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# Multifractal structure of small and large scales fluctuations of interplanetary magnetic fields

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A B S T R A C T

We consider the multifractal spectrum of fluctuations of the interplanetary magnetic field strengths observed by Advanced Composition Explorer at the Earth's orbit. We have found that the multifractal scaling of magnetic fields is observed both on small and large scales from minutes to days. The obtained multifractal spectrum is asymmetric for small scales, in contrast to a rather symmetric spectrum observed at scales larger than a day. Moreover, we show that the degree of multifractality of the magnetic fields on large scales is correlated with the solar activity and greater than that at the small scales, where the magnetic turbulence may become roughly monofractal.



## 1. Introduction

Starting from seminal works of Kolmogorov (1941) and Kraichnan (1965) many authors have attempted to recover the observed scaling exponents, using phenomenological multifractal models of turbulence describing distribution of the energy flux between cascading eddies at various scales (Meneveau and Sreenivasan, 1987; Carbone, 1993; Frisch, 1995). In particular, the multifractal spectrum has been investigated using magnetic field data measured by Voyager in the outer heliosphere (Burlaga, 1991, 1995, 2001; Burlaga et al., 1993) and using Helios (plasma) data in the inner heliosphere (Marsch et al., 1996). The multifractal scaling has also been investigated using Ulysses observations (e.g., Horbury et al., 1997; Horbury, 1999; Horbury and Balogh, 2001; Wawrzaszek and Macek, 2010) and with Advanced Composition Explorer (ACE) and WIND data (e.g., Hnat et al., 2003, 2007; Kiyani et al., 2007; Szczepaniak and Macek, 2008). In addition, the times series of the magnetic field strengths measured *in situ* by Voyager 1 spacecraft to very large distances from the Sun and even in the heliosheath have already been analysed. It is known that fluctuations of the solar magnetic fields at large scales from 2 to 16 (32) days may exhibit multifractal scaling laws (Burlaga, 2004; Burlaga et al., 2006; Macek and Wawrzaszek, 2010).

To quantify scaling of solar wind turbulence, we have developed a generalized two-scale weighted Cantor set model using the partition technique (Macek, 2007; Macek and Szczepaniak, 2008). We have already studied the inhomogeneous rate of the transfer of the energy flux indicating multifractal and intermittent behaviour of solar wind turbulence in the inner and outer heliosphere using fluctuations of the velocity of the flow of the solar wind at small scales. We have investigated the resulting spectrum of generalized dimensions and the corresponding multifractal singularity spectrum depending on one probability measure parameter and two rescaling parameters (Macek and Szczepaniak, 2008; Szczepaniak and Macek, 2008; Macek and Wawrzaszek, 2009).

In particular, we have shown that the generalized dimensions for the solar wind are consistent with the generalized two-scale weighted Cantor set model for both positive and negative indices of the generalized dimensions, $q$ (Macek and Szczepaniak, 2008). In general, given the multiplicative process of cascading eddies, each breaking into new ones, but not necessarily equal, we consider two different rescaling parameters for sizes of eddies and one probability measure parameter, or weight $p$, at each step of turbulence cascade. In particular, using one-scale weighted Cantor set with equal eddies, the usual well-known so-called $p$ model is recovered, which can only reproduce the spectrum for $q \geq 0$. But we have demonstrated that in this way for the two-scale model a better agreement with the solar wind velocity data is obtained. It is worth noting that the multifractal scaling is often rather asymmetric. Both the degree of multifractality and degree of asymmetry are correlated with the heliospheric distance and we observe the evolution of multifractal scaling in the outer heliosphere (Macek and Wawrzaszek, 2009).

The aim of this study is to examine the question of scaling properties of intermittent fluctuations of the magnetic field embedded in the solar wind on both small and large scales using

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our two-scale weighted Cantor set model in comparison with the simple one-scale multifractal spectrum (Macek and Szczepaniak, 2008). In particular, we show that the degree of multifractality for fluctuations of the interplanetary magnetic field strengths at small scales is smaller than that at the large scales. Moreover, we demonstrate that the multifractal spectrum is asymmetric for small scale fluctuations, in contrast to the rather symmetric spectrum observed on large scales. It is worth noting that for the multifractal two-scale Cantor set model a good agreement with the data is obtained. Hence we propose this new model as a useful tool for analysis of intermittent fluctuations of the interplanetary magnetic field strength on both small and large scales.

## 2. Solar magnetic field data

In this paper we would like to test the multifractal scaling of the interplanetary magnetic field strengths, $B$, for the wealth of data provided by Advanced Composition Explorer (ACE) mission, located in the ecliptic plane near the libration point $L1$, i.e., approximately at a distance of $R$ = 1 AU from the Sun. In case of ACE the sampling time resolution of 16 s for the magnetic field is much better than that for the Voyager data, which allow us to investigate the scaling on small scales of the order of minutes.

The calculated energy spectral density as a function of frequency for the data set of the magnetic field strengths $|B|$ consisting of about $2 \times 10^6$ measurements for (a) the whole year 2006 during solar minimum and (b) the whole year 2001 during solar maximum using the Welch window ($40 \times 1024$) is shown in Fig. 1. The dashed line shows the spectrum of the type $E(f) \propto f^{-5/3}$ for comparison. As one can see, the spectrum density is roughly consistent with this well-known power-law at wide range of frequency, $f$, suggesting a self-similar fractal turbulence model often used for looking at scaling properties of plasma fluctuations (e.g., Burlaga and Klein, 1986). However, it is clear that the spectrum alone, which is based on a second moment (or a variance), cannot fully describe fluctuations in the solar wind turbulence (cf. Alexandrova et al., 2007). Admittedly, intermittency, which is deviation from self-similarity (e.g., Frisch, 1995), usually results in non-Gaussian probability distribution functions. However, the multifractal powerful method generalizes these scaling properties by considering not only various moments of the magnetic field, but also the whole spectrum of scaling indices (Halsey et al., 1986).

Therefore, here we further analyse time series of the magnetic field of the solar wind on both small and large scales using multifractal methods. To investigate scaling properties in fuller detail, using basic 64-s sampling time for small scales, we have selected long time intervals of $|B|$ of interpolated samples, each of $2^{18}$ data points, from day 1 to 194. Similarly, for large scales we use daily averages of samples from day 1 to 256 of $2^8$ data points. The data for both small and large scale fluctuations during solar minimum (2006) and maximum (2001) are shown in Fig. 2.

## 3. Multifractal model

The generalized dimensions $D_q$ as a function of a continuous index $q$ are important characteristics of complex dynamical systems; they quantify multifractality of a given system (Grassberger, 1983; Grassberger and Procaccia, 1983; Hentschel and Procaccia, 1983; Halsey et al., 1986; Ott, 1993). In the case of magnetic field fluctuations these generalized measures are related to inhomogeneity with which the scaling exponents depend on various scales. In this way the generalized dimensions provide information about dynamics of magnetic field turbulence. As usual in the solar wind a position $x$ in space can be associated with time $t$ by Taylor's hypothesis and $x = v_{sw}t$, where $v_{sw}$ is the average speed of the flow.

Assume a stationary magnetic field $B$ in the equatorial plane normal to the flow of the solar wind. We like to decompose this signal into time intervals of size $\tau$, corresponding to the spatial scales $l = v_{sw}\tau$. Then to each time interval can be associated a magnetic flux past the cross-section perpendicular to the plane during that time. Namely, a discrete time series, sampled with a basic time difference $\Delta t$, can be normalized so that we have for its average $\langle B(t_i) \rangle := (1/N)\sum_{i=1}^{N} B(t_i) = 1$. Then given the normalized time series $B(t_i)$, where $i$ = 1, …, $N = 2^n$, to each interval of temporal scale $\tau$, we associate a measure $\mu(t_i, \tau)$ calculated by using the average values $\langle B(t_i, \tau) \rangle$ of $B(t_i)$ between $t_i$ and $t_i + \tau$, as

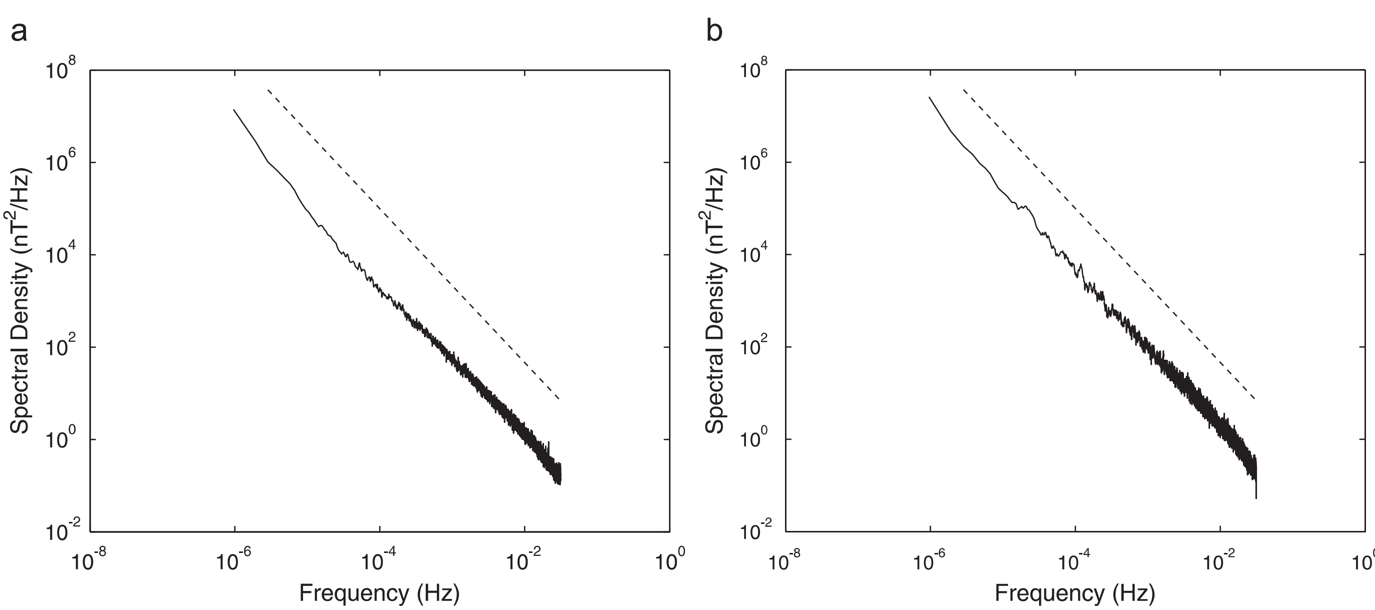


**Fig. 1.** The logarithm of energy spectral density as a function of frequency for the magnetic field strengths $|B|$ observed by ACE at 1 AU using 16-s sampling time during (a) solar minimum (2006) and (b) solar maximum (2001), correspondingly. The dashed line shows the spectrum of $f^{-5/3}$ type for comparison.

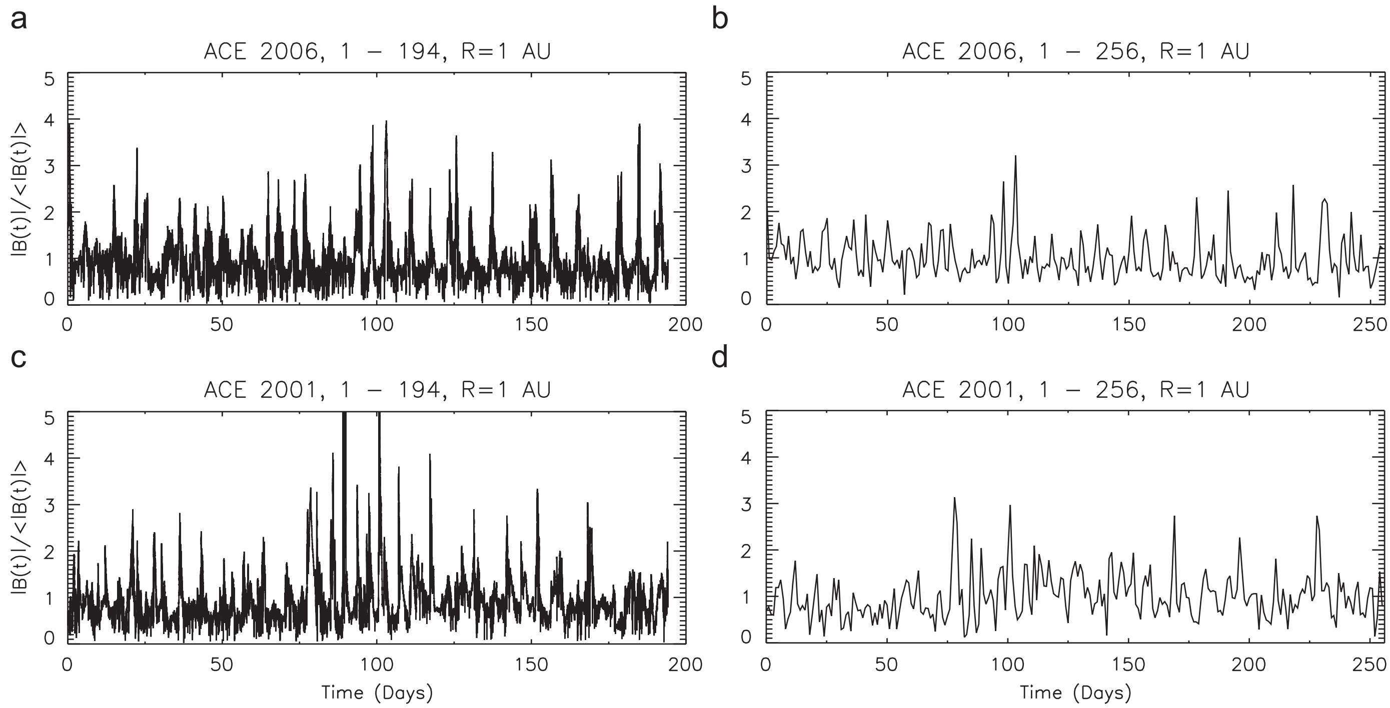


**Fig. 2.** The magnetic field strengths $|B|$ as a function of time observed by ACE at 1 AU using 64-s averages on the small scales (a) and (c), and daily averages on large scales (b) and (d) during solar minimum (2006) and maximum (2001), correspondingly.

discussed by Burlaga (1995)

$$\mu(t_i,\tau) = \langle B(t_i,\tau)\rangle \frac{\tau}{N} = \mu_i(\tau). \tag{1}$$

In fact, this (normalized) quantity can be interpreted as a probability that at time $t_i$ the magnetic flux is transferred to a segment of a temporal size $\tau$. In our case we have $n$ = 18 with $\Delta t = 64$ s and $n$=8 with $\Delta t = 1$ h for small and large scales, correspondingly.

Naturally, multifractal scaling can be characterized by the generalized dimensions ($l = v_{sw}\tau$), which are defined for any continuous index $q$, $-\infty < q < \infty$, by (e.g., Ott, 1993)

$$D_q := \frac{1}{q-1}\lim_{\tau\to 0}\frac{\log\sum_{i=1}^{N}\mu_i^q(\tau)}{\log\tau}. \tag{2}$$

In particular, high positive values of $q$ emphasize regions of intense fluctuations larger than the average, while negative values of $q$ accentuate fluctuations lower than the average. Therefore, given a time series $B(t)$ it can be argued that in some region the average value of its $q$ th moment at various times scales $\tau$ should scale with the exponent $s(q)$ = $(q-1)$ $(D_q-1)$ as obtained by Burlaga (1991)

$$\langle B^q(t,\tau)\rangle \sim \tau^{s(q)}. \tag{3}$$

Now, as a one-dimensional phenomenological model of turbulence let us consider a binomial multiplicative fragmentation process. Namely, we take a closed unit interval, where the probability of choosing one closed subinterval of size $l_1$ is $p$, and the other subinterval of size $l_2$ is $1-p$ as considered by Macek and Szczepaniak (2008), Szczepaniak and Macek (2008) and Macek and Wawrzaszek (2009). At $n$-stage of this iterative process with the weighting parameter $p$, we have $\binom{n}{k}$ intervals each of width $\tau_k = l_1^{n-k} l_2^k$, where $k = 0,1,\ldots,n$, with various associated probability measures

$$\mu(\tau_k) = \left(\frac{p}{l_1}\right)^{n-k}\left(\frac{1-p}{l_2}\right)^k. \tag{4}$$

The resulting set of all $2^n$ closed intervals (more and more narrow segments of various widths and probabilities) for $n\to\infty$ becomes the two-scale weighted Cantor set. Hence in this limit one obtains $D_q$ by solving numerically the following transcendental equation (e.g., Ott, 1993)

$$\frac{p^q}{l_1^{\gamma(q)}} + \frac{(1-p)^q}{l_2^{\gamma(q)}} = 1, \tag{5}$$

where $\gamma(q)$ is the scaling exponent related to $D_q$ by $\gamma(q) \equiv (q-1)D_q$, similarly as the exponent $s(q)$ in Eq. (3).

The dependence of the resulting spectrum of the generalized dimensions $D_q$ for the two-scale weighted Cantor set model has been thoroughly discussed by Macek and Szczepaniak (2008) and Macek and Wawrzaszek (2009). It can be proved that this is a monotonically decreasing function of $q$, except for a constant value in case of a monofractal, $p$=0.5, $l_1$ = $l_2$= 0.5. Therefore, a degree of multifractality could be defined as $\Delta := D_{-\infty} - D_{\infty}$. Hence for given two scales the parameter $p$ quantifies multifractality, $\Delta = |\log(1-p)/\log l_2 - \log(p)/\log l_1|$ (Macek, 2007).

In addition, the singularity multifractal spectrum $f(\alpha) = q\alpha - \gamma(q)$ as a function of the singularity strength $\alpha$, which is the derivative of the scaling function, $\alpha = \gamma'(q)$, can also be obtained by using Legendre transformation (Ott, 1993), or directly from the slopes or generalized measures (Macek and Wawrzaszek, 2009). Using $\alpha_0$, where $f(\alpha_0) = 1$, we can define a measure of asymmetry $A \equiv (\alpha_0 - \alpha_{min})/(\alpha_{max} - \alpha_0)$ (Macek and Wawrzaszek, 2009). In particular, with two equal scales $l_1$ = $l_2$ = 0.5 we have the symmetric multifractal spectrum, $A$ = 1.

## 4. Results

For a given $q$, using the slopes $s(q)$= $(q-1)$ $(D_q-$ 1) of $\log_{10}\langle B^q\rangle$ versus $\log_{10}\tau$ as given in Eq. (3) one can obtain the values of $D_q$ according to Eq. (2), which indicate multifractal scaling behaviour. The results for the generalized dimensions $D_q$ as a function of $q$ obtained using the ACE data of the magnetic field strengths at distances of 1 AU from the Sun for the small (a) and (c) and large scales (b) and (d) averages during solar minimum (2006) and maximum (2001), correspondingly, are presented in Fig. 3. In addition, the multifractal spectrum $f(\alpha)$ as a function of scaling indices $\alpha$ derived from the ACE observations of the magnetic field strengths together with the one-scale $p$ model fit (dashed curve) and the two-scale model (solid curve) is presented in Fig. 4. We also confirm the characteristic shapes of both the generalized dimension and the multifractal spectrum, as seen in Figs. 3 and 4. In fact, one can see that $D_q$ is a decreasing function of $q$ and the singularity spectrum $f(\alpha)$ is an (upward) concave function of $\alpha$, as schematically depicted by, e.g., Ott (1993, Fig. 9.1). It is worth noting the universal character of the shape of the function $f(\alpha)$ in multifractal theory. The width of this function, $\alpha_{\max}-\alpha_{\min}$, which is equal to $D_{-\infty}-D_{\infty}$, can be here identified as the degree of multifractality and intermittency, $\Delta$ (Macek, 2007; Macek and Wawrzaszek, 2009), which is somehow related to other measures of intermittency in the literature, e.g., flatness and kurtosis (Frisch, 1995; Carbone, 1994; Szczepaniak and Macek, 2008). The degree of multifractality $\Delta$ and the degree of asymmetry $A$ are listed in Table 1. As we see, the generalized two-scale weighted Cantor set model is a convenient tool to investigate the asymmetry of this function; in a usual one-scale Cantor set model this function is necessarily symmetric.

For small scales we have identified a scaling region from $\sim 2$ min $\sim 18$ h provided by times series from $2^1$ to $2^{10}$ points ($n=1,\ldots,10$ with $\Delta t = 64$ s), and for large scales from 2 to 16 days ($n=1,\ldots,4$ for daily averages), correspondingly. The obtained values in the large scaling region are consistent with those calculated by Burlaga (2004). We also see that the fit to the generalized two-scale weighted Cantor set model according to Eq. (5) with $p$ = 0.03 or $p$ = 0.08 together with unequal two scales $l_1$ = 0.02 and $l_2$ = 0.98 or $l_1$ = 0.10 and $l_2$ = 0.90 denoted by continuous line shows a good agreement with the data. One should note that space filling magnetic turbulence is recovered, $l_1 + l_2 = 1$ (Burlaga et al., 1993).

As seen from Table 1 using our two-scale Cantor set model we have obtained somewhat smaller value of the degree of multifractality $\Delta$ for small scales as compared with those for the large scales. In addition, on small scales this value is not very much dependent on the solar cycle. However, the degree of multifractality for large scales seems to be correlated with the phase of the solar cycle, increasing with the intensity of the solar magnetic field activity. We have already demonstrated that the multifractal scaling is asymmetric for the energy transfer rate in the turbulence cascade (Macek and Szczepaniak, 2008; Macek and Wawrzaszek, 2009). Here for the magnetic field, the multifractal spectrum is also asymmetric for small scales with the calculated degree of asymmetry of $A$ = 0.3–2.0.

The results for the generalized dimensions $D_q$ and the multifractal spectrum $f(\alpha)$ for large scales obtained using the ACE data of magnetic fields at 1 AU (diamonds), are presented in Figs. 3 and 4(b) or (d) during solar minimum or maximum, correspondingly. Now the fits to the generalized two-scale model with $p \simeq 0.33$ or 0.48 and nearly equal scales, $l_1$ = 0.42 and $l_2$ = 0.58 or $l_1$ = 0.61 and $l_2$ = 0.39 are depicted by continuous lines. This means that for large scales our model provides similar results as the one-scale $p$ model with $p$ = 0.40 or $p$ = 0.38 (dashed curve) confirming

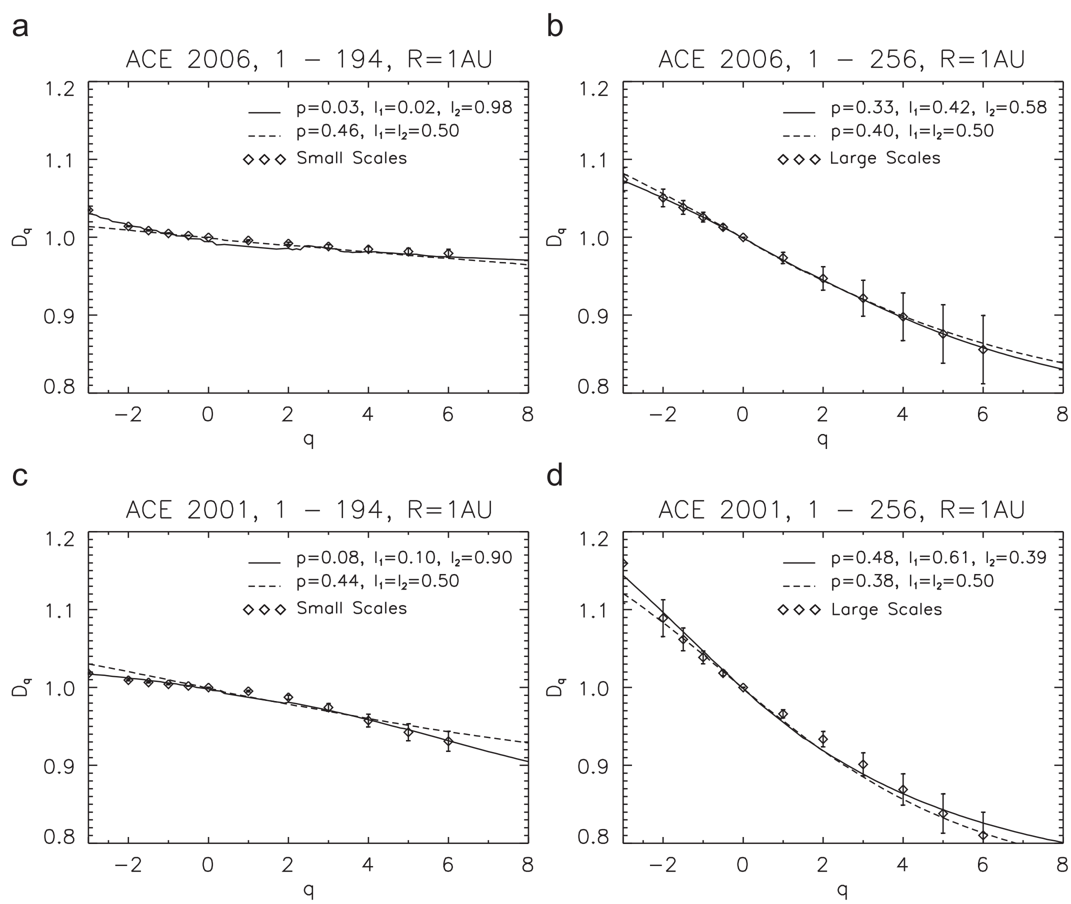


**Fig. 3.** The generalized dimensions $D_q$ as a function of $q$. The values are derived from the magnetic field strengths observed by ACE at 1 AU (diamonds) for the small (a) and (c) and large scale (b) and (d) averages during solar minimum (2006) and maximum (2001), correspondingly, together with fits using the usual one-scale (dashed lines) $p$-model and the generalized two-scale (continuous lines) model.

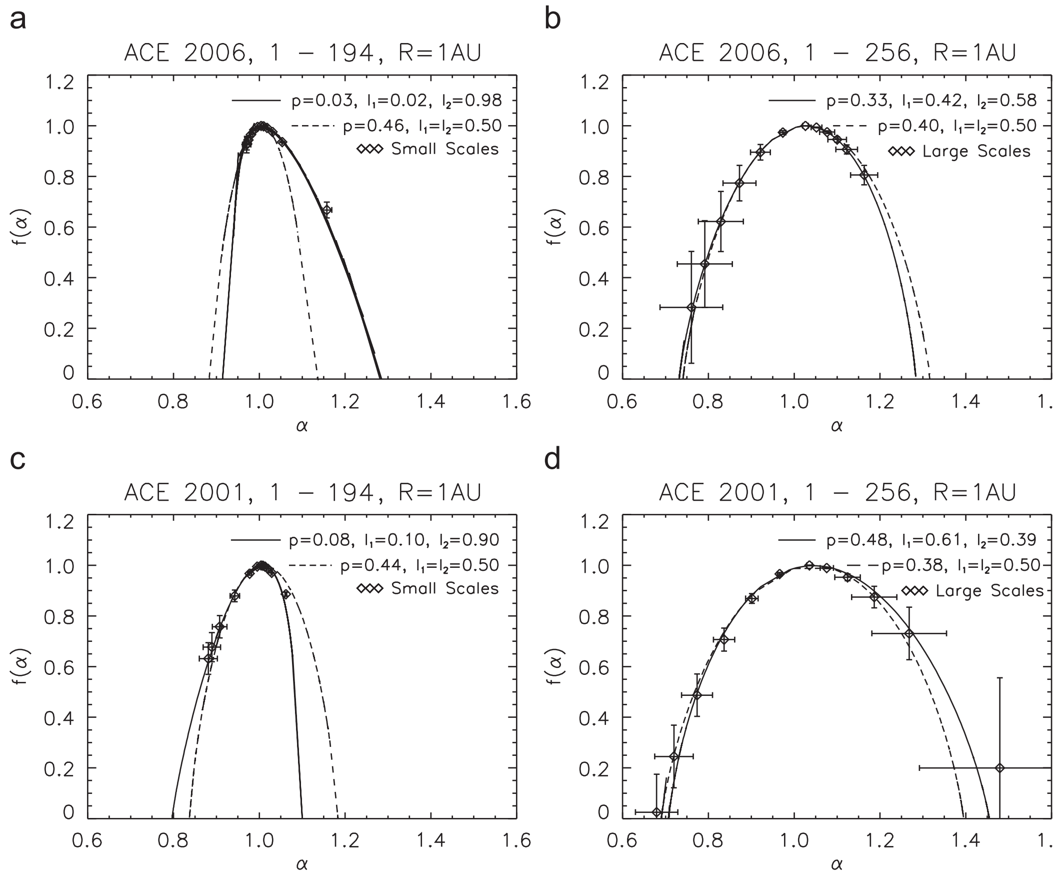


**Fig. 4.** The corresponding singularity spectrum $f(\alpha)$ as a function of $\alpha$.

**Table 1**
Degree of multifractality $\Delta$ and asymmetry $A$ for the magnetic field strengths observed at 1 AU during solar minimum and maximum.

| | Small scales | | Large scales | |
|---|---|---|---|---|
| | $\Delta$ | $A$ | $\Delta$ | $A$ |
| Minimum (2006) | $0.37 \pm 0.02$ | $0.32 \pm 0.02$ | $0.54 \pm 0.11$ | $1.35 \pm 0.55$ |
| Maximum (2001) | $0.30 \pm 0.03$ | $2.00 \pm 0.25$ | $0.75 \pm 0.21$ | $0.88 \pm 0.48$ |

approximately symmetric character of the multifractal singularity spectrum (cf. Burlaga et al., 2006). We see that in contrast to the asymmetric spectrum observed on small scales the spectrum becomes symmetric on large scales at $\sim 1$ AU. Therefore the obtained values of $A = 1.35 \pm 0.55$ or $0.88 \pm 0.48$ are consistent with unity, Table 1.

One sees from Table 1 that the degree of multifractality for fluctuations of the interplanetary magnetic field strengths is generally smaller than that for the energy transfer rate in the turbulence cascade (cf. Burlaga, 1991; Macek and Szczepaniak, 2008; Macek and Wawrzaszek, 2009). However, it is worth noting that the values obtained for large scales, $\Delta = 0.5-0.8$, are somewhat greater than that for small scales $\Delta \sim 0.3$, contrary to that for the energy transfer rates, where the degree of multifractality rises with the decreasing scale (e.g., Marsch et al., 1996). This means that the magnetic field behaviour on large scales, may exhibit a significant multifractal scaling. On the other hand, for smaller scales smaller values of $\Delta$ indicate possibility toward a monofractal behaviour. This supports a recent suggestion that at scales smaller than the proton gyroscale the magnetic field fluctuations in quiet interplanetary solar wind turbulence are monofractal (Kiyani et al., 2009).

## 5. Conclusions

We show that the degree of multifractality for magnetic field fluctuations of the solar wind at $\sim 1$ AU for large scales from 2 to 16 days is greater than that for the small scales from 2 min to 18 h. In particular, we have demonstrated that on small scales the multifractal scaling is strongly asymmetric in contrast to a rather symmetric spectrum on the large scales, where the evolution of the multifractality with the solar cycle is also observed.

Our results provide some new evidence for multifractal structure of the magnetic field strengths in the heliosphere, possibly also on very small scales, where the degree of multifractality is rather small. One can expect that the fluctuations of the magnetic field strength of the solar wind should contain information about the dynamic variations of the solar wind plasma at a broad range of scales from minutes to days. In general, the proposed generalized two-scale weighted Cantor set model should also be valid for magnetic field turbulence. Therefore we propose this new turbulence model describing intermittent fluctuations of plasma parameters for analysis of turbulence in various environments.

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