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On the magnetic field fluctuations during magnetospheric tail current disruption: A statistical approach

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[1] The objective of this study is to investigate the nature of the magnetic field relaxation process associated with tail current disruption on the basis of magnetic field measurements collected in the near-Earth tail regions. In detail, using magnetic field data for three current disruption (CD) events as observed by AMPTEE/CCE spacecraft, we investigate the scaling features of the probability distribution functions (PDFs) of the magnetic field fluctuations at different timescales τ. The PDFs of magnetic field fluctuations in non-MHD domain (i.e., below the proton gyroperiod) show non-Gaussian tails and the probability of return $P_\tau(0)$ scales in this domain as $\tau^{-\alpha}$ with $\alpha > 1/2$, which is compatible with a Lévy statistics. Conversely, the scaling of the PDFs of the CD magnetic fluctuations in the magnetohydrodynamic (MHD) regime is compatible with a classical Brownian motion $\alpha \sim 1/2$. These findings are discussed in terms of an anomalous diffusion process, involving the magnetic field relaxation during CD. Furthermore, the relevance of these results of a non-Gaussian statistics at the shorter timescales is discussed in connection with the non-MHD nature of the CD phenomenon.

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1. Introduction

[2] The Earth's magnetosphere has been now recognized to act as a dynamical nonlinear system, which shows a complex behavior in response to the changes of the solar wind conditions. One of the major displays of the magnetospheric dynamics is the magnetic substorm, a set of phenomena involving a vast region of the near-Earth space [*Akasofu*, 1968].

[3] Without any doubt, one of the most relevant phenomena occurring at the substorm onset is the development of a current wedge, which is responsible for the magnetosphere-ionosphere coupling during magnetic substorms. This current wedge is generally associated with the diversion or disruption of the near cross-tail current system, which is converted into a system of field-aligned currents (FAC) [*Akasofu*, 1972; *McPherron et al.*, 1973; *Baumjohann et al.*, 1981]. The magnetospheric substorm onset is, indeed, associated with a change of the tail magnetic configuration from a stressed to a more dipolar shape involving the decrease of the cross-tail current intensity [*Atkinson*, 1967; *Akasofu*, 1972]. In more detail, during substorm expansion phase the near-Earth magnetic field configuration has been observed to dipolarize.

[4] In the last years this near-Earth dipolarization phenomenon has been the subject of several observation as well as simulation studies, which suggested a multiscale and a non-MHD nature of the phenomenon [*Lui et al.*, 1999; *Sitnov et al.*, 2000; *Miura*, 2000].

[5] As previously mentioned, in association with the dipolarization at the substorm onset a phenomenon, named current disruption (CD), is observed. In the past, tail current disruption was widely investigated, especially in connection to the onset location, the spatial extension, and the magnetic field and particle flux variations [*Lopez and Lui*, 1990; *Ohtani et al.*, 1991, 1993]. During tail current disruption, large-amplitude ($\Delta B/B > 1$) and

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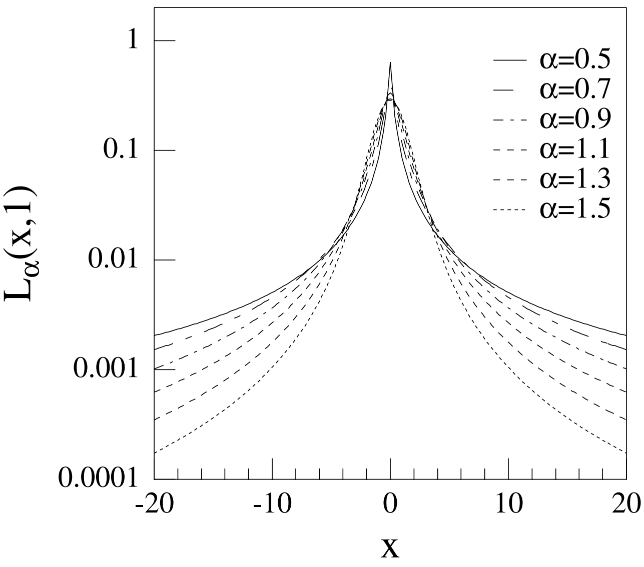

Figure 1. Behavior of the Lévy distribution $L_\alpha(x, \gamma)$ with $\gamma = 1$ and $\frac{1}{2} \leq \alpha \leq \frac{3}{2}$.

turbulent magnetic field fluctuations were observed in the near-Earth magnetotail [*Takahashi et al.*, 1987; *Lui et al.*, 1988]. The features of these large-amplitude magnetic field fluctuations were widely studied in the last decade (see for a review *Lui* [2002]). These previous studies pointed out the nonlinear nature of CD and underlined that a broadband spectrum of fluctuations is intermittently excited during CD, with timescales ranging from below to above the ion (proton) cyclotron timescale $T_{ci} = 1/f_{ci}$ [*Lui and Najmi*, 1997; *Ohtani et al.*, 1995, 1998]. Moreover, it has been shown that in situ magnetic field fluctuations, associated with CD, are characterized by intermittency at the smaller timescales and nonlinear intermittent cross-coupling among fluctuations at different timescales [*Consolini and Lui*, 2000]. All these previous studies point to a non-MHD origin of the observed CD magnetic field fluctuations, in which an extremely relevant role is played by ions. Furthermore, *Ohtani et al.* [1998] described the tail current disruption in terms of "a system of chaotic filamentary electric currents."

[6] In this work, we investigate the nature of the magnetic field relaxation from a stressed configuration toward a dipolar one during CDs by investigating the statistical features of the magnetic field fluctuations. In detail we present some results on the Wiener-Lévy-like nature of the fluctuations of the magnetic field magnitude at different scales in the study of the distribution function and the associated return probability $P_\tau(0)$. The statistical features of CD fluctuations are compared with the concept of crossover of the scaling properties between non-MHD and MHD domain and are discussed in connection with the suggested bimodal intermittent turbulence scenario encompassing symmetry breaking phenomenon and nonlinear crossover process between forced and/or self-organized criticalities generated by MHD (Alfvenic) and kinetic (whistler) coherent structures [*Chang*, 1998a, 1999; *Chang et al.*, 2004]. A leading candidate for the observed fluctuations and the onset of CD is suggested to be the kinetic current-driven instability above the ion gyrofrequency [*Lui et al.*, 1991, 1999].

2. Lévy Distribution Function: A Brief Outlook

[7] A large class of dynamical processes in complex systems may be described in term of strange kinetics and anomalous diffusion [*Bouchaud and Georges*, 1990; *Shlesinger et al.*, 1993; *Klafter et al.*, 1996; *Sornette*, 2000]. Under certain conditions the description of the fluctuations of dynamical processes falls outside the domain of the well-known gaussian and poissonian statistics, showing distributions characterized by scale invariance in the central part with power law tails. As a matter of fact, a wide variety of stochastic phenomena in physical systems is controlled by non-Gaussian distributions characterized by power law tails. A special class of non-Gaussian distributions, which is quite ubiquitous in nature, is the Lévy distribution. Lévy-like distributions have been quite ubiquitously observed in several dynamical processes, as anomalous diffusion processes, chaotic transport in laminar fluid flows, particle diffusion in turbulent magnetic fields, gravitational force resulting from randomly and homogeneously distributed point masses, etc.

[8] Stable Lévy distribution follows from the generalization of the Central Limit Theorem (CTL) for distributions characterized by an infinite second moment and may be generally defined through its characteristic function $L_\alpha(k, \gamma)$:

$$\ln L_\alpha(k,\gamma) = -\gamma|k|^\alpha(1 + i\beta\omega(\alpha,k)), \tag{1}$$

where $\alpha \in (0,2]$ is a characteristic index that controls the tail shape, $\beta \in [-1, 1]$ determines the distribution symmetry features, $\gamma > 0$ is a scale factor, and

$$\omega(\alpha,k) = \begin{cases} sgn(k)\tan\left(\frac{\alpha\pi}{2}\right) \leftrightarrow \alpha \neq 1 \\ \left(\frac{2}{\pi}\right)\ln|k| \leftrightarrow \alpha = 1 \end{cases} \tag{2}$$

[9] The probability distribution function (PDF) is obtained by Fourier inverse transform of the characteristic function $L_\alpha(k, \gamma)$. Here, we will consider only the case of symmetrical Lévy-stable distribution ($\beta = 0$)

$$L_\alpha(x,\gamma) = \frac{1}{\pi}\int_0^\infty \exp(-\gamma k^\alpha)\cos(kx)dk. \tag{3}$$

[10] Closed analytical forms of symmetrical Lévy distribution may be recovered only in the case of $\alpha = 1$, the Cauchy distribution, and in the limit of $\alpha = 2$, the Gaussian distribution. Figure 1 shows the behavior of the Lévy distribution with the same scaling factor $\gamma = 1$ and for a set of different Lévy exponents α in the range $[\frac{1}{2}, \frac{3}{2}]$.

[11] The class of Lévy distributions represents a special class of stable distributions, which has also nontrivial

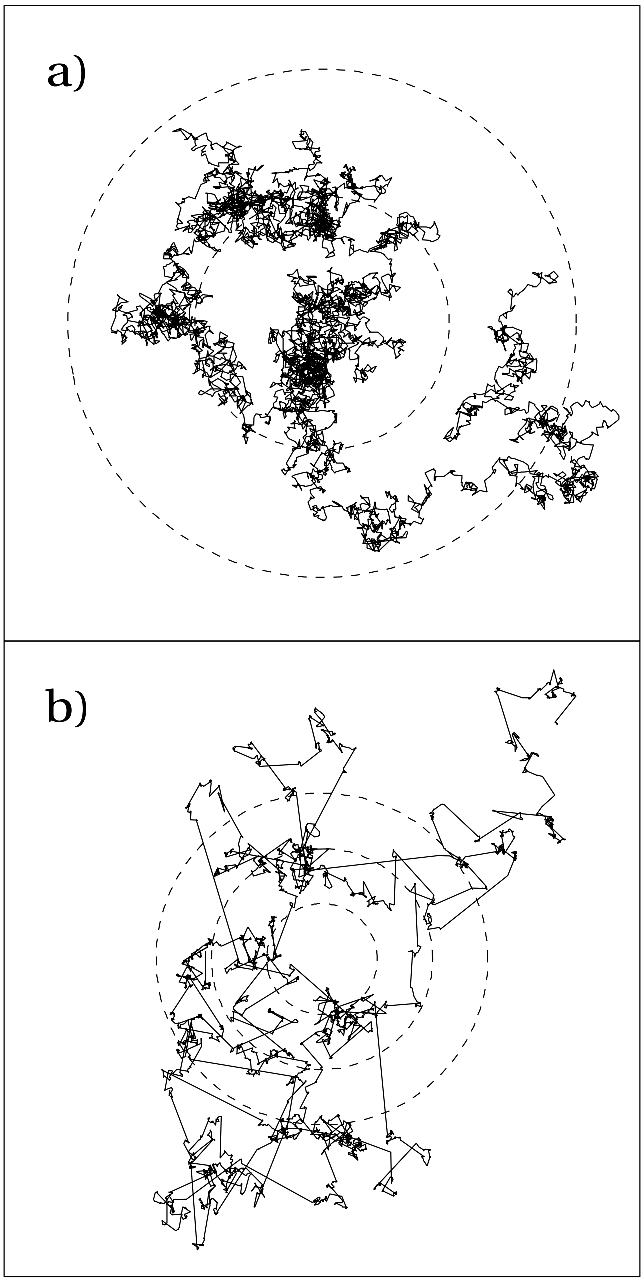

Figure 2. A sample (a) of Brownian two-dimensional (2-D) random walk and (b) of a 2-D Lévy flight simulated using a 2-D version of equation (7). Dashed circles have a radius r = 10 and 20, and r = 20, 40, and 60 in Figures 2a and 2b, respectively.

scaling features. As a matter of fact, if we consider the following scale transformation:

$$\begin{cases} x & \rightarrow & \lambda^a x \\ \gamma & \rightarrow & \lambda^b \gamma \end{cases}, \tag{4}$$

where a and b are real numbers (generally named scaling powers) and λ is a positive parameter, then it is possible to show that symmetrical stable Lévy distributions satisfy the following functional equation:

$$L_\alpha\left(\lambda^a x; \lambda^b \gamma\right) = \lambda L_\alpha(x;\gamma) \tag{5}$$

with $a = -1$ and $b = -\alpha\ \forall\lambda \in R^+$, i.e., Lévy distribution has the property of a generalized homogeneous function (see equation (5)). Thus it is possible by choosing $\lambda = \gamma^{\frac{1}{\alpha}}$ to define a *scaling function* $F(\tilde{x})$,

$$\frac{L_\alpha(x;\gamma)}{\gamma^{-\frac{1}{\alpha}}} = L_\alpha\left(\frac{x}{\gamma^{\frac{1}{\alpha}}};1\right) = F(\tilde{x}), \tag{6}$$

where $\tilde{x} = x/\gamma^{\frac{1}{\alpha}}$. In other words, if we plot $(L_\alpha(x;\ \gamma)/\gamma^{-\frac{1}{\alpha}})$ versus $(x/\gamma^{\frac{1}{\alpha}})$, we observe data collapsing of different PDFs characterized by the same Lévy exponent α. To illustrate this property, let us consider the following stochastic process:

$$x(i+1) = x(i) + \eta(i), \tag{7}$$

where $x(0) = 0$ and $\eta(i)$ is a δ-correlated noise distributed according to a symmetric Lévy distribution $L_\alpha(x;\ \gamma)$ ($\beta = 0$). This process is equivalent to a discrete-time random walk (i.e., a diffusion process) of a particle under the influence of stochastic pulses and is generally named Lévy flight.

[12] Figure 2 shows a comparison between a standard Brownian two-dimensional (2-D) random walk and a 2-D Lévy flight, where the jumps are distributed according to a Gaussian distribution and to a Cauchy distribution (Lévy index $\alpha = 1$), respectively.

[13] Furthermore, the process described by equation (7) is equivalent to a sum of N independent identically distributed stochastic variables, i.e.,

$$x(N) = \sum_{i=0}^{N-1} \eta_i. \tag{8}$$

Thus the probability distribution function $P_N(x;\ \gamma)$ of the stochastic variable x after N steps, written in terms of the characteristic function as

$$P_N(k;\gamma) = \prod_i L_\alpha(k,\gamma) = \exp(-N\gamma|k|^\alpha) \tag{9}$$

reads

$$P_N(x;\gamma) = \frac{1}{N^{\frac{1}{\alpha}}} L_\alpha\left(\frac{x}{N^{\frac{1}{\alpha}}};\gamma\right). \tag{10}$$

In passing we note how equation (10) is equivalent to equation (6).

[14] In Figure 3 we show the evolution of the PDF with increasing N, as evaluated using expression (9). Furthermore, by means of equation (10), we may collapse data reported in Figure 3 in a single master curve. This is exactly what is shown in Figure 4. As a consequence of equation (10), we can write a scaling relationship for the probability of return $P_N(0;\ \gamma)$,

$$P_N(0;\gamma) \sim N^{-\frac{1}{\alpha}}. \tag{11}$$

The equation (11) has a straightforward consequence: if we read the stochastic process described by equation (7) as a diffusion process, then, being $0 < \alpha \leq 2$ (or $\frac{1}{2} \leq \frac{1}{\alpha} < \infty$), we

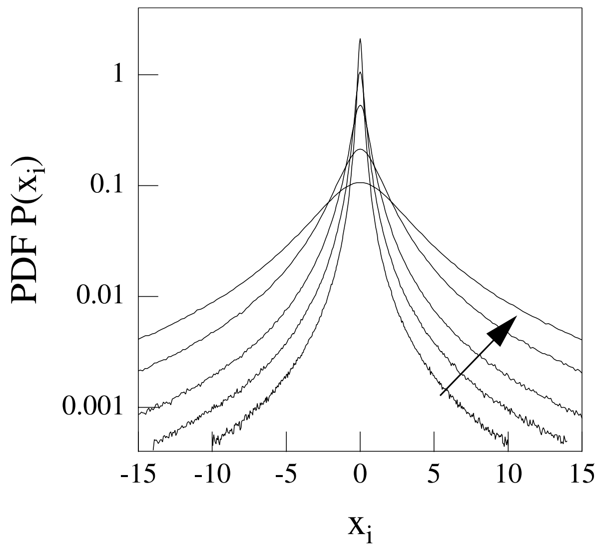

Figure 3. Evolution of the probability distribution functions (PDF) $P(x_i)$ relative to the stochastic process described by equation (7) at different time steps. The arrow indicates increasing time steps (i = 1, 2, 4, 10, and 20). Here, the Lévy exponent is $\alpha = 1$.

are dealing with an anomalous superdiffusive process [*Bouchaud and Georges*, 1990]. Anomalous transport processes can be described as Lévy random walks in terms of integral equations and fractal dynamics [*Klafter et al.*, 1987; *Metzler and Klafter*, 2000]. Also, approaches involving fractional derivatives were developed [*Zaslavsky*, 1994; *Metzler and Klafter*, 2000; *Sokolov and Metzler*, 2003]. *Yanovsky et al.* [2000] have recently shown that in the case of Lévy distributed pulses an anomalous diffusion process can be associated with a fractional Fokker-Planck equation (FFPE) of the form:

$$\frac{\partial p(x,t)}{\partial t} = -\gamma\left[(-\Delta)^{\frac{\alpha}{2}} p(x,t)\right], \tag{12}$$

where $(-\Delta)^{\frac{\alpha}{2}}$ is the Riesz's fractional differential operator, defined as

$$(-\Delta)^{\frac{\alpha}{2}} f(x) = F^{-1}\left(|k|^{\alpha}\hat{f}(k)\right), \tag{13}$$

where F^{-1} denotes the inverse Fourier transform and $\hat{f}(k)$ the Fourier transform of the function $f(x)$. From equation (12) we may note that the scale factor γ of the Lévy distribution related with the stochastic noise in the equation (7) plays the role of a diffusion coefficient in the FFPE.

[15] Although in recent years the Lévy distributions and the corresponding Lévy processes have encountered a good success in the description of the statistical features of a wide variety of observed complex phenomena, the application of Lévy distributions to real data is limited by the divergence of the second and higher moments, which is in contrast with the finite moments of real empirical data. As a matter of fact, in many cases the Lévy shape is limited to small and intermediate values of the variables, while at very large values some cutoff is present so that the moments are not divergent. To overcome such a problem associated with stable Lévy distribution, a different class of distributions, known as truncated Lévy distributions (TLD), was introduced [*Mantegna and Stanley*, 1994; *Koponen*, 1995; *Sornette*, 2000]. TLDs are Lévy-like in the central regime and truncated in the far tail by a function decaying faster than the Lévy distribution. Thus the TLD has finite variance, and as a consequence of the the Central Limit Theorem (CLT), the distribution of the sums of independent variable distributed according to the TLD approaches a Gaussian distribution. Thus it should exist a characteristic scale separating Lévy and Gaussian regimes. As a consequence of its definition, TLD is no longer stable to convolution, and therefore the convolutions of TLDs cannot be collapsed using a simple expression as equation (6) and/or equation (10). In such a case, TLDs will involve multi-scaling properties. As an example of a TLD, we will consider a Lévy distribution with an exponential cutoff [*Koponen*, 1995]:

$$p_{\alpha}^{T}(x) \sim \frac{C\exp\left(-\frac{|x|}{\delta}\right)}{x^{\alpha+1}}, \tag{14}$$

where the expression is valid in the limit $x \to \pm\infty$, C is a coefficient, and δ is the exponential cutoff scale. The corresponding characteristic function is

$$\ln\hat{P}(k) = -\gamma\frac{\left(k^2+\delta^{-2}\right)^{\frac{\alpha}{2}}\cos[\alpha\arctan(\delta|k|)] - \delta^{-\alpha}}{\cos\left(\frac{\pi\alpha}{2}\right)}. \tag{15}$$

[16] Figure 5 shows a comparison between a Lévy distribution and a truncated Lévy distribution characterized by the same characteristic index $\alpha = 1.2$ and the same scaling factor $\gamma = 0.1$. The TLD has a exponential cutoff $\delta = 3$. To show the convergence of a TL process to a

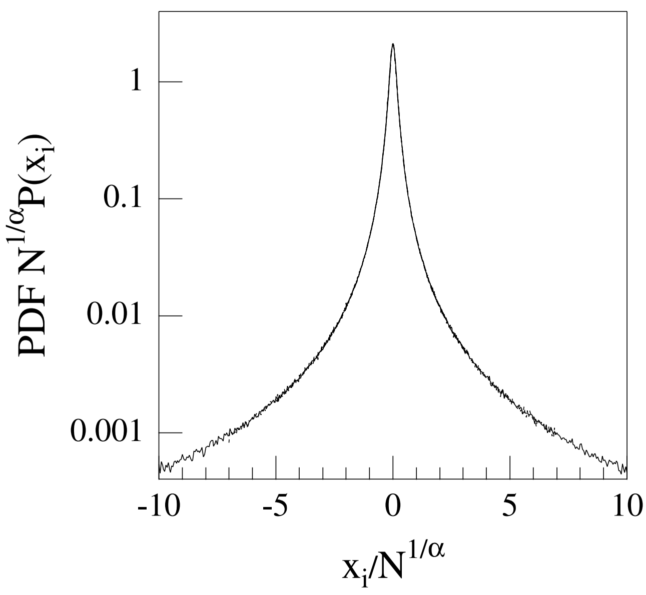

Figure 4. The data collapsing of the PDFs showed in Figure 3 according to equation (6). Here, α = 1 - Cauchy or Lorentz distribution.

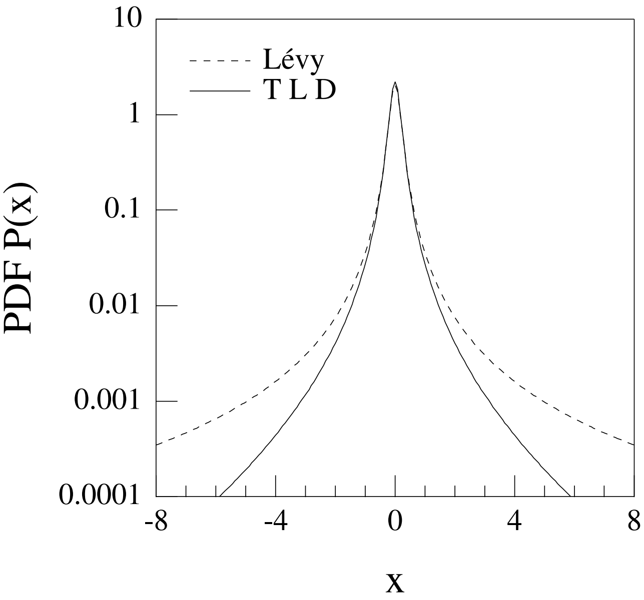

Figure 5. A comparison between a Lévy distribution (dashed line) and a truncated Lévy distribution (solid line) characterized by the same characteristic index α. The used parameters are $\alpha = 1.2$, $\gamma = 0.1$, and $\delta = 3$.

Gaussian process, we have investigated the evolution of the probability of return $P_N(0)$ in the case of a process analogous to the one described by equation (7) or (8). Figure 6 shows the evolution of $P_N(0)$ as a function of N for a simple Lévy flight and a TL process. While for a simple Lévy flight $P_N(0)$ scales according to equation (11) ($P_N(0) \sim N^{-s}$ with $s = 1/\alpha$), for the TL process two asymptotic regimes are found:

$$P_N(0) \sim \begin{cases} N^{-s} & \rightarrow \quad N \ll N_c \\ N^{-\frac{1}{2}} & \rightarrow \quad N \gg N_c \end{cases}, \tag{16}$$

where $s = 1/\alpha$ and the crossover scale $N_c \sim \delta^{\alpha}$. The absence of a single scaling regime implies that in such a case it is not possible to get a single parameter data collapsing as described for the Lévy distribution. Data collapsing is possible only in the asymptotic limit (i.e., $N \ll N_c$ or $N \gg N_c$). This is the evidence of multiscaling features [*Nakao*, 2000].

3. Data Description and Analysis

[17] To investigate the dynamics of magnetic field reconfiguration during magnetotail current disruption, we have considered the best three events of current disruption, as observed by the AMPTE/CCE spacecraft on 13 May 1985 (85/133) at ~2111 UT, on 1 June 1985 (85/152) at ~2314 UT, and on 28 August 1986 (86/240) at ~1152 UT. These CD events have been previously widely studied [*Ohtani et al.*, 1995; *Lui and Najmi*, 1997; *Consolini and Lui*, 1999, 2000].

[18] Data time resolution is $\Delta t \simeq 0.125$ s (see *Potemra et al.* [1985] for details of the instrument). In all the analyzed events the magnetic field latitude angle remains very high during all the period under investigation, indicating that AMPTE/CCE spacecraft was in the current sheet. Thus we will assume that the observed fluctuations are mainly temporal and not spatial. Furthermore, to investigate the dynamics of magnetic field during the CD, we will restrict our study only to the time intervals during which the events take place, and we will concentrate our attention to the evolution of the magnetic field magnitude $B = \sqrt{\sum_i B_i^2}$.

[19] In Table 1 we report the selected time intervals for the three CD events along with some relevant features of the CD locations (distance from the Earth R, the magnetic local time MLT, and the distance from the neutral sheet dZ) and the corresponding average proton cyclotron frequency $\langle f_{ci} \rangle$, evaluated using the average magnetic field intensity $\langle B \rangle$ on the selected time intervals.

[20] Figure 7 shows the actual magnetic field magnitude, observed during the 85/152 CD event. As clearly shown by this figure, during CD the magnetic field relaxes from one configuration (stretched), characterized by an average field of $\sim 8 \div 9$ nT, to a different one (dipolar), with an average field of ~30 nT. Furthermore, if we look at the short timescale dynamics, we may notice the occurrence of intermittent variations of the magnetic field magnitude, i.e., the occurrence of periods of relative stasis punctuated by activity bursts. To underline this point, we have studied intermittency during CD using the Extended Self-Similarity (ESS) analysis [*Benzi et al.*, 1993], which is based on the investigation of the relative scaling of the qth-order generalized structure functions $S_q(\tau)$, defined as follows:

$$S_q(\tau) = \langle |B(t+\tau) - B(t)|^q \rangle, \tag{17}$$

where $\langle \ \rangle$ denotes time average. In detail, ESS consists of determining the relative scaling exponents $\eta_p(q)$ of the qth-order generalized structure functions in respect of the pth-order generalized structure function, i.e.,

$$S_q(\tau) \sim \left[S_p(\tau)\right]^{\eta_p(q)}. \tag{18}$$

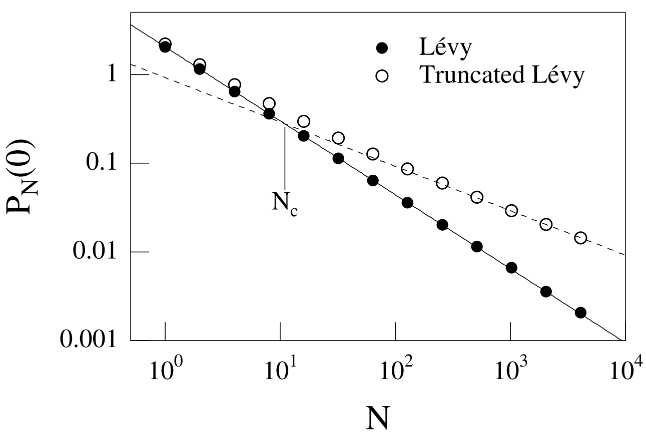

Figure 6. The behavior of the probability of return $P_N(0)$ for a Lévy and a truncated Lévy process as the one described by equation (7). Here, $\alpha = 1.2$, $\gamma = 0.1$, and $\delta = 3$. Solid and dotted lines are power law best fit with scaling exponents $s \sim 0.833$ and $s \sim 0.500$, respectively. N_c indicates the location of the crossover scale.

Table 1. Selected Current Disruption (CD) Events[a]

Event	Time Interval, UT	R, R_E	MLT, hour	dZ, R_E	$\langle f_{ci} \rangle$, Hz
85/133	2111:40–2119:20	7.5	23.9	−0.62	~0.77
85/152	2313:45–2320:15	8.8	0.3	0.22	~0.40
86/240	1152:30–1157:30	8.1	23.4	0.03	~0.40

[a]Here $\langle f_{ci} \rangle$ is the average proton-cyclotron frequency during CD. (Some information is extracted from *Ohtani et al.* [1995].)

[21] In Figure 8 we report the result of the ESS analysis in the case of the CD event 85/133. Figure 8a shows the relative scaling of the qth-order structure functions versus the second-order structure function during CD. This choice may be justified by the fact that for a standard brownian motion the second-order structure function scales linearly with time. A generalized scale invariance in the structure functions is observed. Figures 8b and 8c show the relative scaling exponents $\eta_2(q)$ for a period before and during CD. While before CD $\eta_2(q)$ depends linearly on q, during CD an anomalous scaling (convex dependence on q) is found. This anomalous scaling is the signature of intermittency during CD [*Benzi et al.*, 1993], and the comparison with the linear trend of $\eta_2(q)$ before CD suggests that time intermittency is a peculiar feature of CD. Furthermore, time intermittency is a general behavior of CD as clearly shown in Figure 9, where the dependence of $\eta_2(q)$ on q is shown for the three events here investigated. Thus the magnetic field relaxation toward a dipolar configuration is sustained by fast, intermittent, and stochastic events and may be viewed in terms of a transient motion from one configuration toward another.

[22] This point suggests that we can approach CD by investigating the statistical features of fluctuations of the magnetic field intensity B at different timescales or equivalently the scaling properties of the PDFs of the magnetic field fluctuations $P(\delta B, \tau)$ at different timescales τ. Here, intensity fluctuations are defined as follows:

$$\delta B(\tau) = B(t + \tau) - B(t). \tag{19}$$

[23] In Figure 10 we show the PDFs $P_\tau(\delta B)$ of the magnetic fluctuations $\delta B(\tau)$, evaluated at the shortest available timescale τ = 0.125 s for the three events here considered. The PDFs are almost symmetrical and leptokurtic, i.e., characterized by a non-Gaussian shape with enhanced wings. This result again supports the occurrence of intermittent fast processes during CD. For comparison, we show in Figure 11 the PDFs of the magnetic field fluctuations at the same timescale for a selected interval before CD. In contrast to the PDFs during CD, the PDFs before CD are almost Gaussian, thus confirming that the occurrence of large intermittent small-scale fluctuations is a

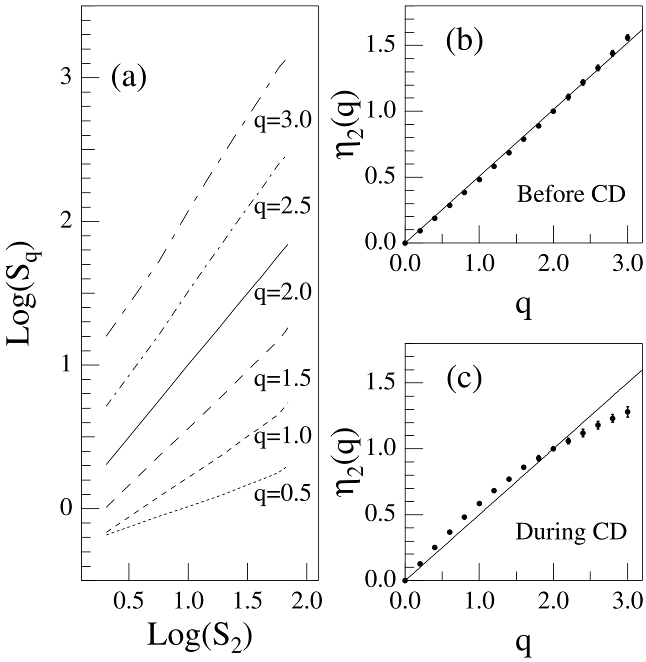

Figure 8. Extended Self-Similarity (ESS) analysis for the 85/133 CD event. (a) The relative scaling of the qth-order structure function $S_q(\tau)$ as a function of the second-order structure function $S_2(\tau)$, for q = 0.5, 1.0, 1.5, 2.0, 2.5, and 3.0, during the CD event. (b) The relative scaling exponents $\eta_2(q)$ versus the moment order q for a period just before CD. Solid line is a linear best fit. (c) The relative scaling exponents $\eta_2(q)$ versus the moment order q during CD. Solid line refers to the linear best fit of Figure 8b.

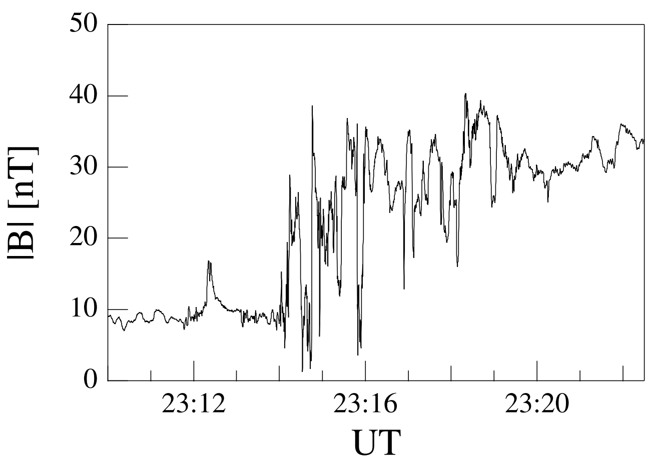

Figure 7. The actual magnetic field magnitude as observed during the current disruption (CD) event (85/152).

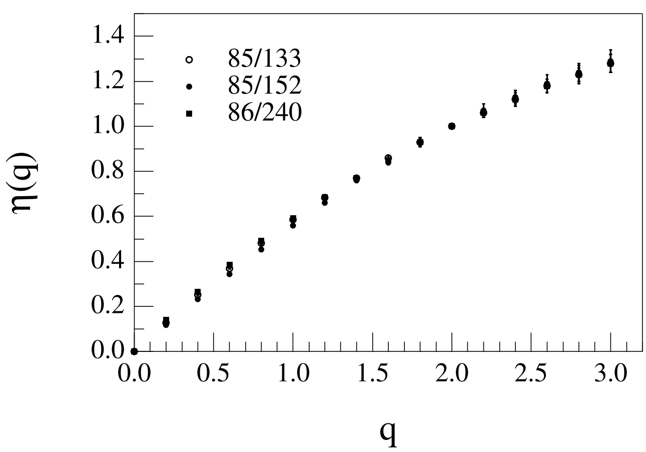

Figure 9. The relative scaling exponents $\eta_2(q)$ versus the moment order q for the three CD events here investigated.

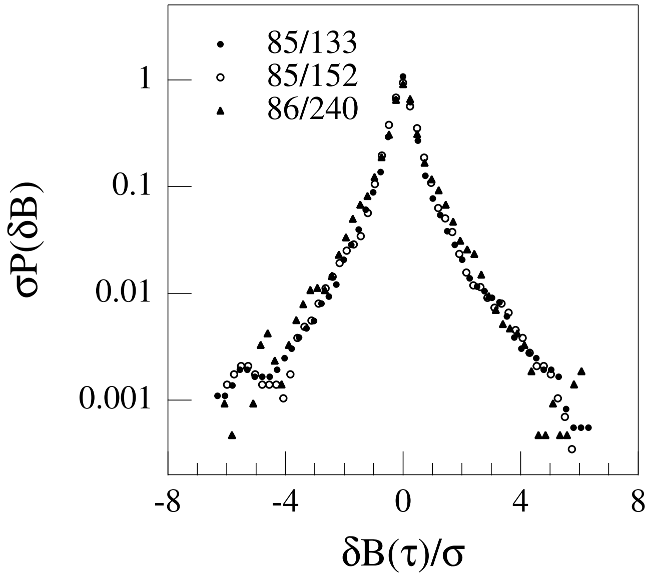

Figure 10. The normalized PDF of the magnetic field fluctuations at the timescale $\tau = 0.125$ s during the CD for the events here considered.

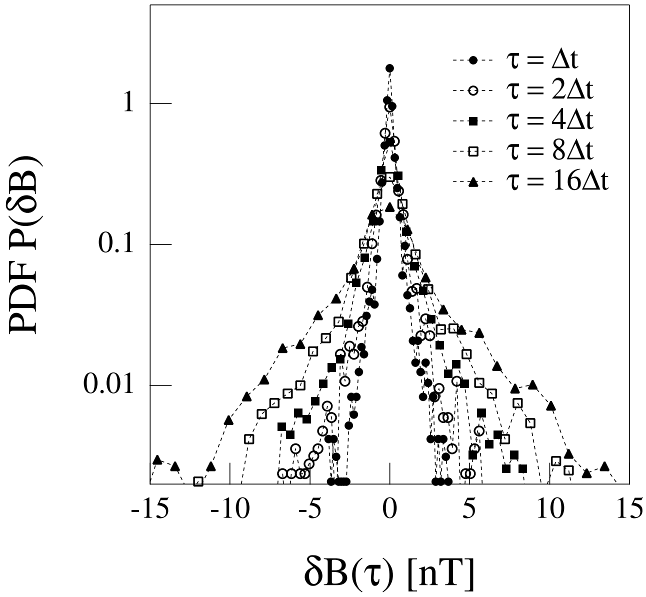

Figure 12. Evolution of the PDF of the magnetic field fluctuations at different timescales τ in the case of the CD event 85/152. τ is in units of data time resolution $\Delta t \simeq 0.125$ s.

peculiar feature of the CD dynamics. Another relevant aspect of these non-Gaussian fluctuations is that they occur on timescales which are below the characteristic proton gyroperiod, thus involving processes in the non-MHD domain.

[24] Figure 12 shows the evolution of the PDF of the magnetic field fluctuations with the scale τ in the case of the event 85/152. PDFs evolve with the timescale τ from a non-Gaussian leptokurtic shape at the smaller non-MHD scales toward a more Gaussian shape at the MHD scales. This behavior resembles that of a TL process, as described in section 2. To characterize the evolution of the PDFs on different timescales, we have studied the scaling features of the so-called probability of return $P_\tau(0)$ with τ.

[25] Figure 13 shows the scaling features of the probability of return P_τ (0) with τ in the case of the 85/152 CD event. Two different asymptotic regimes characterized by different power laws (i.e., $P_\tau\ (0) \sim \tau^{-s}$) are found in the limit of small and large timescales, respectively. In particular, while the large timescale scaling is quite well in agreement with the typical scaling features of P_τ (0) for a normal Gaussian (Brownian) process (here $s \sim 0.5$), in the limit of small timescale the observed scaling resembles the behavior of a Lévy process with $\alpha \sim 1.1$ ($s = \frac{1}{\alpha} \sim 0.89$).

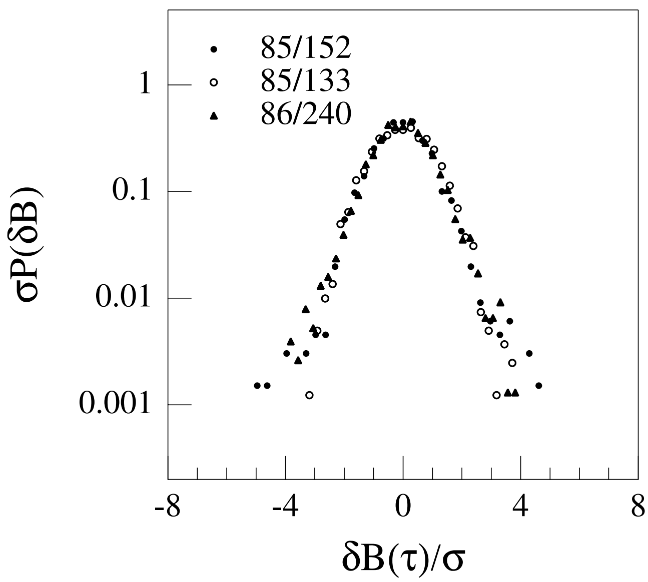

Figure 11. The normalized PDF of the magnetic field fluctuations at the timescale $\tau = 0.125$ s before CD for the events here considered.

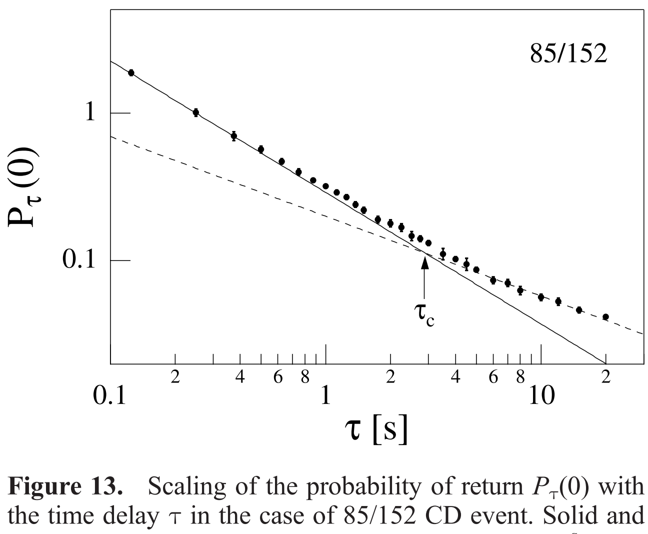

Figure 13. Scaling of the probability of return $P_\tau(0)$ with the time delay τ in the case of 85/152 CD event. Solid and dashed line are power law best fits, $P_\tau(0) \sim \tau^{-s}$, with scaling exponents $s = [0.89 \pm 0.04]$ and $s = [0.54 \pm 0.03]$, respectively. $\tau_c \sim 3.1$ s indicates a crossover time among the two asymptotic regimes.

Table 2. Characteristic Parameters of the Scaling Features of the Probability of Return $P_\tau(0)$ With τ for the Three CD Events

Event	s_1	s_2	τ_c, s
85/133	0.97 ± 0.06	0.51 ± 0.04	1.5 ± 0.7
85/152	0.89 ± 0.06	0.54 ± 0.03	3.0 ± 1.0
86/240	0.99 ± 0.04	0.50 ± 0.04	1.9 ± 0.6

Furthermore, we can identify a crossover characteristic timescale $\tau_c \sim 3$ s. This crossover is quite well in agreement with the average ion gyrofrequency, as evaluated using the average magnetic field during CD and reported in Table 1. In other words the ion gyrofrequency sets a crossover timescale among two different asymptotic regimes: the former related with Lévy statistics of the fluctuations and the latter with Gaussian statistics.

[26] The observed behavior is not peculiar just for the case of the 85/152 CD event but is a general behavior of all the three events here investigated. In Table 2 we report the scaling exponents s_1 and s_2 in the two asymptotic regimes (small and large timescales, respectively) and the corresponding crossover timescale τ_c. This table suggests the existence of a universal behavior in the way how the magnetic field relaxes toward the dipolar configuration during CD. In particular we may observe how at non-MHD scale (i.e., above the ion gyrofrequency) the asymptotic regime is that of Lévy, thus involving an anomalous diffusion process, while at the MHD scales (i.e., below the ion gyrofrequency) a standard diffusion regime is recovered. We are thus observing a gradual change of the diffusion regime as it occurs for the case of a TL process. To stress this point, in Figure 14 we have plotted the trend of the probability of return $P_\tau(0)$ with τ, rescaled as follows:

$$\begin{array}{ccc} \tau & \rightarrow & \tau' = \tau/\tau_c \\ P_\tau(0) & \rightarrow & P_\tau(0)/P_{\tau_c}(0) \end{array}, \tag{20}$$

where $\tau_c \sim 1/\langle f_{ci} \rangle$, for the three CD events. The collapse of the scaled probability of return onto one single curve is the signature of a general and universal behavior of the dynamical features of the processes responsible for the

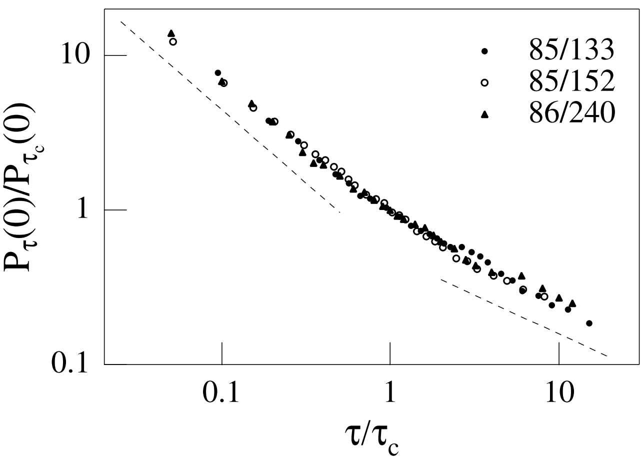

Figure 14. Universal trend of the scaled probability of return (see equation (20)). Dashed lines are power law with scaling exponents $s = 0.95$ and $s = 0.50$, respectively.

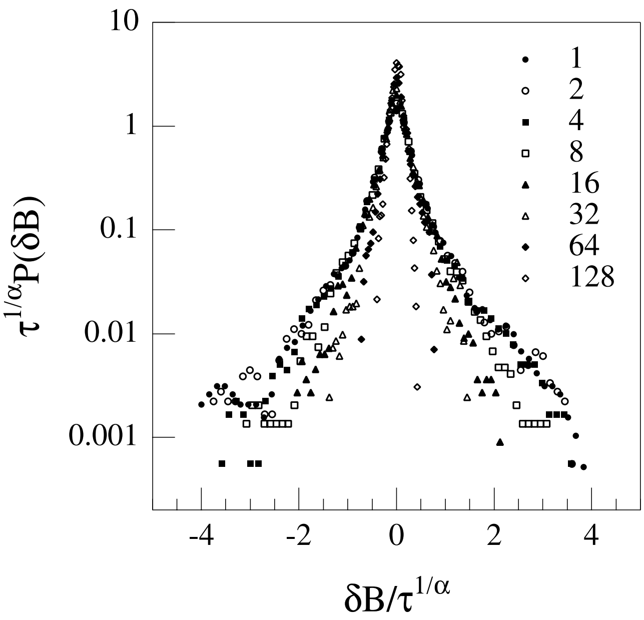

Figure 15. Collapsing of PDFs of magnetic fluctuations at different timescales for the 85/152 CD event. The legend numbers (#) refer to the scale according to the following relationship: $\tau = \#\Delta t$.

Earth's dipolarization phenomenon occurring at the onset of the geomagnetic substorms. The average scaling exponents are s = [0.95 ± 0.05] and s = [0.52 ± 0.02] in the non-MHD and in the MHD domain, respectively. In passing let us underline how this universal behavior of the probability of return could suggest the occurrence of a symmetry breaking. We will return to this point in the next section.

[27] The evidence of a symmetry breaking and of two different scaling regimes separated by a crossover region is also shown by the difficulty to get a global data collapsing, as clearly shown in Figure 15. As a matter of fact, data collapsing should be possible only in the asymptotic limits for $\tau \ll \tau_c$ or $\tau \gg \tau_c$. The consequences of this point will be discussed later in the next section.

[28] In Figure 16 we show a comparison among the PDFs of the smallest timescale fluctuations, Lévy and truncated Lévy (TL) distribution. The good agreement with the TL distribution confirms the previous hypothesis that magnetic field relaxation during CD take the form of a TL process.

4. Discussion

[29] Let us start with a brief resume of the results of the previous section. The study of the relaxation process observed during CD evidenced the existence of two different asymptotic regimes for the probability of return $P_\tau(0)$, characterized by Lévy and Gaussian scaling features, respectively. In detail, this process resembles the behavior of a stochastic truncated Lévy diffusion process involving anomalous diffusion at the smaller timescales (non-MHD domain), a continuous symmetry breaking, and standard diffusion at the longer timescales (MHD domain). Furthermore, because fluctuations at the shorter timescale (i.e., below the crossover timescale τ_c) are intermittent and the

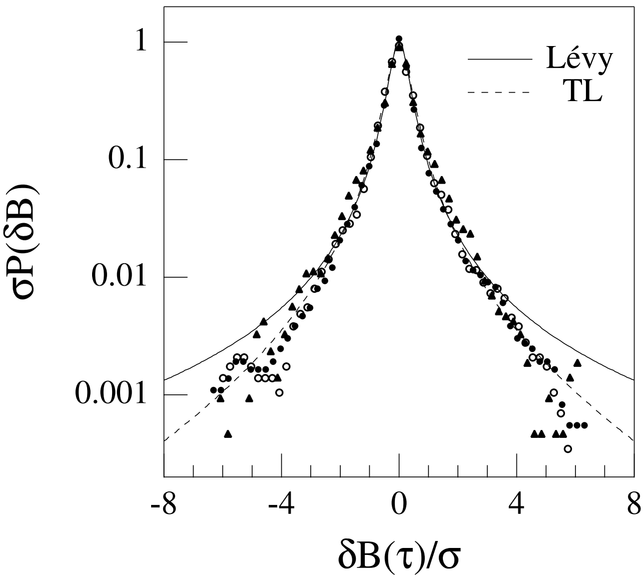

Figure 16. Comparison among shortest timescale PDFs, Lévy distribution with α = 1.05, and a truncated Lévy distribution (TL) with α = 1.05. Solid and dashed lines are nonlinear best fit using expressions (3) and (15), respectively. Fitting parameters are $\gamma = [0.29 \pm 0.01]$, and $\gamma = [0.35 \pm 0.01]$ $\delta = [5.5 \pm 0.3]$ for Lévy and TL PDF, respectively.

magnetic field relaxation toward a dipolar configuration is analogous to an anomalous superdiffusive process, CD seems to be mediated by a sequence of spotty, fast, and intermittent fluctuations occurring in the non-MHD domain. Let us now discuss the physical implication of our results.

[30] First of all, we can comment that the change of the statistics from Gaussian to Lévy (obtained when going from the MHD domain to the non-MHD domain) could be an evidence of the fact that the underlying relaxation process is changing from Markovian (simple) to non-Markovian (complex), involving an increase of the range of interaction and correlation that manifests as fast processes. We underline that the occurrence of fast relaxation process at frequencies above the characteristic ion cyclotron frequency (i.e., in the non-MHD domain where the plasma dynamics is governed by electron motions) again points toward a non-MHD nature of the CD phenomenon [*Lui et al.*, 1999] involving fast processes in high-β noncollisional plasmas.

[31] A candidate mechanism for the CD is the current-driven kinetic instability known as the cross-field current instability, which excites oblique whistler waves [*Lui et al.*, 1991]. Several observational features are consistent with this theory, such as the observed wave frequencies and the timescale for the onset of CD. Quasi-linear calculations of the instability show the resultant anomalous resistivity and current reduction to be consistent with those inferred from observation as well [*Lui et al.*, 1993; *Yoon and Lui*, 1993]. Since the excited waves are oblique whistlers, magnetic reconnection arising from the generated anomalous resistivity is expected to be mediated by whistler waves, a feature which is also consistent with recent findings [*Deng and Matsumoto*, 2001]. These instability analyses were based on the notion that just prior to CD onset the plasma environment renders the ions unmagnetized while electrons remain magnetized.

[32] The above approach is quite compatible with another approach [*Kingsep et al.*, 1990; *Fruchtman and Gomberoff*, 1993] to treat plasma phenomena occurring on timescales fast in comparison to the "ideal" MHD timescales. This new plasma description, named electron-magnetohydrodynamics (EMHD), is a fluid theory describing the behavior of high-β plasmas at timescales shorter than the ion cyclotron period and spatial scales smaller than the ion inertial length d_i, where most of the dynamics of the plasma is governed by electrons. Such a theory is of special interest to model the collisionless reconnection events [*Bulanov et al.*, 1992; *Avinash et al.*, 1998; *Attico et al.*, 2000], which seem to be responsible for the activity observed both in the solar corona and in the Earth's magnetosphere. In this framework, *Biskamp et al.* [1996] introduced a novel type of turbulence in dissipationless plasmas, arising from electron-magnetohydrodynamics. Such a novel EMHD turbulence exhibits some interesting features when it is compared with standard MHD-turbulence. For example, in contrast to the Alfvén effect in MHD turbulence [*Kraichnan*, 1965], the whistler effect is not relevant to the spectral energy transfer. Moreover, two relevant spectral regimes are expected in the limit of long and short wavelength. In detail, in the limit of $kd_e < 1$ (here d_e is the electron inertial length), neglecting the whistler effect, a steeper energy spectrum is recovered, $E_k \sim k^{-7/3}$ [*Biskamp et al.*, 1996]. However, we remark that EMHD is still a fluid theory that cannot address kinetic effects, e.g., Landau damping, finite Larmor radius effect, etc.

[33] In Figure 17 we show the average energy spectrum $E(f)$ in the case of the three CD events here analyzed. A $f^{-7/3}$ region is found below the ion cyclotron frequency. Neglecting the whistler effect and assuming that it could be possible to invoke Taylor's hypothesis (i.e., a linear

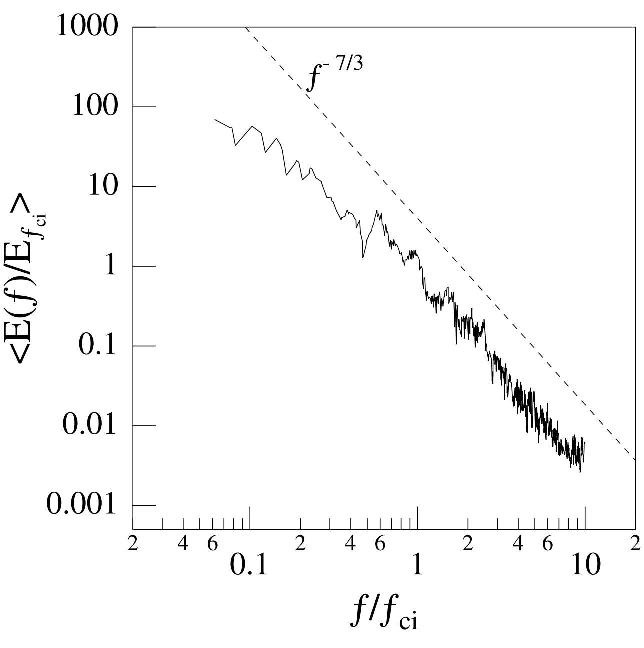

Figure 17. The average energy spectrum scaled to the ion-cyclotron frequency. The dashed line refers to the expected behavior for electron-magnetohydrodynamics (EMHD) turbulence.

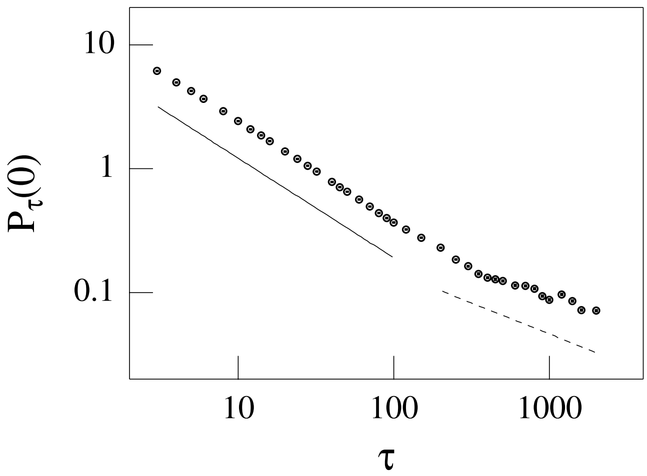

Figure 18. The behavior of the probability of return $P_\tau(0)$ for the PDFs of the fluctuations of the process described by equation (21). Solid and dashed line refer to power laws with scaling exponents $s = -0.83$ and $s = 0.5$, respectively.

relationship between k and f), this result suggests that the turbulent and intermittent magnetic field fluctuations observed during CD might be compatible with the occurrence of EMHD turbulence. Thus the anomalous scaling of the return probability, observed by analyzing the statistical features of the magnetic field fluctuations at the smaller timescales, could be the consequence of fast relaxation events in a turbulent medium governed by electron dynamics.

[34] A slightly different framework for interpreting the results of section 3 could be the stochastic intermittent turbulent scenario proposed by *Chang* [1998a, 1998b, 1998c, 1999, 2001] and his coauthors [*Consolini and Chang*, 2001; *Chang et al.*, 2003, 2004]. Such a scenario approaches the localized and intermittent processes and the anomalous global transport phenomena observed in the magnetotail plasma environment in terms of the development, interaction merging, and evolution of coherent magnetic and plasma structures. These structures, which are bundles of nonpropagating multiscale (i.e., covering a very wide range of frequencies from non-MHD to MHD regimes) fluctuations localized at plasma resonance sites, take the form of cross-tail current filaments in the neutral sheet region. In such a scenario, the stochastic evolution and interaction of these coherent structures would characterize the turbulent dynamics of the tail regions.

[35] As shown by the results of our analysis, the relaxation of the tail magnetic field from a stressed configuration toward a dipolar one is strongly mediated by fast and intermittent processes below the ion gyroperiod which appear in an anomalous scaling of the return probability $P_\tau(0)$ at the corresponding scales. Thus we could conjecture that observed intermittent fluctuations result from the intermixing and interactions of coherent whistler structures, whose coelescence could be responsible for the reconfiguration of the filamentary current structures. In other words, the observed relaxation of the tail magnetic field toward a dipolar one may be viewed in terms of a stochastic coarse-grained dissipation associated with the interaction of these coherent whistler structures. In this picture, the coarse-grained dissipation associated with the spotty fluctuations would involve a strong cross-coupling among fluctuations at different timescales, without involving local $k - \omega$ space interactions as generally suggested in the view of turbulent cascading processes, as well as a sort of inverse cascading process. This point is supported by previous findings [*Consolini and Lui*, 2000] on the occurrence of strong short-lived phase coupling among different high-frequency fluctuations. Furthermore, if we assume that the general dynamical state of the plasma and current sheet is that of a forced and/or self-organized criticality (FSOC) [*Chang*, 1999], the scaling features, observed in the structure functions $S_q(\tau)$ and in the probability of return $P_\tau(0)$, would be a consequence of the self-similar and hierarchical topology of the coherent structures [*Milovanov et al.*, 2001].

[36] Let us now discuss the meaning of the TL distribution of the fluctuations and its connection with the occurrence of a sort of continuous symmetry-breaking as evidenced by the scaling features of the probability of return $P_\tau(0)$. As clearly stated in section 2, for a Lévy distribution with characteristic scaling index α, moments higher than α diverge. In detail, we cannot define the second moment, which means that fluctuations involving infinite energy could be possible. This is clearly nonsense when we refer to any physical system in which simple consideration on the finiteness of these systems corresponds to a finiteness of the fluctuations. In other words, we should observe a cutoff in the power law tails of the Lévy distribution. The existence of a crossover or a rollover to a second branch of the distribution, involving the finiteness of the second moment, will lead to a not stable distribution for the fluctuations that flows from an ideal Lévy distribution below a certain crossover scale to a Gaussian distribution at scales much larger than the crossover.

[37] To clarify the aforementioned points on a possible link of a hierarchy of self-similar structures and the stochastic nature of the observed diffusion process, we have simulated a stochastic process according to the following linear stochastic Langevin equation:

$$\frac{dx(t)}{dt} = -\gamma x(t) + \sigma\eta(t), \tag{21}$$

where γ and σ are parameters related with the decorrelation time ($\gamma = 1/t_c$) and the amplitude of the noise, respectively, and $\eta(t)$ is a δ-correlated noise with zero mean value and distributed according to a power-law distribution $p(\eta)$ defined as follows:

$$P(\eta) \sim \begin{cases} 0 & \rightarrow \quad \eta \le -b \\ |\eta|^{-\mu} & \rightarrow \quad \eta = (-b, -a) \\ \eta^{-\mu} & \rightarrow \quad \eta = (a, b) \\ 0 & \rightarrow \quad \eta \ge b \end{cases}, \tag{22}$$

where $a = 0.01$, $b = 10$, and $\mu = 2.2$. This choice for $P(\eta)$ of small-scale fluctuations distributed according to a power law can be understood in terms of scale-free self-similar hierarchy of whistler coherent structures. In other words, we are assuming that the merging of scale-free self-similar coherent structures in the non-MHD regime will reflect in a scale-free distribution function of the fluctuations at the

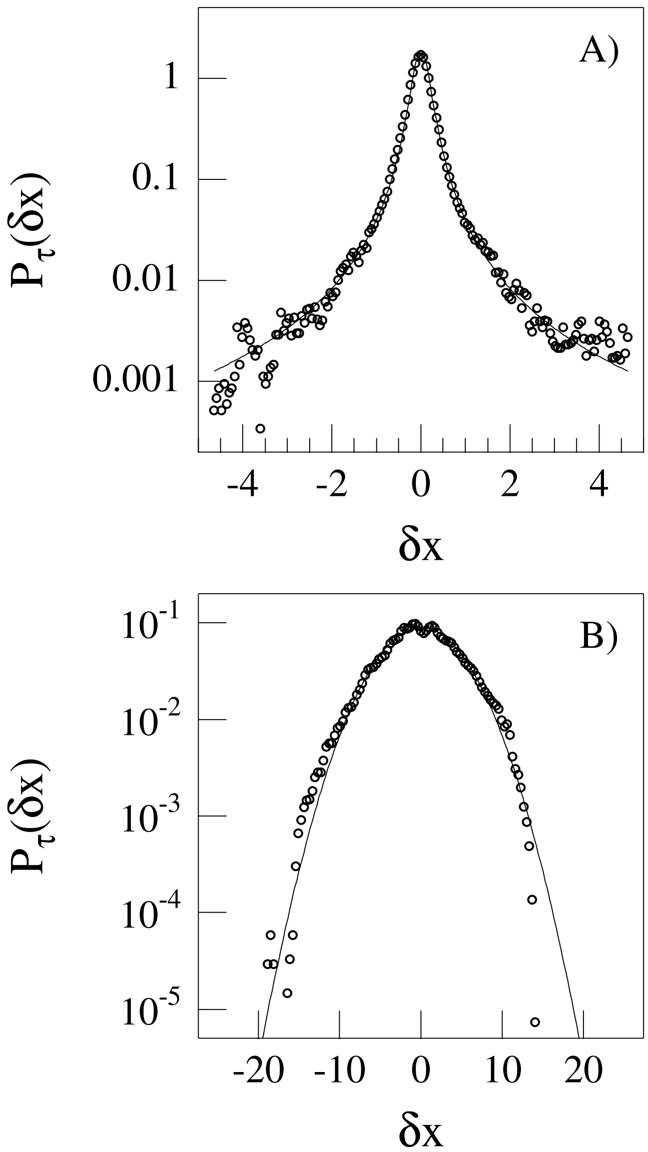

Figure 19. The PDFs of the fluctuations of the process described by equation (21) at two different scales, (a) $\tau = 16$ and (b) $\tau = 1024$. Solid lines refer to nonlinear best fits using respectively a Lévy distribution with characteristic index $\alpha = 1.2$ (Figure 19a) and a Gaussian distribution (Figure 19b).

shorter timescales. Figure 18 shows the behavior of the probability of return $P_\tau(0)$ of the distribution function of fluctuations at different timescales τ. As expected on the basis of what already mentioned, the $P_\tau(0)$ will flow from an anomalous scaling $P_\tau(0) \sim \tau^{-s}$ with $s = 1/(\mu - 1)$ toward the standard scaling of Brownian process $P_\tau(0) \sim \tau^{-1/2}$. Thus the PDF of the fluctuations converges to a Lévy fixed point and successively to a Gaussian stable law (see Figure 19).

[38] The aforementioned simulation clearly shows the relationship between the stochastic coarse-grained dissipation, which follows the coalescence of multiscale coherent structures in a general topological phase transition [*Consolini and Lui*, 1999; *Chang*, 1999, 2001], and the observed symmetry-breaking in the relaxation of the magnetic field toward a dipolar configuration.

5. Summary

[39] In this study we have examined the statistical features of the magnetic field fluctuations and the relaxation of the magnetic field towards a dipolar configuration, as observed by AMPTE/CCE in the near-Earth tail during CD. We applied a rather novel approach based on the analysis of the scaling properties of the probability of return $P_\tau(0)$ at different timescales τ to three events widely studied in the past, with a particular emphasis on the 85/152 event. We found that the scaling of the probability of return $P_\tau(0)$ changes from a nearly Lévy regime to a Gaussian regime from the non-MHD to the MHD domain. This fact indicates that CD is strongly mediated by fast and intermittent relaxation processes occurring in the non-MHD domain, where the nature of the fluctuations and of the relaxation of the tail magnetic field from a stressed configuration toward a dipolar one appears to be a complex phenomenon. Furthermore, the PDFs of the magnetic field fluctuations are very well in agreement with Lévy distribution at the smaller timescales converging to a Gaussian distribution at the larger timescales. This has been interpreted as the occurrence of a dynamical symmetry-breaking related with a truncated Lévy (TL) process.

[40] Our results point toward the non-MHD nature of CD phenomenon, which involves whistler turbulence. Two slightly different physical scenarios have been discussed. One involves the development of EMHD turbulence by a kinetic current-driven instability in generating fluctuations above the ion gyrofrequency. The other invokes stochastic merging of whistler coherent structures. These two scenarios can be considered complementary to each other. The first scenario describes the specific physical process in the EMHD regime responsible for the creation of large magnetic fluctuations at high frequencies. Since multiscale coupling is expected in the CD phenomenon, the simple approach of following the instability growth to its saturation may not provide a precise description of the nonlinear physics at the end of CD, especially in terms of its scaling properties and complexity characteristics. Therefore this first scenario does not preclude that the nonlinear stage of CD could be better described by the FSOC scenario than by the traditional turbulence theory. The FSOC scenario describes well the scaling properties, symmetry breaking, and intermittent relaxation at the nonlinear stage of the observed turbulence. However, it lacks the specification of the physical mechanism for the creation of the coherent whistler structures, which are fundamental entities of the FSOC scenario but that are only present at substorm times.

[41] In closing, although our study does not allow us to identify the trigger mechanism of CD, we believe that our results on the statistical features of the CD fluctuations open a new perspective on the modelling of magnetic substorm initiation based on the relevance of stochasticity in the evolution of such a dynamical process.

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