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Model of line preserving field line motions using Euler potentials

Przemysław Figura [a,*], Wiesław M. Macek [b,a]

[a] Space Research Centre, Polish Academy of Sciences, Warsaw, Poland

[b] Faculty of Mathematics and Natural Sciences, Cardinal Stefan Wyszyński University, Warsaw, Poland

HIGHLIGHTS

- We formulate a line preserving magnetic field flow equation using Euler potentials.
- We find constraints on a non-reconnective general resistivity term in Ohm's Law.
- We propose a new method of detecting magnetic reconnection.

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ABSTRACT

We consider behavior of finite magnetic field lines during reconnection processes. We portray field line motions using Euler potentials representation. Here, we propose a new insight into plasma flow fields related with magnetic reconnection. In this approach reconnection is treated as a breakage of magnetic topology, which results in deviation from the line preserving flow regime. We derive constraints and the general equations for these flows. In our approach the flux preserving flows are treated as a special case of line preserving regime. We also derive a constraint on a non-ideal term in Ohm's Law within diffusion regions, which relates plasma flow with resistivity, and which must hold for non-reconnective diffusion. We also propose a new method of detecting magnetic reconnection.

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1. Introduction

Magnetic reconnection is a very important process in many areas of physics. Applications of this process range from laboratory to astrophysical plasmas. However, the basic mechanism of

* Corresponding author. Tel.: +48 22 49 66 407; fax: +48 22 840 31 31.
E-mail addresses: pfigura@cbk.waw.pl (P. Figura), macek@cbk.waw.pl (W.M. Macek).

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reconnection is still not understood fully. Even its appropriate definition is not commonly agreed and many researchers would like to stress certain aspects of the reconnection processes.

At the beginning of the reconnection studies the subject was usually treated as a two-dimensional problem only. After introducing early concepts (see *e.g.* [1–3]), researchers started to develop also three-dimensional reconnection models. It turned out that this three-dimensionality allowed a wide spectrum of new configuration magnetic fields, but because of that this subject began to be much more complicated [4,5]. As the theory is still under development and no simple description exists it appears that two aspects of reconnection are investigated separately. The first, finite-**B** reconnection is a regime where magnetic reconnection occurs, but there are no magnetic singularities, nor null points (see *e.g.* [6]). The second, zero-**B** reconnection includes regimes with reconnection occurring in magnetic null points or on separator lines (see *e.g.* [7,8]). Kinetic particle studies are usually conducted in zero-**B** regime (see *e.g.* [9]). A recent review on three-dimensional magnetic reconnection can be found in the papers by Yamada et al. [10] or by Pontin [11].

In the present paper we adopt the concept of *general magnetic reconnection* developed by Axford [12] and subsequently by Schindler, Hesse and Birn [13]. Therefore, we consider magnetic reconnection as a process, in which magnetic connection of plasma elements breaks down; this means that a change in magnetic field topology takes place in the reconnection process. We consider only finite-**B** reconnection, that is processes without null points of **B** in the considered region as *e.g.* slip-running reconnection [14].

In particular, we can distinguish two kinds of motions. The first one and more general is a line preserving motion. In this case every point on a given field line remains on the same line. The second motion we can distinguish is a flux preserving motion. It has a property that any loop moving with velocity field $\mathbf{U}(\mathbf{r}, t)$, where $\mathbf{r}$ is a position vector at time t, will preserve field's flux through the loop.

Therefore, the condition for a line preserving motion for a magnetic field **B** is given by

$$\mathbf{B} \times \left[\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{U} \times \mathbf{B}) \right] = 0, \tag{1}$$

whereas for a flux preserving motion we have

$$\frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{U} \times \mathbf{B}) = 0. \tag{2}$$

We see that any flux preserving motion is also line preserving, but not the opposite.

Now if we find actual plasma flow with velocity field $\mathbf{V}(\mathbf{r}, t)$ that satisfies Eqs. (1) or (2), *i.e.* if $\mathbf{U}(\mathbf{r}, t) = \mathbf{V}(\mathbf{r}, t)$, we may say that the flow is line or flux preserving, respectively. According to our definition of magnetic reconnection we can say that the reconnection process occurs when line preserving condition in Eq. (1) is not satisfied.

2. Euler potentials

We use Euler potentials representation of magnetic fields. A comprehensive review on Euler potentials can be found in the paper by Stern [15]. It can be shown that a divergence free vector can be, at least locally, represented as a cross product of two gradients of scalar functions. Therefore, we can write

$$\mathbf{B} = \nabla\alpha \times \nabla\beta, \tag{3}$$

where α and β are functions of coordinates x, y, z. And because **B** has a magnetic vector potential **A**, *i.e.* $\mathbf{B} = \nabla \times \mathbf{A}$, we obtain

$$\mathbf{A} = \alpha\nabla\beta. \tag{4}$$

Eqs. (3) and (4) are equivalent to a local choice of an electromagnetic field gauge

$$\mathbf{A} \cdot \mathbf{B} = 0. \tag{5}$$

One sees from Eq. (3), that magnetic field lines are tangential to families of surfaces defined by $\alpha = \text{const.}$ and $\beta = \text{const.}$ Naturally, a given magnetic field line is defined by the intersection of these surfaces. Admittedly, a choice of α and β is not unique and the same field line may be represented by another potentials pair, say $g(\alpha, \beta)$ and $h(\alpha, \beta)$, provided that

$$\frac{\partial(g, h)}{\partial(\alpha, \beta)} = 1. \tag{6}$$

Obviously Euler potentials representation may not be used when **B** vanishes, *e.g.* in magnetic null points. However, we see that if only $\mathbf{B} \neq 0$, it is always possible to locally derive a set of Euler potentials describing **B** in a form of Eq. (3), see Ref. [16]. For given α and β families, we can also choose a local curvilinear coordinate system (α, β, s), where s is a function of x, y, z and s is an arc length on the magnetic field line. This means that while moving along the s-axis we are always staying on the same magnetic field line.

3. Field line motions

We now derive equations for a more general, line preserving flows following Vasyliunas' approach for a flux preserving case, see Ref. [17]. Substituting $\mathbf{B} = \nabla \times \mathbf{A}$ to Eq. (1) and excluding curl from it we obtain

$$\mathbf{B} \times \left[\nabla \times \left(\frac{\partial \mathbf{A}}{\partial t} - \mathbf{U} \times \mathbf{B}\right)\right] = 0. \tag{7}$$

We see that in this equation the term in square bracket must be a linear combination of vector **B**. Hence one can write

$$\nabla \times \left(\frac{\partial \mathbf{A}}{\partial t} - \mathbf{U} \times \mathbf{B}\right) = \tilde{\zeta}\mathbf{B}, \tag{8}$$

where $\tilde{\zeta} = \tilde{\zeta}(\alpha, \beta)$ is any function of α and β. It is important to note, that $\tilde{\zeta}$ does not depend on arc length s. If it were a function of s, then **B** would change along s-axis, but our coordinate system is chosen so that **B** is constant on s-axis.

Considering vector identity

$$\zeta(\nabla \times \mathbf{A}) = \nabla \times \zeta\mathbf{A} + \mathbf{A} \times \nabla\zeta \tag{9}$$

we see that if ζ is a function of α and β only, *i.e.* $\zeta = \zeta(\alpha, \beta)$ then

$$\mathbf{A} \times \nabla\zeta = \alpha\nabla\beta \times \left(\frac{\partial\zeta}{\partial\alpha}\nabla\alpha + \frac{\partial\zeta}{\partial\beta}\nabla\beta\right) = -\alpha\frac{\partial\zeta}{\partial\alpha}(\nabla \times \mathbf{A}),$$

so

$$\nabla \times \zeta\mathbf{A} = \left(\zeta + \alpha\frac{\partial\zeta}{\partial\alpha}\right)(\nabla \times \mathbf{A}). \tag{10}$$

Without limiting generality we can set

$$\tilde{\zeta} = \zeta + \alpha\frac{\partial\zeta}{\partial\alpha} \tag{11}$$

and by substituting $\mathbf{B} = \nabla \times \mathbf{A}$ into Eq. (8) we obtain

$$\nabla \times \left(\frac{\partial \mathbf{A}}{\partial t} - \zeta\mathbf{A} - \mathbf{U} \times \mathbf{B}\right) = 0. \tag{12}$$

We can integrate this equation to obtain

$$\frac{\partial \mathbf{A}}{\partial t} - \zeta\mathbf{A} - \mathbf{U} \times \mathbf{B} = \nabla\chi, \tag{13}$$

where χ is any scalar function of α, β, and s, *i.e.* $\chi = \chi(\alpha, \beta, s)$, such that the curl of $\nabla\chi$ vanishes.

Now we construct a left hand side dot product of Eq. (13) with $\mathbf{B}$

$$\mathbf{B}\cdot\frac{\partial\mathbf{A}}{\partial t}-\mathbf{B}\cdot\zeta\mathbf{A}-\mathbf{B}\cdot(\mathbf{U}\times\mathbf{B})=\mathbf{B}\cdot\nabla\chi. \tag{14}$$

Note that two terms on the left hand side of Eq. (14) vanish: the second one because of the chosen gauge condition and the third one because of simple vector identity. We have thus the following condition for χ

$$\mathbf{B}\cdot\frac{\partial\mathbf{A}}{\partial t}=\mathbf{B}\cdot\nabla\chi. \tag{15}$$

It can be shown that

$$\mathbf{B}\cdot\frac{\partial\mathbf{A}}{\partial t}=\mathbf{B}\cdot\nabla\left(\alpha\frac{\partial\beta}{\partial t}\right). \tag{16}$$

Without limiting the generality of our considerations, we can set $\chi=\alpha\frac{\partial\beta}{\partial t}-\Lambda$, where Λ is a function of α and β only, but not depending on s. Using Eq. (16) we see that such χ satisfies Eq. (15). Thus from Eq. (13) we obtain the following equation for the case of line preserving flow

$$\begin{aligned}\mathbf{U}\times\mathbf{B}&=\frac{\partial\mathbf{A}}{\partial t}-\zeta\mathbf{A}-\nabla\chi\\&=\frac{\partial\alpha}{\partial t}\nabla\beta-\frac{\partial\beta}{\partial t}\nabla\alpha-\zeta\alpha\nabla\beta+\nabla\Lambda.\end{aligned} \tag{17}$$

In general, Eq. (17) may not be solved explicitly for $\mathbf{U}$. Only in special cases do explicit analytical solutions exist. However, Eq. (17) is a necessary condition for a flow to be line preserving as it is an equivalent of Eq. (1) in finite-$\mathbf{B}$ regime. So if any plasma flow with actual velocity field $\mathbf{V}(\mathbf{r},t)$ satisfies this equation, *i.e.* if $\mathbf{U}(\mathbf{r},t)=\mathbf{V}(\mathbf{r},t)$, we know that reconnection does not occur. Otherwise, if given flow $\mathbf{V}(\mathbf{r},t)$ does not satisfy Eq. (17), then the magnetic connection of plasma elements breaks down and some reconnection processes are present. Note that because Eq. (17) involves two arbitrary functions ζ and Λ, then there exists an infinite number of solutions for the case of the line preserving motions. Furthermore, any line preserving flow can be described by choosing relevant functions ζ and Λ. However, we do not state that those functions in analytical form may always be found.

If we set $\tilde{\zeta}=0$, we recover a special case of line preserving flow, namely a flux preserving flow equation that was obtained by Vasyliunas, see Ref. [17]. Eq. (8) then becomes a flux preserving motion condition as given in Eq. (2).

Setting $\Lambda=\alpha\frac{\partial\beta}{\partial t}+c\Phi$, where Φ is an electric field potential and c is a speed of light, in the flux preserving case we would obtain "$\mathbf{E}\times\mathbf{B}$" drift velocity $\mathbf{U}=\mathbf{V}_B=c\frac{\mathbf{E}\times\mathbf{B}}{B^2}$, see Ref. [17]. In a more general line preserving case, we obtain analogous drift, but with velocity

$$\mathbf{U}=\tilde{\mathbf{V}}_B=c\frac{(\mathbf{E}+\frac{\zeta}{c}\mathbf{A})\times\mathbf{B}}{B^2}. \tag{18}$$

So we have formulated line preserving field line motions using Euler potentials representation. We have shown that our approach generalizes Vasyliunas' formulation for flux preserving motions [17], which is a special case of our line preserving motions approach. Our formulation allows to express reconnection of magnetic fields as a breakage of the line preserving regime.

4. Ohm's law

Finite-$\mathbf{B}$ magnetic reconnection takes place in so called diffusion regions. These are localized zones, where ideal Ohm's Law is not satisfied. However the existence of a diffusion region does not imply occurrence of magnetic reconnection as there are possible line preserving flows within the region.

Let us now consider Ohm's Law. It can be written as

$$\mathbf{E} + \frac{1}{c}\mathbf{U} \times \mathbf{B} = \mathbf{R}, \tag{19}$$

where **R** is a non-ideal term in a general form. We do not consider here any particular form of **R**, but it can consist of many different terms like ion pressure, Hall effect, electron inertia, and other terms. We do not want to focus on physical mechanisms of non-idealness of Ohm's Law, however many researchers study magnetic reconnection on this fundamental levels (see *e.g.* [18]) and do it with great results (*e.g.* [19]). Thus, we would like to address only general properties of **R**.

To see what restrictions are imposed on **R** to satisfy magnetic line preservation we substitute $\mathbf{U} \times \mathbf{B}$ from Eq. (17) into Eq. (19). So we have

$$c\mathbf{R} = -c\nabla\Phi - \nabla\left(\alpha\frac{\partial\beta}{\partial t}\right) - \zeta\alpha\nabla\beta + \nabla\Lambda. \tag{20}$$

We can decompose vector **R** into $\nabla\alpha$, $\nabla\beta$, and ∇s components. In general it has the form

$$\mathbf{R} = R_\alpha\nabla\alpha + R_\beta\nabla\beta + R_s\nabla s, \tag{21}$$

where R_α, R_β, R_s are functions of α, β, and s. From Eq. (20) we write

$$\begin{aligned} c\mathbf{R} = & -c\frac{\partial\Phi}{\partial\alpha}\nabla\alpha - c\frac{\partial\Phi}{\partial\beta}\nabla\beta - c\frac{\partial\Phi}{\partial s}\nabla s + \frac{\partial\Lambda}{\partial\alpha}\nabla\alpha + \frac{\partial\Lambda}{\partial\beta}\nabla\beta \\ & - \left[\frac{\partial}{\partial\alpha}\left(\alpha\frac{\partial\beta}{\partial t}\right)\right]\nabla\alpha - \left[\frac{\partial}{\partial\beta}\left(\alpha\frac{\partial\beta}{\partial t}\right)\right]\nabla\beta - \left[\frac{\partial}{\partial s}\left(\alpha\frac{\partial\beta}{\partial t}\right)\right]\nabla s - \zeta\alpha\nabla\beta. \end{aligned} \tag{22}$$

Cross coordinates derivatives vanish so terms in square brackets are as follows:

$$\frac{\partial}{\partial\alpha}\left(\alpha\frac{\partial\beta}{\partial t}\right) = \frac{\partial\beta}{\partial t}, \tag{23a}$$

$$\frac{\partial}{\partial\beta}\left(\alpha\frac{\partial\beta}{\partial t}\right) = \frac{\partial\alpha}{\partial t}, \tag{23b}$$

$$\frac{\partial}{\partial s}\left(\alpha\frac{\partial\beta}{\partial t}\right) = 0. \tag{23c}$$

Finally, we obtain

$$\begin{cases} R_\alpha = -\dfrac{\partial\Phi}{\partial\alpha} + \dfrac{1}{c}\dfrac{\partial\Lambda}{\partial\alpha} - \dfrac{1}{c}\dfrac{\partial\beta}{\partial t} \\ R_\beta = -\dfrac{\partial\Phi}{\partial\beta} + \dfrac{1}{c}\dfrac{\partial\Lambda}{\partial\beta} - \dfrac{1}{c}\dfrac{\partial\alpha}{\partial t} - \dfrac{1}{c}\zeta\alpha \\ R_s = -\dfrac{\partial\Phi}{\partial s}. \end{cases} \tag{24}$$

Now we differentiate R_α and R_β with respect to β and α, respectively, obtaining

$$\begin{cases} \dfrac{\partial R_\alpha}{\partial\beta} = -\dfrac{\partial^2\Phi}{\partial\alpha\partial\beta} + \dfrac{1}{c}\dfrac{\partial^2\Lambda}{\partial\alpha\partial\beta} - \dfrac{1}{c}\dfrac{\partial}{\partial\beta}\dfrac{\partial\beta}{\partial t} \\ \dfrac{\partial R_\beta}{\partial\alpha} = -\dfrac{\partial^2\Phi}{\partial\alpha\partial\beta} + \dfrac{1}{c}\dfrac{\partial^2\Lambda}{\partial\alpha\partial\beta} - \dfrac{1}{c}\dfrac{\partial}{\partial\alpha}\dfrac{\partial\alpha}{\partial t} - \dfrac{1}{c}\zeta - \dfrac{1}{c}\alpha\dfrac{\partial\zeta}{\partial\alpha} \end{cases} \tag{25}$$

and then subtracting both sides of those equations we obtain

$$\zeta + \alpha\frac{\partial\zeta}{\partial\alpha} + c\frac{\partial R_\beta}{\partial\alpha} - c\frac{\partial R_\alpha}{\partial\beta} = 0 \tag{26}$$

or shortly

$$\tilde{\zeta} + c\frac{\partial R_\beta}{\partial \alpha} - c\frac{\partial R_\alpha}{\partial \beta} = 0. \tag{27}$$

Eq. (26) relates function ζ with resistivity. This equation has to be satisfied in order to have line preserving flow. Thus setting one of the two quantities, ζ or $\mathbf{R}$, we obtain the constraint on the second quantity.

Differentiating Eq. (27) with respect to s gives

$$\frac{\partial \tilde{\zeta}}{\partial s} + c\frac{\partial^2 R_\beta}{\partial s \partial \alpha} - c\frac{\partial^2 R_\alpha}{\partial s \partial \beta} = 0 \tag{28}$$

and remembering that $\tilde{\zeta}$ does not depend on s we obtain

$$\frac{\partial^2 R_\beta}{\partial s \partial \alpha} = \frac{\partial^2 R_\alpha}{\partial s \partial \beta}. \tag{29}$$

Eq. (29) is equivalent to the Eq. (26) from the paper by Hesse and Schindler [20]. Differentiating R_α and R_β from (24) with respect to s we obtain

$$\begin{cases} \dfrac{\partial R_\alpha}{\partial s} = -\dfrac{\partial^2 \Phi}{\partial s \partial \alpha} + \dfrac{1}{c}\dfrac{\partial^2 \Lambda}{\partial s \partial \alpha} - \dfrac{1}{c}\dfrac{\partial}{\partial s}\dfrac{\partial \beta}{\partial t} \\ \dfrac{\partial R_\beta}{\partial s} = -\dfrac{\partial^2 \Phi}{\partial s \partial \beta} + \dfrac{1}{c}\dfrac{\partial^2 \Lambda}{\partial s \partial \beta} - \dfrac{1}{c}\dfrac{\partial}{\partial s}\dfrac{\partial \alpha}{\partial t} - \dfrac{1}{c}\zeta\dfrac{\partial \alpha}{\partial s} - \dfrac{1}{c}\alpha\dfrac{\partial \zeta}{\partial s}, \end{cases} \tag{30}$$

which gives us

$$\begin{cases} \dfrac{\partial R_\alpha}{\partial s} = \dfrac{\partial R_s}{\partial \alpha} \\ \dfrac{\partial R_\beta}{\partial s} = \dfrac{\partial R_s}{\partial \beta}. \end{cases} \tag{31}$$

Eqs. (31) are the same as Eqs. (27a) and (27b) from the same paper by Hesse and Schindler, see Ref. [20]. They provided a proof that in such a general form $\mathbf{R}$ satisfies

$$\mathbf{B} \times (\nabla \times \mathbf{R}) = 0, \tag{32}$$

i.e. the line preservation condition.

Let us now consider a flux preserving flow. We see that Eq. (27) becomes simply

$$\frac{\partial R_\beta}{\partial \alpha} = \frac{\partial R_\alpha}{\partial \beta}. \tag{33}$$

We see that differentiating with respect to s gives us Eq. (29) and similarly it can be shown that our flow is obviously line preserving. We have that Eq. (24) become

$$\begin{cases} R_\alpha = -\dfrac{\partial \Phi}{\partial \alpha} + \dfrac{1}{c}\dfrac{\partial \Lambda}{\partial \alpha} - \dfrac{1}{c}\dfrac{\partial \beta}{\partial t} \\ R_\beta = -\dfrac{\partial \Phi}{\partial \beta} + \dfrac{1}{c}\dfrac{\partial \Lambda}{\partial \beta} - \dfrac{1}{c}\dfrac{\partial \alpha}{\partial t} \\ R_s = -\dfrac{\partial \Phi}{\partial s}. \end{cases} \tag{34}$$

To verify flux preservation we need to show that

$$\nabla \times \mathbf{R} = 0. \tag{35}$$

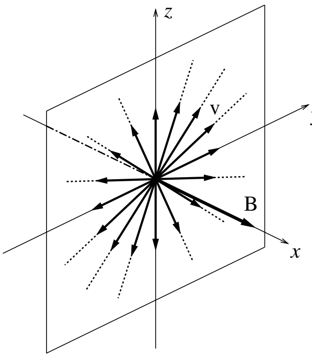

Fig. 1. Configuration of magnetic field and plasma velocity field in the example.

We can write $\nabla \times \mathbf{R}$ in $\nabla\alpha$, $\nabla\beta$, ∇s decomposition

$$\nabla \times \mathbf{R} = \left(\frac{\partial R_\beta}{\partial \alpha} - \frac{\partial R_\alpha}{\partial \beta}\right)\nabla\alpha \times \nabla\beta + \left(\frac{\partial R_s}{\partial \alpha} - \frac{\partial R_\alpha}{\partial s}\right)\nabla\alpha \times \nabla s + \left(\frac{\partial R_s}{\partial \beta} - \frac{\partial R_\beta}{\partial s}\right)\nabla\beta \times \nabla s. \quad (36)$$

By substituting (34) into this equation we obtain required condition of Eq. (35).

Summarizing, we have shown that our model is consistent with the *general magnetic reconnection* concept developed by Axford [12] and by Schindler, Hesse and Birn [13]. Within the concept we have formulated Ohm's Law in a general form and constraints on resistivity term, particularly as given by Eq. (26). We think it can be used as a new way of detecting reconnection. An example below presents this method.

5. Example

All actual measurements are restricted to a certain level of accuracy only. Estimated layout of magnetic field and plasma flow structure might be different from the real values. Therefore we can expect that reconnection appearance is possible even though from direct inaccurate measurement and anticipations it should not be. Given that, we can use Eq. (26) and estimates of resistivity to validate measurements from experiments and to check whether magnetic reconnection occurs in the analyzed case.

Note that usage of Euler potentials representation is limited to such configurations in which Euler potentials can be defined globally or at least within the whole region of interest. As an example we inspect a simple laboratory plasma case.

We consider a plane potential compressible flow in a form of a source described by the velocity potential $\varphi(y, z) = \frac{-Q}{\sqrt{y^2+z^2}}$, where Q is a constant. We add uniform magnetic field $\mathbf{B}$ perpendicular to the flow plane. We suppose that $\mathbf{B}$ is weak enough not to disturb the flow velocity $\mathbf{V}$. The flow is a line preserving flow, but it is not flux preserving one. We choose the (x, y, z) coordinate system so that $\mathbf{B}$ is aligned with the x-axis and the plasma flows in the yz-plane as illustrated in Fig. 1.

In this case $\mathbf{V} = [0, v_y, v_z]$ and $\mathbf{B} = [B_x, 0, 0]$. This gives us the opportunity to choose such Euler potentials coordinate system (α, β, s) that both systems have parallel axes—x to s, y to α, and z to β, respectively. Therefore the transformation between coordinate systems is given simply by

$$\begin{cases} \alpha = ay \\ \beta = bz \\ s = rx, \end{cases} \quad (37)$$

where a, b, and r are some constants.

We now set $B_x = ab$, $v_y = \frac{Qy}{(y^2+z^2)^{3/2}}$ and $v_z = \frac{Qz}{(y^2+z^2)^{3/2}}$. It can be shown that flow **V** in field **B** in such a form is line preserving, but is not flux preserving. We also see from Eq. (17) that setting **V** and **B** in this form is equivalent with setting

$$\begin{cases} \zeta = \dfrac{-2Qab^3}{\beta^2(a^2\beta^2 + b^2\alpha^2)^{1/2}} \\ \Lambda = \dfrac{2Qab^3\alpha}{\beta(a^2\beta^2 + b^2\alpha^2)^{1/2}}. \end{cases} \tag{38}$$

We can now solve flow–resistivity equation (26). In this simple case we have

$$-\frac{2Qa^3b^3}{c(a^2\beta^2 + b^2\alpha^2)^{3/2}} + \frac{\partial R_\beta}{\partial \alpha} - \frac{\partial R_\alpha}{\partial \beta} = 0. \tag{39}$$

This equation holds if $R_\alpha = \frac{kQa^3b\beta}{c\alpha^2(a^2\beta^2+b^2\alpha^2)^{1/2}}$ and $R_\beta = \frac{(k+2)Qab^3\alpha}{c\beta^2(a^2\beta^2+b^2\alpha^2)^{1/2}}$, where k is a constant. Because R_s can have any value, we can simply set $R_s = 0$. Then resistivity expressed using (x, y, z) coordinates is

$$\begin{cases} R_\alpha = \dfrac{kQbz}{cy^2\sqrt{z^2+y^2}} \\ R_\beta = \dfrac{(k+2)Qay}{cz^2\sqrt{z^2+y^2}} \\ R_s = 0. \end{cases} \tag{40}$$

We see that sets of values of R_α and R_β define specific surfaces. This calculated resistivity is a constraint for the given flow to be line preserving. If we know that we are dealing with the given flow and magnetic field, then the actual resistivity must be as resistivity presented in Eqs. (40) with R_s having any value. Therefore, any deviations of measured resistivities from above dependencies indicate that reconnection occurs.

6. Conclusions

We have explained magnetic line preserving flows during finite-**B** reconnection processes by using Euler potentials representation. In this paper we have obtained some general equations for these processes. In particular, a flux preserving flow appears to be a special case of our newly proposed more general model.

We have also found constraints on general resistivity term in Ohm's Law for a general line preserving condition. It relates plasma flow field with the local resistivity. We have shown that this condition is consistent with a concept of magnetic reconnection originally proposed by Hesse and Schindler [20]. Applying this relation we have proposed a new method of detecting magnetic reconnection.

Admittedly, application of Euler potentials to magnetic reconnection processes should be used with care. It is important to bear in mind mathematical restrictions of this representation. In general Euler potentials can be defined only locally. Its global usage is restricted to instances of appropriate magnetic field configurations or suitable boundary conditions. Whereas in laboratory plasmas this should not be of a major problem as specific experiments can be suitably designed; in space plasmas however this might be a greater concern.

The number of magnetic reconnection studies based on three-dimensional numerical modeling has been growing recently (for finite-**B** reconnection see *e.g.* [21,22]). Although Euler potentials formulation is not common, we think that our flow–resistivity constraint might be useful, for instance as an indicator of reconnection occurrence in appropriately designed numerical models.

It is known that due to new space missions growth of observational data contribute to a better understanding of reconnection problems (see *e.g.* [23]). We therefore hope that our flow–resistivity constraint might be used as a new way of detecting reconnection sites, provided plasma flow in given magnetic field and resistivity are derived from *in situ* measurements and magnetic field configuration appears to be suitable for Euler potentials application. We are aware that further study of the theory as well as its application is still required.

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