Observation of the multifractal spectrum at the termination shock by Voyager 1

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[1] We look at the multifractal scaling of fluctuations of the interplanetary magnetic field strength. We analyze the multifractal spectrum in a wide range of heliospheric distances from 7 to 107 AU, including spectra observed by Voyager 1 before and after crossing the termination heliospheric shock. It is worth noting a change of the asymmetry of the spectrum at the termination shock: the spectrum is right-skewed in the outer heliosphere, in contrast to the left-skewed or possibly symmetric spectrum in the heliosheath. We argue that the degree of multifractality falls steadily with the distance from the Sun and is apparently modulated by the solar activity. Our analysis also brings significant additional support to earlier results suggesting that the degree of multifractality of solar wind magnetic turbulence before the shock crossing is greater than that in the heliosheath, where the plasma is in equilibrium, and hence the turbulence may become roughly monofractal. **Citation:** Macek, W. M., A. Wawrzaszek, and V. Carbone (2011), Observation of the multifractal spectrum at the termination shock by Voyager 1, *Geophys. Res. Lett.*, *38*, L19103, doi:10.1029/2011GL049261.

1. Introduction

[2] Starting from seminal works of *Kolmogorov* [1941] and *Kraichnan* [1965] many authors have attempted to recover the observed scaling laws, by using multifractal phenomenological models of turbulence describing distribution of the energy flux between cascading eddies at various scales [*Meneveau and Sreenivasan*, 1987; *Carbone*, 1993; *Frisch*, 1995]. In particular, multifractal scaling of this flux in solar wind turbulence using Helios (plasma) data in the inner heliosphere has been analyzed by *Marsch et al.* [1996]. It is known that fluctuations of the solar magnetic fields may also exhibit multifractal scaling laws. The multifractal spectrum has been investigated using magnetic field data measured *in situ* by Voyager in the outer heliosphere up to large distances from the Sun [*Burlaga*, 1991, 1995, 2001, 2004] and even in the heliosheath [*Burlaga and Ness*, 2010; *Burlaga et al.*, 2005, 2006].

[3] To quantify scaling of solar wind turbulence we have developed a generalized two-scale weighted Cantor set model using the partition technique [*Macek*, 2007; *Macek and Szczepaniak*, 2008], which leads to complementary information about the multifractal nature of the fluctuations as the rank-ordered multifractal analysis [cf. *Lamy et al.*, 2010]. We have investigated the spectrum of generalized dimensions and the corresponding multifractal singularity spectrum depending on one probability measure parameter and two rescaling parameters. In this way we have looked at the inhomogeneous rate of the transfer of the energy flux indicating multifractal and intermittent behavior of solar wind turbulence. In particular, we have studied in detail fluctuations of the velocity of the flow of the solar wind, as measured in the inner heliosphere by Helios [*Macek and Szczepaniak*, 2008], Advanced Composition Explorer (ACE) [*Szczepaniak and Macek*, 2008], and Voyager in the outer heliosphere [*Macek and Wawrzaszek*, 2009], including Ulysses observations at high heliospheric latitudes [*Wawrzaszek and Macek*, 2010].

[4] In December 2004 at distances of 94 AU from the Sun Voyager 1 crossed the termination heliospheric shock separating the Solar System plasma from the surrounding heliosheath and entered the subsonic solar wind, where it encountered quite unusual conditions. It is worth noting that magnetic fields in the heliosheath are normally distributed in contrast to the lognormal distribution in the outer heliosphere [*Burlaga et al.*, 2005]. It has also appeared that the magnetic field in the near heliosheath, at ~95 AU, has a multifractal structure [*Burlaga et al.*, 2006].

[5] The results of the generalized dimensions and the multifractal spectrum obtained using the Voyager 1 data of the magnetic field strength have been discussed at distances of 83.4–85.9 AU from the Sun, i.e., before the termination shock crossing [*Burlaga*, 2004], and in the heliosheath at 94.2–97.2 AU [*Burlaga et al.*, 2006] and 108.5–112.1 AU [*Burlaga and Ness*, 2010], correspondingly. The comparison of the weighted one-scale and two-scale Cantor set models and other formulae fitting the experimental data have been presented by *Macek and Wawrzaszek* [2010, Figures 3–6], where the dependence on the various parameters of these models and the resulting degrees of multifractality and asymmetry have also been thoroughly discussed. In particular, space filling turbulence has been recovered [*Burlaga et al.*, 1993].

[6] The aim of this study is to examine the question of scaling properties of intermittent solar wind turbulence using our weighted two-scale Cantor set model at a wide range of the heliospheric distances focusing on the solar cycle variations. In particular, we show that the degree of multifractality modulated by the solar activity is also decreasing with distance: before shock crossing is greater than that in the heliosheath. Moreover, we demonstrate that the multifractal spectrum is asymmetric before shock crossing, in contrast to

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the nearly symmetric spectrum observed in the heliosheath; the solar wind in the outer heliosphere may exhibit strong asymmetric scaling.

2. Multifractal Model

[7] The generalized dimensions D_q as a function of a continuous index q, $-\infty < q < +\infty$, and the singularity multifractal spectrum $f(\alpha)$ as a function of the singularity strength α quantify multifractality of a given system [*Grassberger*, 1983; *Grassberger and Procaccia*, 1983; *Hentschel and Procaccia*, 1983; *Halsey et al.*, 1986]. Consequently, the multifractals are described by an (uncountably) infinite number of dimensions, D_q, and by the related multifractal spectrum $f(\alpha)$. In multifractal theory it can be proved that D_q is a monotonically decreasing function of q, which is constant only for a monofractal. It is also worth noting a surprisingly universal character of $f(\alpha)$, which is a (downward) concave function of α [e.g., *Ott*, 1993, Figure 9.1]. The width of this function, $\Delta \equiv \alpha_{\max} - \alpha_{\min} = D_{-\infty} - D_{\infty}$, has been identified as the degree of multifractality and intermittency [*Macek*, 2007; *Macek and Wawrzaszek*, 2009], which is somehow related to other measures of intermittency, e.g., skewness and kurtosis [*Frisch*, 1995; *Carbone*, 1994; *Szczepaniak and Macek*, 2008].

[8] Let us now consider a simple multifractal. We take an initial unit closed interval divided into any two closed subintervals, where the probability of choosing one subinterval of size l_1 is p and for the other subinterval of size l_2 is $1 - p$ [*Macek and Szczepaniak*, 2008; *Macek and Wawrzaszek*, 2009]. The generalized (weighted with a parameter p) Cantor (uncountable) set is obtained iterating this procedure infinitely many (countable) number of times. In this limit for any q one obtains the dimensions D_q by solving numerically the following transcendental equation

$$\frac{p^q}{l_1^{\gamma(q)}} + \frac{(1-p)^q}{l_2^{\gamma(q)}} = 1, \qquad (1)$$

where $\gamma(q)$ is the scaling exponent related to D_q by $\gamma(q) \equiv (q-1)D_q$, and $\Delta = |\log(1-p)/\log l_2 - \log(p)/\log l_1|$.

[9] Since D_q is constant only for $p = 0.5$, $l_1 = l_2 = 0.5$ (monofractal), we see that for unequal two scales the parameter p quantifies multifractality [*Macek*, 2007]. In addition, the singularity multifractal spectrum $f(\alpha) = q\alpha - \gamma(q)$ as a function of the singularity strength α, which is the derivative of the scaling function, $\alpha = \gamma'(q)$, can be obtained directly [*Chhabra and Jensen*, 1989] from the slopes of generalized measures [e.g., *Macek and Wawrzaszek*, 2009, Figure 7]. Using α_0 where $f(\alpha_0) = 1$, we can define a measure of asymmetry $A \equiv (\alpha_0 - \alpha_{\min})/(\alpha_{\max} - \alpha_0)$ [*Macek and Wawrzaszek*, 2009]. In particular, using two equal scales $l_1 = l_2 = 0.5$ we have the symmetric multifractal spectrum, $A = 1$.

[10] In the case of magnetic field fluctuations the generalized probability measures are related to inhomogeneity with which the scaling exponents depend on various scales. In this way the generalized dimensions provide information about dynamics of magnetic field turbulence. In particular, high positive values of q emphasize regions of intense fluctuations larger than the average, while negative values of q accentuate fluctuations lower than the average [*Burlaga*, 1995].

3. Data Analysis

[11] Here we would like to test the multifractal scaling of the interplanetary magnetic field strength for the wealth of data provided by deep space missions. We analyze time series of the magnetic field fluctuations measured by Voyager 1 at a wide range of distances before the termination shock crossing (during years 1980–2003), namely between 7 and 89 AU from the Sun, and subsequently (2005–2008) at 94–107 AU, i.e., in the heliosheath.

[12] Let us take a stationary magnetic field B in the equatorial plane. Nevertheless, the velocity of the solar wind plasma, $v_{\rm sw}$, flowing in the radial direction is reduced at the termination shock, the magnetic field is mainly in the azimuthal direction, this means, perpendicular to the flow. Therefore, Taylor's hypothesis can still be invoked also in the heliosheath, so that we can decompose this signal into time intervals of size τ corresponding to the spatial scales $l = v_{\rm sw}\tau$. Then to each time interval one can associate a magnetic flux past the cross-section perpendicular to the plane during that time. In every considered year we use a discrete time series of daily averages, which is normalized so that we have $\langle B(t) \rangle = \frac{1}{N}\sum_{i=1}^{N} B(t_i) = 1$, where $i = 1,\ldots, N = 2^n$ (we take $n = 8$). Next, given this (normalized) time series $B(t_i)$, to each interval of temporal scale τ (using $\tau = 2^k$, with $k = 0, 1,\ldots, n$) we associate some probability measure $p_j(\tau) = \frac{1}{N}\sum_{i=1+(j-1)\tau}^{j\tau} B(t_i)$, where $j = 2^{n-k}$, i.e., calculated by using the successive average values $\langle B(t_i, \tau)\rangle$ of $B(t_i)$ between t_i and $t_i + \tau$ [*Burlaga et al.*, 2006]. Further, in some region the q-order total probability measure (the partition function) should scale as $\sum p_j^q(\tau) \sim \tau^{\gamma(q)}$, with $\gamma(q)$ given in equation (1). In this case *Burlaga* [1995] has shown that the average value of the qth moment of $B(t)$ at various times scales τ scales as

$$\langle B^q(t,\tau)\rangle \sim \tau^{s(q)}, \qquad (2)$$

with the similar exponent $s(q) = (q-1)(D_q-1)$.

4. Results

[13] For a given q, using the slopes $s(q)$ of $\log_{10}\langle B^q\rangle$ versus $\log_{10}\tau$ in the range of scales from 2 to 16 days, one can obtain the values of D_q as a function of q according to equation (2). Equivalently, as discussed in Sec. 2, the multifractal spectrum $f(\alpha)$ as a function of scaling indices α indicates universal multifractal scaling behavior. In this paper the results for the multifractal spectrum $f(\alpha)$ obtained using the Voyager 1 data of the solar wind magnetic fields in the distant heliosphere beyond the planets, at 50 AU (1992, diamonds) and 90 AU (2003, triangles), and after crossing the heliospheric shock, at 95 AU (2005, diamonds) and 105 AU (2008, triangles), are presented in Figures 1a and 1b, correspondingly. It is worth noting a change of the symmetry of the spectrum at the shock relative to its maximum at a critical singularity strength $\alpha = 1$. Because the density of the measure $\varepsilon \propto \tau^{\alpha-1}$, this is related to changing properties of the magnetic field density ε at the termination shock. Consequently, a concentration of magnetic fields shrinks resulting in thinner flux tubes or stronger current concentration in the heliosheath.

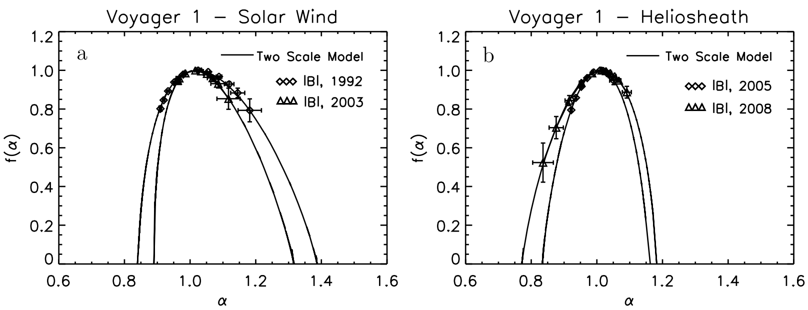

Figure 1. The multifractal singularity spectrum of the magnetic fields observed by Voyager 1 (a) in the solar wind near 50 and 90 AU (1992, diamonds, and 2003, triangles) and (b) in the heliosheath near 95 and 105 AU (2005, diamonds, and 2008, triangles) together with a fit to the two-scale model (solid curve), suggesting change of the symmetry of the spectrum at the termination shock.

[14] We are looking for the degree of multifractality Δ in the heliosphere as a function of the heliospheric distances during solar minimum (MIN), solar maximum (MAX), declining (DEC) and rising (RIS) phases of solar cycles. The obtained values of Δ roughly follow the fitted periodically decreasing function of time (in years, dotted), $20.27 - 0.00992t + 0.06 \sin((t - 1980)/(2\pi(11)) + \pi/2)$, with the corresponding averages shown in Figure 2 by continuous lines. The crossing of the termination shock (TS) by Voyager 1 is marked by a vertical dashed line. Below are shown the Sunspot Numbers (SSN) during years 1980–2008. We see that the degree of multifractality falls steadily with distance and is apparently modulated by the solar activity, as noted by *Burlaga et al.* [2003].

[15] We have already demonstrated that the multifractal scaling is asymmetric in the outer heliosphere [*Macek and Wawrzaszek*, 2009]. Now, the degree of asymmetry A of this multifractal spectrum in the heliosphere as a function of the phases of solar cycles is shown in Figure 3; the value $A = 1$ (dotted) corresponds to the one-scale symmetric model. One sees that in the heliosphere only one of three points above unity is at large distances from the Sun. In fact, inside the outer heliosphere prevalently $A < 1$ and only once (during the declining phase) the left-skewed spectrum ($A > 1$) was clearly observed. Anyway, it seems that the right-skewed spectrum ($A < 1$) before the crossing of the termination shock is preferred. As expected the multifractal scaling is asymmetric before shock crossing with the calculated degree of asymmetry at distances $70 - 90$ AU equal to $A = 0.47 - 0.96$. It also seems that the asymmetry is probably changing when crossing the termination shock ($A = 1.0 - 1.5$) as is also illustrated in Figure 1, but owing to large errors bars and a very limited sample, symmetric spectrum is still locally possible in the heliosheath [cf. *Burlaga and Ness*, 2010].

[16] For comparison, the values calculated from the papers by *Burlaga et al.* [2006] and *Burlaga and Ness* [2010] (LB) are also given in Table 1. One sees that the degree of multifractality for fluctuations of the interplanetary magnetic field strength obtained from independent types of studies are in surprisingly good agreement; generally these values are smaller than that for the energy rate transfer in the turbulence

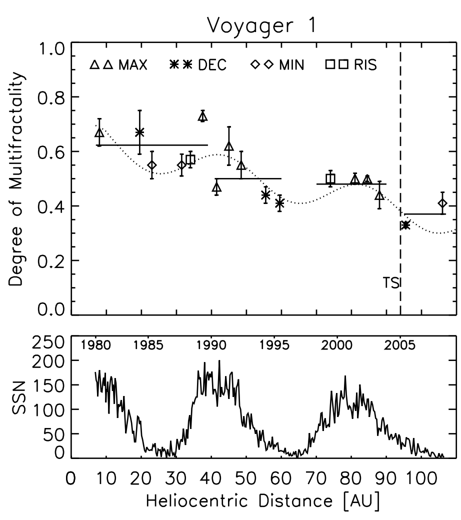

Figure 2. The degree of multifractality Δ in the heliosphere versus the heliospheric distances compared to a periodically decreasing function (dotted) during solar minimum (MIN) and solar maximum (MAX), declining (DEC) and rising (RIS) phases of solar cycles, with the corresponding averages shown by continuous lines. The crossing of the termination shock (TS) by Voyager 1 is marked by a vertical dashed line. Below is shown the Sunspot Number (SSN) during years 1980–2008.

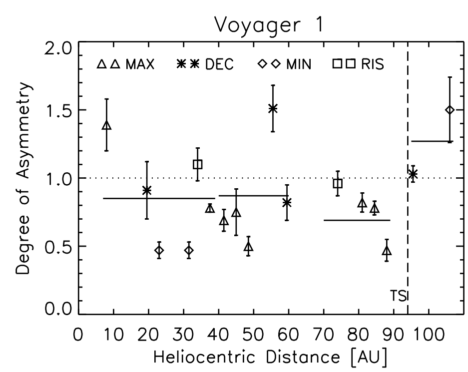

Figure 3. The degree of asymmetry A of the multifractal spectrum in the heliosphere as a function of the heliospheric distance during solar minimum (MIN) and solar maximum (MAX), declining (DEC) and rising (RIS) phases of solar cycles, with the corresponding averages denoted by continuous lines; the value $A = 1$ (dotted) corresponds to the one-scale symmetric model. The crossing of the termination shock (TS) by Voyager 1 is marked by a vertical dashed line.

Table 1. The Degree of Multifractality Δ and Asymmetry *A* of the Multifractal Spectrum for the Magnetic Field Strength Observed by Voyager 1 at Various Heliospheric Distances[a]

Heliocentric Distance	Year	Multifractality Δ(WM)	Δ(LB)	Asymmetry *A*(WM)	*A*(LB)
7 – 40 AU	1980–1989	0.55 – 0.73	0.64	0.47 – 1.39	0.69
40 – 60 AU	1990–1995	0.41 – 0.62	0.69	0.51 – 1.51	0.63
70 – 90 AU	1999–2003	0.44 – 0.50	0.69	0.47 – 0.96	0.63
95 – 107 AU	2005–2008	0.33 – 0.41	0.34	1.03 – 1.51	0.89

[a]Before and after crossing the termination shock, as calculated by L. Burlaga (LB) and by the authors of this paper (WM).

cascade (Δ = 2 − 3) [*Macek and Wawrzaszek*, 2009]. Moreover, it is worth noting that our values obtained before the shock crossing, Δ = 0.4 − 0.7, are somewhat greater than those for the heliosheath Δ = 0.3–0.4. This confirms the results presented by *Burlaga et al.* [2006] and *Burlaga and Ness* [2010]. In this way we have provided a supporting evidence that the magnetic field behavior in the outer heliosphere, even in a very deep heliosphere, may exhibit a multifractal scaling, while in the heliosheath smaller values indicate possibility toward a monofractal behavior, implying roughly the same density of the probability measure.

5. Conclusions

[17] Using our weighted two-scale Cantor set model, which is a convenient tool to investigate the asymmetry of the multifractal spectrum, we confirm the characteristic shape of the universal multifractal singularity spectrum. In fact, as seen in Figure 1, $f(\alpha)$ is a downward concave function of scaling indices α.

[18] Basically, here for the first time we show that the degree of multifractality for magnetic field fluctuations of the solar wind falls steadily with the distance from the Sun and seems to be modulated by the solar activity. Moreover, in contrast to the right-skewed asymmetric spectrum with singularity strength $\alpha > 1$ inside the heliosphere, the spectrum becomes more left-skewed, $\alpha < 1$, or approximately symmetric after the shock crossing in the heliosheath, where the plasma is expected to be roughly in equilibrium in the transition to the interstellar medium. In particular, we also confirm the results obtained by *Burlaga et al.* [2006] that before the shock crossing, especially during solar maximum, turbulence is more multifractal than that in the heliosheath. Admittedly, the multifractal spectrum could possibly be owing to the interactions among shocks and similar relatively large-scale features that are produced by dynamical processes, which are quite distinct from turbulence.

[19] In addition, outside the heliosphere during solar minimum the spectrum seems to be dominated by values of $\alpha < 1$. This is very interesting because it represents a first direct *in situ* information of interest in the astrophysical context beyond the heliosphere. In fact, a density of measure dominated by $\alpha < 1$ implies that the magnetic field in the very local interstellar medium is roughly confined in thin filaments of high magnetic density. That is the heliosheath is possibly more dominated by 'voids' of magnetic fields, thus implying that magnetic turbulence tends to be more 'passive' [cf. *Frisch*, 1995] in the very local interstellar medium. This information, obtained *in situ* rather than through scintillations observations, is relevant in the context of interstellar turbulence confirming stellar formation modeling [e.g., *Spangler*, 2009] related to the presence of very localized intense magnetic field structures.

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