Multifractal Turbulence at the Termination Shock

Wiesław M. Macek[*†,**] and Anna Wawrzaszek[†]

[*]*Faculty of Mathematics and Natural Sciences, Cardinal Stefan Wyszyński University, Wóycickiego 1/3, 01-938 Warsaw, Poland*
[†]*Space Research Centre, Polish Academy of Sciences, Bartycka 18 A, 00-716 Warsaw, Poland*
[**]*macek@cbk.waw.pl*

Abstract. We consider the multifractal spectra of fluctuations of the interplanetary magnetic field strength before and after shock crossing by Voyager 1 near 85 and 95 AU from the Sun, correspondingly. We show that the multifractal scaling of magnetic fields is asymmetric in the outer heliosphere, in contrast to the symmetric spectrum observed in the heliosheath. Moreover, we show that the degree of multifractality of the solar wind before shock crossing is greater than that in the heliosheath, where the turbulence may become roughly monofractal.

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INTRODUCTION

Starting from seminal works of Kolmogorov (1941) and Kraichnan (1965) many authors have attempted to recover the observed scaling exponents, using multifractal phenomenological models of turbulence describing distribution of the energy flux between cascading eddies at various scales [1, 2, 3, 4, 5]. It is known that fluctuations of the solar magnetic fields may also exhibit multifractal scaling laws. In particular, the multifractal spectrum has been investigated using magnetic field data measured by Voyager in the outer heliosphere [6, 7, 8, 9] and using Helios (plasma) data in the inner heliosphere [10]. In addition, the times series of the magnetic field strengths measured in situ by Voyager 1 spacecraft at very large distances from the Sun and even in the heliosheath have already been analyzed [11, 12].

To quantify scaling of solar wind turbulence on small scales, we have developed a generalized weighted two-scale Cantor set model using the partition technique [13, 14]. We have already studied the inhomogeneous rate of the transfer of the energy flux indicating multifractal and intermittent behavior of solar wind turbulence in the inner and outer heliosphere using fluctuations of the velocity of the flow of the solar wind. We have investigated the resulting spectrum of generalized dimensions and the corresponding multifractal singularity spectrum depending on one probability measure parameter and two rescaling parameters [14, 15, 16].

In particular, we have shown that the generalized dimensions for solar wind are consistent with the generalized p model for both positive and negative indices of the generalized dimensions, q, but rather with different scaling parameters for sizes of eddies, while the usual p model can only reproduce the spectrum for $q \geq 0$ [14]. We have demonstrated that in this way a much better agreement with the solar wind velocity data is obtained. It is worth noting that the multifractal scaling is often rather asymmetric. Both the degree of multifractality and degree of asymmetry are correlated with the heliospheric distance and we observe the evolution of multifractal scaling in the outer heliosphere [16].

In 2004 at distances of 85 AU from the Sun Voyager 1 crossed the termination heliospheric shock separating the Solar System plasma from the surrounding heliosheath, and entered the subsonic solar wind, where it has encountered quite unusual conditions [17]. It is worth noting that magnetic fields in the heliosheath are normally distributed in contrast to the lognormal distribution in the outer heliosphere [17]. It also appears that the magnetic field in the inner heliosheath, at $\sim$95 AU, has a multifractal structure on large scales from $\sim$2 to 16 days [12].

The aim of this study is to examine the question of scaling properties of intermittent solar wind turbulence on large scales using our weighted two-scale Cantor set model. We compare the Burlaga's fit to the multifractal spectrum [11] with the two-scale model at the heliospheric boundary [14]. In particular, we show that the degree of multifractality for fluctuations of the interplanetary magnetic field strength before shock crossing is greater than that in the heliosheath. Moreover, we demonstrate that the multifractal spectrum is asymmetric before shock crossing, in contrast to the symmetric spectrum observed in the heliosheath; the solar wind in the outer heliosphere may exhibit strong asymmetric scaling. It is worth noting that for the multifractal two-scale Cantor set model a good agreement with the data is obtained. Hence we propose this new model as a useful tool for analysis of intermittent turbulence also at the heliospheric boundaries.

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SOLAR WIND DATA

Here we would like to test asymmetry of the multifractal scaling of the interplanetary magnetic field strength for the wealth of data provided by Voyager mission. Namely, we analyze time series of the magnetic field fluctuations measured by Voyager 1, as used by Burlaga (2004, 2006) at distances of 83.4 – 85.9 AU from the Sun (from day 1 to 256 (2^8), 2002), i.e. before the termination shock crossing [11], and in the heliosheath at 94.2 – 97.2 AU (from day 1 to 256, 2005) [12], correspondingly.

MULTIFRACTAL MODEL

The generalized dimensions D_q as a function of index q are important characteristics of complex dynamical systems; they quantify multifractality of a given system [18, 19, 20, 21, 22]. In the case of magnetic field fluctuations these generalized measures are related to inhomogeneity with which the scaling exponents depend on various scales. In this way the generalized dimensions provide information about dynamics of magnetic field turbulence. In particular, high positive values of q emphasize regions of intense fluctuations larger than the average, while negative values of q accentuate fluctuations lower than the average [7].

Therefore, given a (normalized) time series $B(t)$ it can be argued that in some region the average value of the qth moment at various times scales $\tau_i = 2^i$, where $i = 0, 1, \ldots, 8$, should scale with the exponent $s(q) = (q-1)(D_q - 1)$ as [7]

$$\langle B_i^q \rangle \sim \tau_i^{s(q)}. \tag{1}$$

Let us now consider the generalized weighted Cantor set, where the probability of visiting one segment of size l_1 is p and for the other segment of size l_2 is $1-p$ as used in [14, 15, 16]. For any q one obtains $D_q \equiv \gamma(q)/(q-1)$ by solving numerically the following transcendental equation, e.g., [22]

$$\frac{p^q}{l_1^{\gamma(q)}} + \frac{(1-p)^q}{l_2^{\gamma(q)}} = 1. \tag{2}$$

The dependence of the resulting spectrum of the generalized dimensions D_q for the two-scale weighted Cantor set model are illustrated in Figures 1 and 2. It can be proved that this is a monotonic function, which is constant only for a monofractal, $p = 0.5$, $l_1 = l_2 = 0.5$. For unequal two scales the parameter p quantifies multifractality. Therefore a degree of multifractality could be defined as $\Delta = D_{-\infty} - D_{\infty}$ [13]. In addition, the singularity multifractal spectrum $f(\alpha) = q\alpha - \gamma(q)$ as a function of $\alpha = \gamma'(q)$ can also be obtained by using Legendre transformation [22], or directly from the slopes or generalized

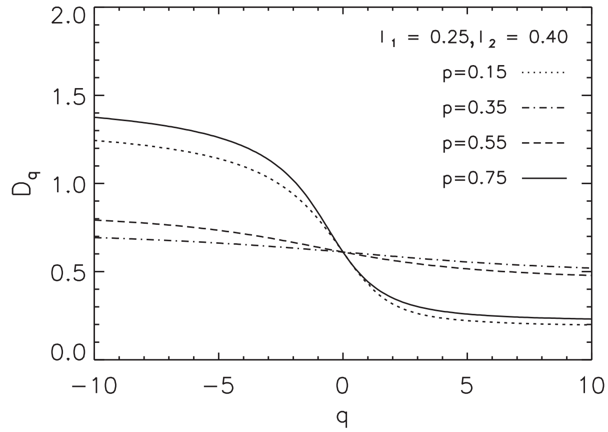

FIGURE 1. The generalized dimensions D_q. The values of D_q are calculated numerically for the two-scale weighted Cantor set using different values of p.

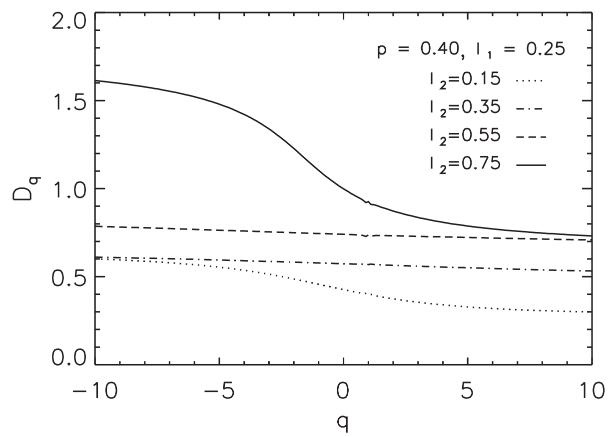

FIGURE 2. The values of D_q and $f(\alpha)$ are calculated numerically for the two-scale weighted Cantor set using different values of l_2.

measures [16]. Using α_0, where $f(\alpha_0) = 1$, we can define a measure of asymmetry $A \equiv (\alpha_0 - \alpha_{\min})/(\alpha_{\max} - \alpha_0)$ [16]. In particular, using two equal scales $l_1 = l_2 = 0.5$ we have the symmetric multifractal spectrum, $A = 1$.

RESULTS

For a given q, using the slopes $s(q)$ of $\log_{10}\langle B_i^q \rangle$ versus $\log_{10}\tau_i$ one can obtain the values of D_q according to Equation (1), which indicate multifractal scaling behavior. The results for the generalized dimensions D_q as a function of q obtained using the Voyager 1 data of the magnetic field strength at distances of 83.4–85.9 AU from the Sun are presented in Figure 3 and the degree of multifractality Δ and the degree of asymmetry A are listed in Table 1. These obtained values in the scaling re-

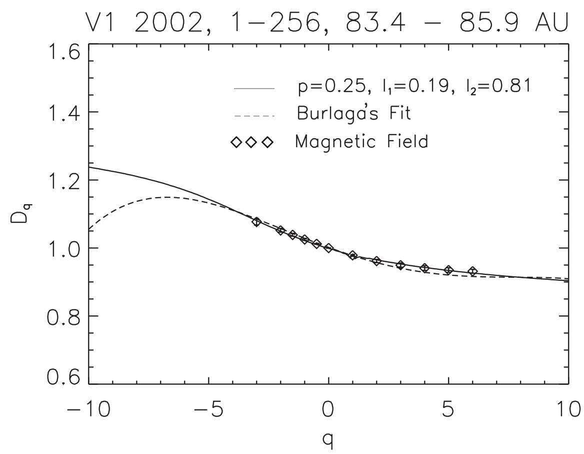

FIGURE 3. The generalized dimensions derived from the Voyager 1 observations of magnetic field strengths near 85 AU (diamonds), together with fits using a fourth order polynomial (dashed line) [11] and the two-scale model (continuous line).

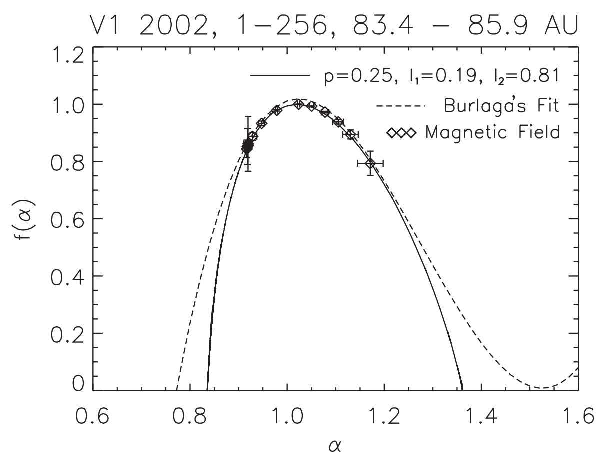

FIGURE 4. The multifractal spectrum of the magnetic fields observed by the Voyager 1 near 85 AU (diamonds) together with a polynomial fit (dashed curve) [11]. The fit to the two-scale model (solid curve) shows a good agreement with the data, exhibiting smaller degree of multifractality.

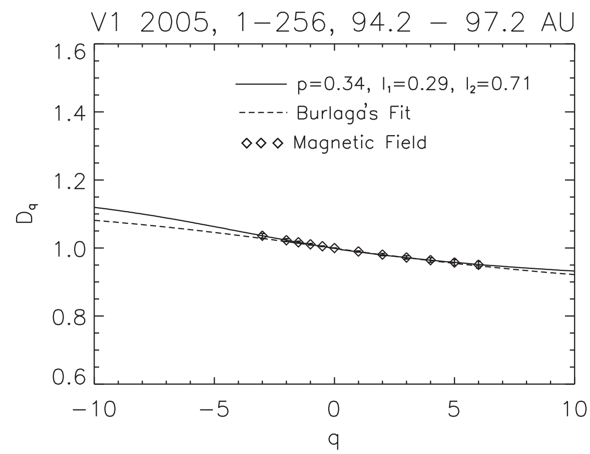

FIGURE 5. The generalized dimensions derived from the Voyager 1 observations of magnetic fields in the heliosheath at 95 AU (diamonds) with fits to the p model, $p = 0.56$, [12] and the equivalent two-scale model (continuous line).

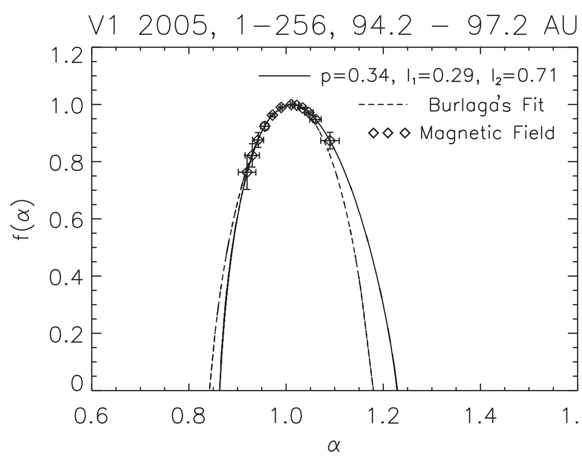

FIGURE 6. The multifractal spectrum of the magnetic fields observed by the Voyager 1 in the heliosheath at 95 AU (diamonds) together with a fit to the p model (dashed curve) [12]. The fit to the two-scale model (solid curve) confirms symmetric character of the spectrum.

gion from ~ 2 to 16 days are shown by diamonds, while the dashed line is a fit using a fourth order polynomial as given in Ref. [11]. We also see that the fit to the generalized weighted two-scale Cantor set model according to Equation (2) with $p = 0.25$ together with unequal two scales $l_1 = 0.19$ and $l_2 = 0.81$ denoted by continuous line shows a good agreement with the data. We see that space filling turbulence is recovered, $l_1 + l_2 = 1$, Ref. [8].

As seen from Table 1 using our two-scale Cantor set model we have calculated somewhat smaller degree of multifractality Δ as compared with the simplified fit of Ref. [11]. We have already demonstrated that the multifractal scaling is asymmetric in the outer heliosphere [16]. Hence as expected the multifractal scaling is also asymmetric before shock crossing with the calculated degree of asymmetry of $A = 0.5$. In addition, the multifractal spectrum $f(\alpha)$ as a function of scaling indices α derived from the Voyager 1 observations of the magnetic field strength at 85 AU (diamonds) together with a fourth order polynomial fit of Ref. [11] (dashed curve) and the two-scale model (solid curve) is presented in Figure 4.

The results for the generalized dimensions D_q and the multifractal spectrum $f(\alpha)$ obtained using the Voyager 1 data of magnetic fields at 94.2–97.2 AU (diamonds), i.e. after crossing the termination heliospheric shock, are presented in Figures 5 and 6, correspondingly. Now the

TABLE 1. Degree of multifractality Δ and asymmetry A for the magnetic field strengths at the termination shock.

	$\sim$ **85 AU**	$\sim$ **95 AU**
Burlaga's Fit	$\Delta = 0.68$	$\Delta = 0.34$
Two-scale model	$\Delta = 0.53 \pm 0.04$	$\Delta = 0.37 \pm 0.04$
Asymmetry	$A = 0.51 \pm 0.12$	$A = 0.80 \pm 0.30$

fits to the generalized two-scale model with $p = 0.34$ and scales, $l_1 = 0.29$ and $l_2 = 0.71$, are depicted by continuous lines. This also means that after shock crossing our model provides similar results as the p model with $p = 0.56$ (dashed curve) [12], confirming approximately symmetric character of the multifractal singularity spectrum. We see that in contrast to the asymmetric spectrum observed in the distant heliosphere the spectrum becomes symmetric in the heliosheath, where the plasma is expected to be roughly in equilibrium. Therefore the obtained value of $A = 0.8$ is closer to unity, Table 1.

One sees from Table 1 that the degree of multifractality for fluctuations of the interplanetary magnetic field strengths is generally smaller that that for the energy rate transfer in the turbulence cascade [16]. However, it is worth noting that the values obtained before shock crossing, $\Delta = 0.5$–0.6, are substantially greater than that for the heliosheath $\Delta \sim 0.3$. This means that the magnetic field behavior in the outer heliosphere, even in the distant solar wind, may exhibit a multifractal scaling, while in the heliosheath smaller values indicate possibility toward a monofractal behavior.

CONCLUSIONS

We show that the degree of multifractality for magnetic field fluctuations of the solar wind before shock crossing is greater than that in the heliosheath. In particular, we demonstrate that the multifractal scaling is rather asymmetric before shock crossing. In contrast to the asymmetric spectrum inside the heliosphere the spectrum becomes symmetric after the shock crossing, i. e. in the heliosheath, where the plasma is expected to be roughly in equilibrium.

Our results provide supporting evidence for multifractal structure of the solar wind in the outer heliosphere and even in the heliosheath. One can expect that the fluctuations in the solar wind magnetic field should contain information about the dynamic variations of the solar wind plasma also at the heliospheric boundaries. In general, the proposed generalized two-scale weighted Cantor set model should also be valid for non space filling turbulence. Therefore we propose this new turbulence model describing intermittent magnetic field fluctuations for analysis of turbulence in various environments.

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