, 2009).

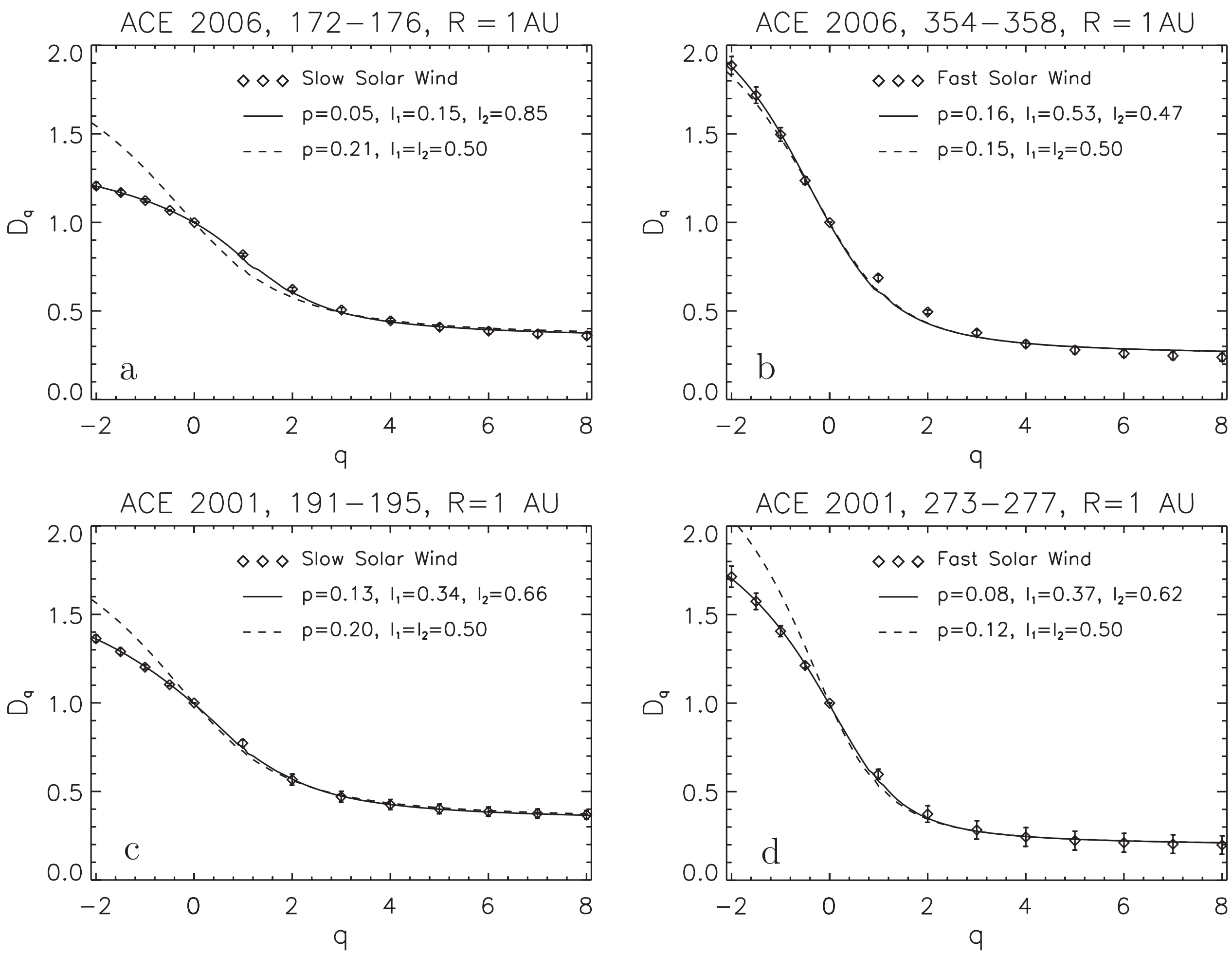


Fig. 4. The generalized dimensions D_q as a function of q. The values obtained for one-dimensional turbulence are calculated for the usual one-scale (dashed lines) *p*-model and the generalized two-scale (continuous lines) model (Macek *et al.*, 2009).

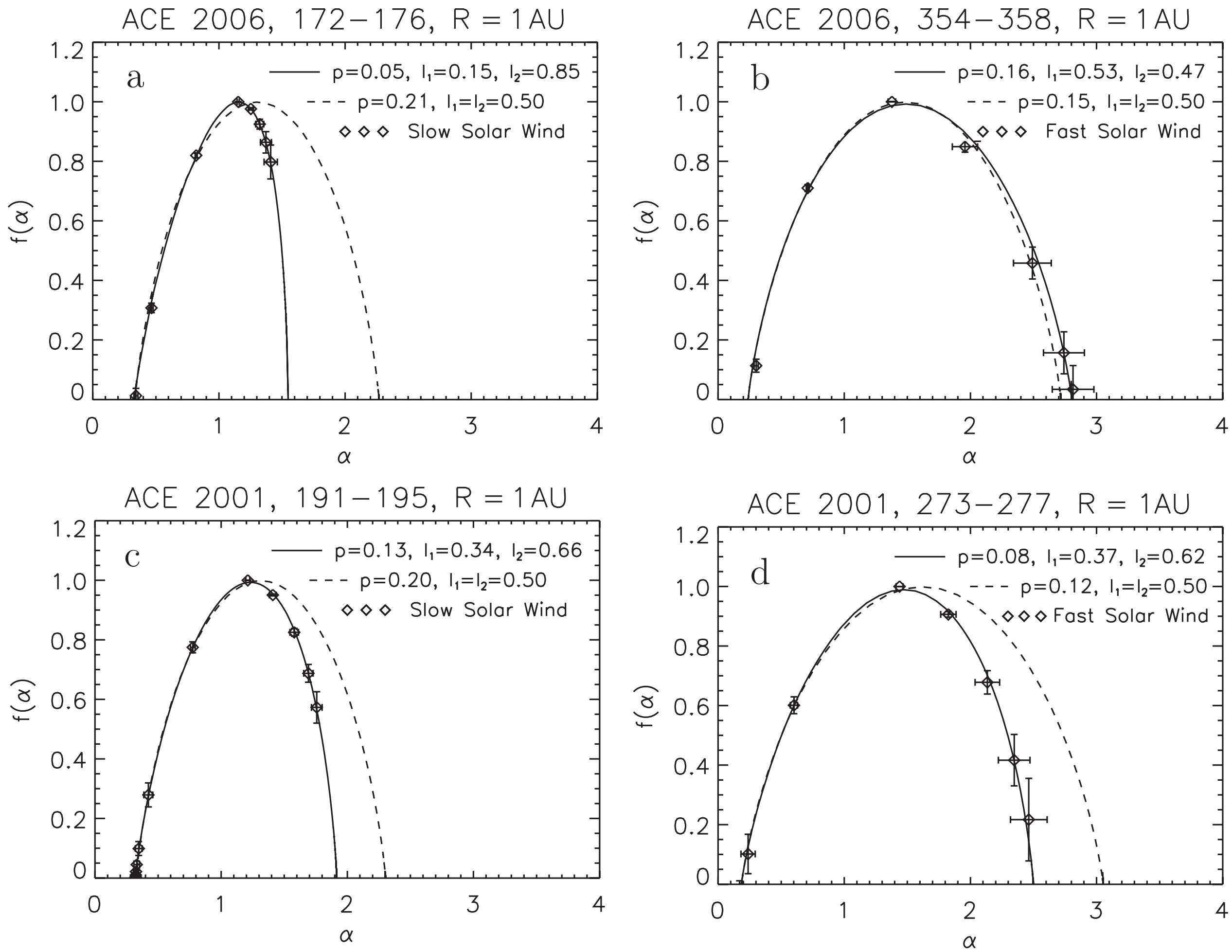
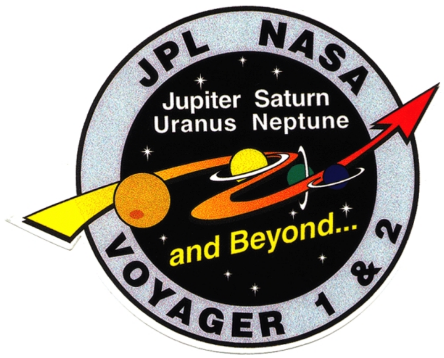


Fig. 7. The singularity spectrum $f(\alpha)$ as a function of α. The values obtained for one-dimensional turbulence are calculated for the usual one-scale (dashed lines) *p*-model and the generalized two-scale (continuous lines) model (Macek, 2009).

Table 1: Degree of multifractality Δ and asymmetry A for solar wind data in the inner heliosphere (1 AU) during solar minimum and maximum.

	Slow Solar Wind	Fast Solar Wind
Solar Minimum	$\Delta = 1.22, A = 2.21$	$\Delta = 2.56, A = 0.95$
Solar Maximum	$\Delta = 1.60, A = 1.33$	$\Delta = 2.31, A = 1.25$

Voyager Spacecraft



2.5 AU (1978)
25 AU (1987-1988)
85 AU (1996-1997)

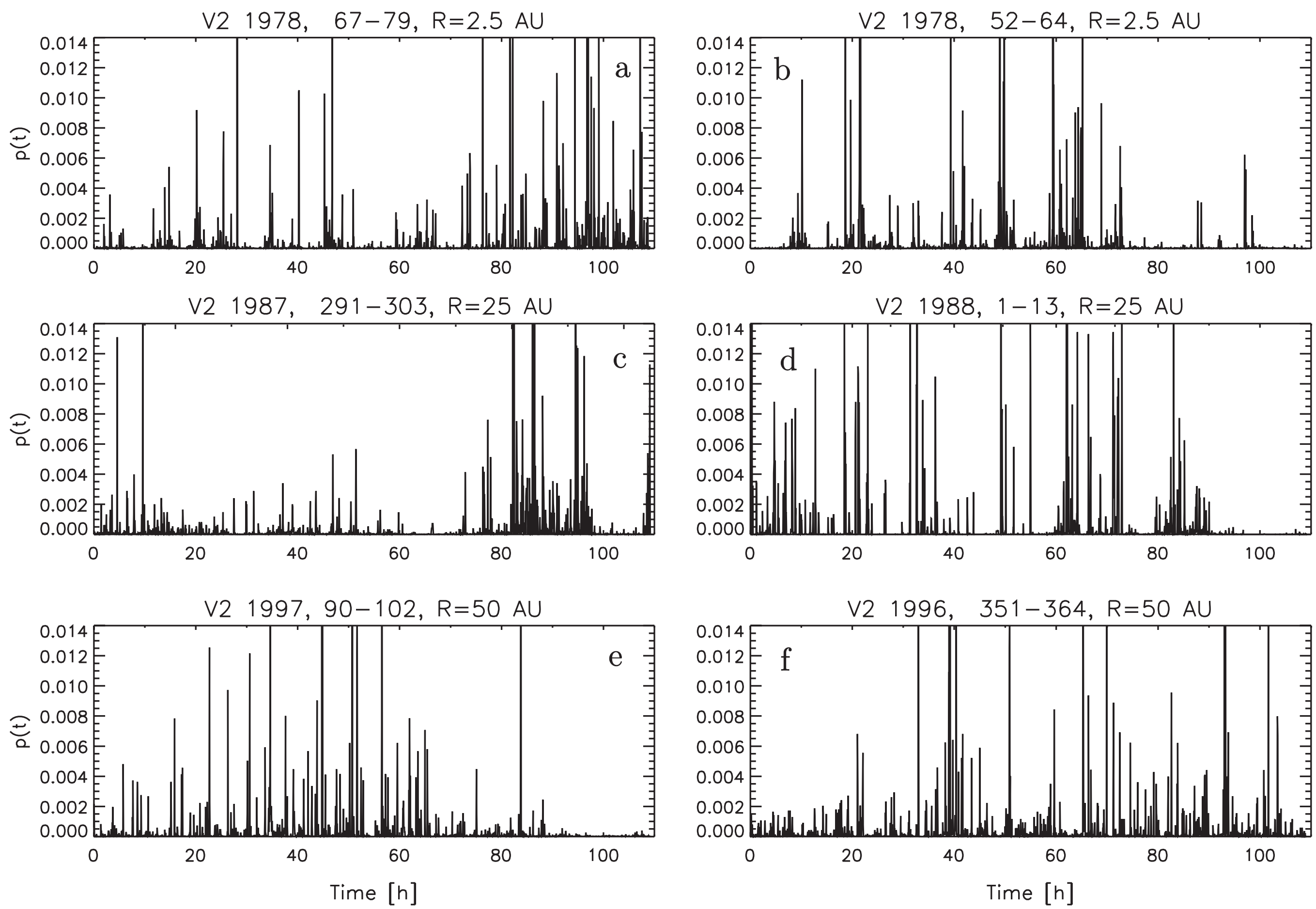


Fig. 2. The normalized transfer rate of the energy flux $p(t) = \varepsilon_i(t) \ / \ \sum \varepsilon_i(t)$ obtained using data of the v_x velocity components measured by Voyager 2 during solar minimum (1978, 1987–1988, 1996–1997) at 2.5, 25, and 50 AU for the slow (a), (c), and (e) and fast (b), (d), and (f) solar wind, correspondingly (Macek and Wawrzaszek, 2009).

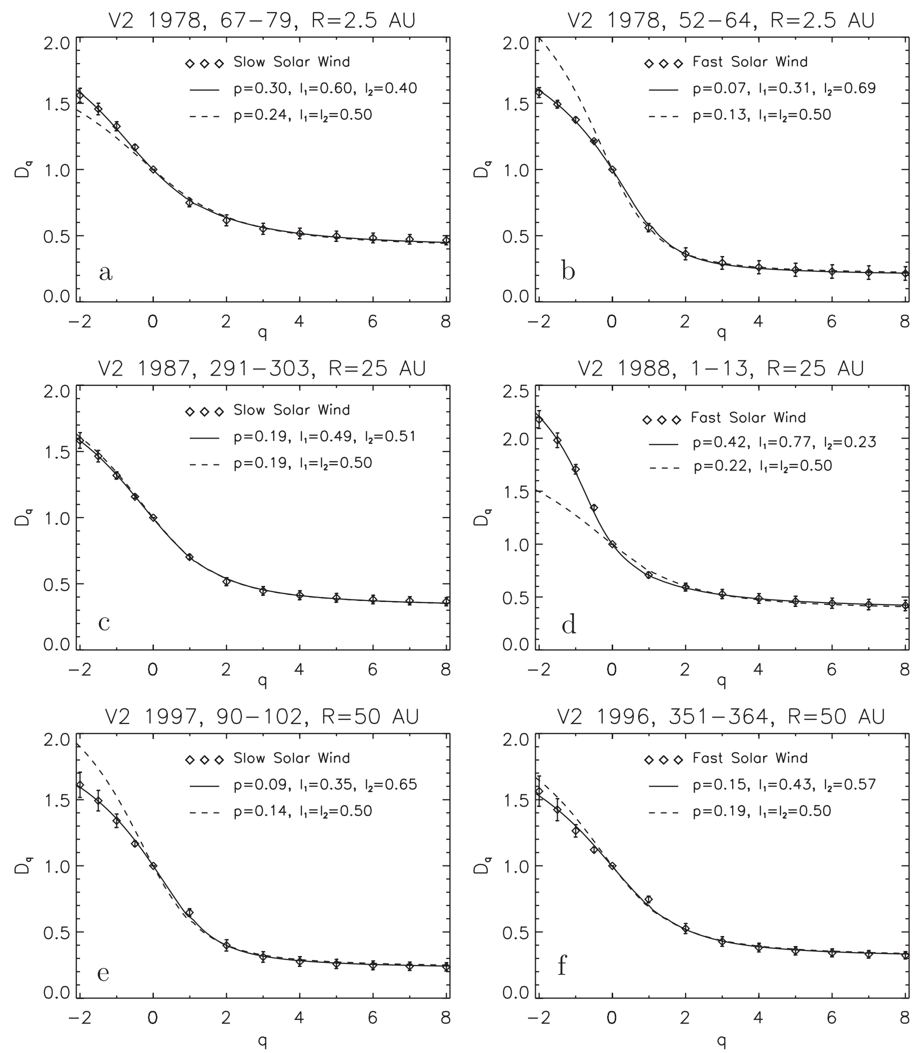


Fig. 4. The generalized dimensions D_q for the one-scale *p*-model (dashed) and the generalized two-scale (continuous lines) model (Macek and Wawrzaszek, 2009).

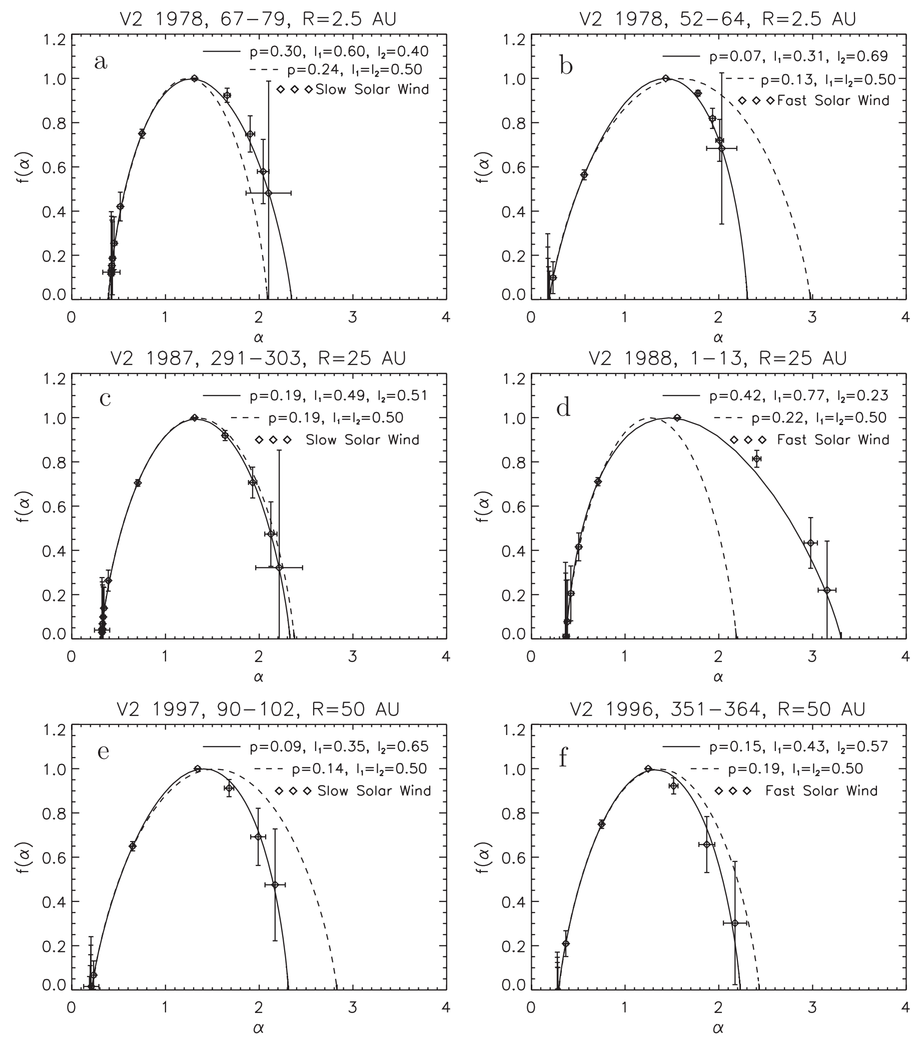


Fig. 7. The singularity spectrum $f(\alpha)$ for the one-scale *p*-model (dashed) and the generalized two-scale (continuous lines) model (Macek and Wawrzaszek, 2009).

Table 2: Degree of multifractality Δ and asymmetry A for solar wind data in the outer heliosphere during solar minimum.

Heliospheric distance (year)	Slow Solar Wind	Fast Solar Wind
2.5 AU (1978)	$\Delta = 1.95, A = 0.91$	$\Delta = 2.12, A = 1.54$
25 AU (1987-1988)	$\Delta = 2.02, A = 0.98$	$\Delta = 2.93, A = 0.66$
50 AU (1996-1997)	$\Delta = 2.10, A = 1.14$	$\Delta = 1.94, A = 0.95$

Conclusions

- We have studied the inhomogeneous rate of the transfer of the energy flux indicating multifractal and intermittent behavior of solar wind turbulence in the inner and outer heliosphere.
- Basically, the generalized dimensions for solar wind are consistent with the generalized *p*-model for both positive and negative q, but rather with different scaling parameters for sizes of eddies, while the usual *p*-model can only reproduce the spectrum for $q \geq 0$. We have demonstrated that a much better agreement of the two-scale model with the real data is obtained, especially for $q < 0$.
- It is worth noting that the multifractal scaling is often rather asymmetric. In particular, the fast wind during solar minimum exhibits strong asymmetric scaling (Helios, ACE, and Voyager).
- We have also shown that intermittent pulses are stronger for the model with asymmetric scaling.

- The degree of multifractality for the solar wind during solar minimum is greater for fast streams than that for the slow streams (Helios, ACE, and Voyager).
- As the solar activity increases the slow solar wind becomes somewhat more multifractal, and the fast wind is slightly less multifractal. On the other hand, it seems that the degree of asymmetry of the dimension spectrum for the slow wind is rather weakly correlated with the phase of the solar activity (ACE).
- Both the degree of multifractality and degree of asymmetry are correlated with the heliospheric distance and we observe the evolution of multifractal scaling in the outer heliosphere (Helios and Voyager).
- In general, the proposed generalized two-scale weighted Cantor set model should also be valid for non space filling turbulence. Therefore we propose this new cascade model describing intermittent energy transfer for analysis of turbulence in various environments.
- Our results provide supporting evidence for **multifractal** structure of the solar wind in the inner heliosphere. One can expect the fluctuations in the solar wind plasma should contain information about the dynamic variations of the coronal streamers.
- The multifractal structures, convected by the solar wind, might probably be related to the complex topology shown by the magnetic field at the source regions of the solar wind. It is also possible that it represents a structure of the time sequence of near-Sun coronal fine-stream tubes (see, Macek, 1998, 2006, 2007), and references therein.

Epilogue

Within the complex dynamics of the solar wind's fluctuating intermittent plasma parameters, there is a detectable, hidden order described by a generalized Cantor set that exhibits a multifractal structure.

This means that the observed **intermittent** behavior of the solar wind's velocity and Alfvénic fluctuations results from intrinsic *nonlinear* dynamics rather than from random external forces.

Thank you!