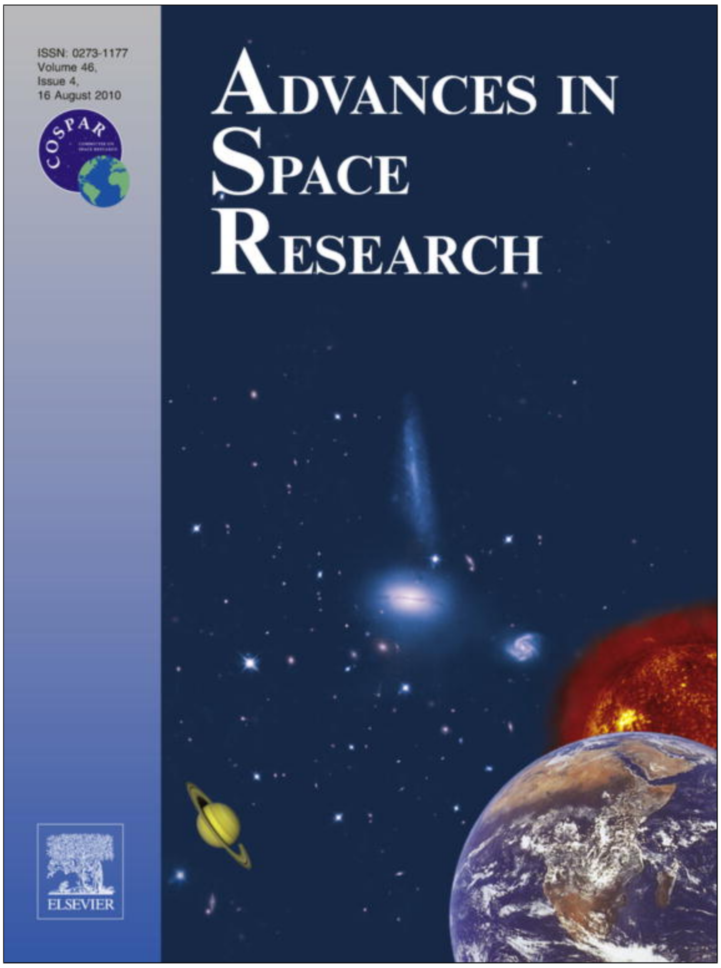
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Chaos and multifractals in the solar wind

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Abstract

By using the false-nearest-neighbours method, we have argued that the deterministic component of solar wind plasma dynamics should be low-dimensional. In fact, the results we have obtained using the method of topological embedding indicate that the behaviour of the solar wind can be approximately described by a low-dimensional chaotic attractor in the inertial manifold, which is a subspace of system phase space. We have also shown that the multifractal spectrum of the solar wind attractor is consistent with that for the multifractal measure of the self-similar generalized weighted Cantor set with two different scaling parameters and one probability measure parameter responsible for nonuniform compression in phase space and multifractality. The values of the parameters fitted also demonstrate that the complex solar wind system could only be weakly non-conservative (small dissipation) and quantify nonlinear dynamics; some parts of the attractor in phase space are visited much more frequently than other parts. In addition, to quantify the multifractality of space plasma intermittent turbulence, we consider that generalized Cantor set also in the context of scaling properties of solar wind turbulence. We investigate the resulting multifractal spectrum of a one-dimensional phenomenological model of turbulence cascade depending on its parameters, especially for asymmetric scaling. In particular, we have shown that intermittent pulses are stronger for the cascade model with two different scaling parameters. Even thought solar wind turbulence appears to be rather space filling, a better agreement with the data is obtained, especially for the negative index of generalized dimensions. Therefore we argue that there is a need to use a two-scale asymmetric cascade model. We hope that this generalized multifractal model will be a useful tool for analysis of intermittent turbulence in space plasmas. We thus believe that fractal analysis of chaotic systems could lead us to a deeper understanding of their nature, and maybe even to predict their seemingly unpredictable behaviour.

Keywords: Nonlinear dynamics and chaos; Fractal and multifractal systems; Turbulence; Solar wind plasma

1. Importance of chaos and multifractality

Nonlinear dynamical systems are often highly sensitive to initial conditions resulting in chaotic phenomena. Chaos is thus an aperiodic long-term behaviour in a deterministic system that exhibits sensitivity to initial conditions (e.g. Strogatz, 1994). In non-conservative systems (with dissipation) the trajectories describing its evolution in the phase space may asymptotically converge towards a certain invariant set that is called an *attractor*. In such a system chaos requires the low-dimensional attractor and deterministic *nonlinear* time evolution. In infinite-dimensional systems or systems with very large dimension it is often the case that one can show that there exists a low-dimensional subspace, the so-called inertial manifold, to which the orbit tends and on which the attractor lies (e.g. Ott, 1993).

We remind that a fractal is a rough or fragmented geometrical object that can be subdivided in parts, each of which is (at least approximately) a reduced-size copy of the whole. Strange attractors are often fractal sets, which exhibits a hidden order within chaos. Fractals are generally *self-similar* and independent of scale (with a fractal dimension). A multifractal is an object that demonstrate

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various self-similarities, described by a multifractal spectrum of dimensions. One can say that self-similarity of multifractals is scale dependent resulting in singularity spectrum. A multifractal is therefore in a certain sense like a set of intertwined fractals. More precisely, one may distinguish a probability measure from its geometrical support, which may or may not have fractal geometry. Then, if the measure has different fractal dimensions on different parts of the support, the measure is multifractal (Mandelbrot, 1989).

The nature of the fluctuations in solar wind plasma parameters is still little understood. The slow solar wind most likely originates from nonlinear processes in the solar corona. The fast wind associated with coronal holes is relatively uniform and stable, while the slow wind is more turbulent and consequently quite variable in terms of velocities. Fortunately, it appears that a certain kind of order does lie concealed within the irregular solar wind fluctuations, which can be described using methods of nonlinear time series analysis, based on fractal analysis and the theory of deterministic chaos. This involves the notions of fractal and multifractal sets, which could be presumably strange attractors in a certain state space of a given complex dynamical system. By employing the so-called false-nearest-neighbours method, we argue that the deterministic component of solar wind plasma dynamics should be low-dimensional (e.g. Macek and Strumik, 2006). In fact, the results we have obtained using the method of topological embedding indicate that the behaviour of the solar wind can be approximately described by a low-dimensional chaotic attractor in the inertial manifold, which is a subspace of system phase space.

A direct determination of a solar wind attractor from the data is known to be a difficult problem. Indication of a chaotic dynamics in the magnetic field fluctuations has been provided by Polygiannakis and Moussas (1994a), Polygiannakis and Moussas (1994b) using Heos (at 1 AU) and Pioneer (5–10 AU) data in the solar wind, and ICE data in the cometary environment. The chaotic strange attractor has been identified by Macek (1998) using Helios measurements of velocity fluctuations in the solar wind as further examined by Macek and Redaelli (2000). In particular, Macek (1998) has calculated the correlation dimension of the reconstructed attractor in the solar wind and has provided tests for this measure of *complexity* including statistical surrogate data tests (Theiler et al., 1992). Further, Macek and Redaelli (2000) have shown that the Kolmogorov entropy of the attractor is *positive* and finite, as it holds for a *chaotic* system.

We have also considered the spectrum of generalized dimensions D_q as a function of a continuous index, $-\infty < q < \infty$, for the solar wind attractor, using a multifractal model with a measure of the self-similar weighted Cantor set with one parameter describing uniform compression and another parameter for the probability measure of the attractor of the system. The spectrum has been found to be consistent with the data, at least for positive index q of the generalized dimensions D_q (Macek, 2002; Macek, 2003; Macek, 2006; Macek et al., 2005; Macek et al., 2006). However, the full singularity spectrum is necessary to quantify the degree of multifractality. Notwithstanding of the well-known statistical problems with negative q (Macek, 2006), we have recently succeeded in estimating the entire spectrum for solar wind attractor using a generalized weighted Cantor set with two different scales describing nonuniform compression (Macek, 2007).

The question of multifractality is also of great importance because it allows us to investigate the nature of interplanetary hydromagnetic turbulence in the solar wind (e.g. Burlaga, 1991; Carbone, 1993; Marsch et al., 1996; Marsch and Tu, 1994; Marsch and Tu, 1997; Bruno et al., 2001). Starting from Richardson's version of turbulence, many authors tried to recover the observed scaling exponents, using some simple and more advanced models of turbulence describing distribution of the energy flux between cascading eddies at various scales. In particular, the multifractal spectrum has been investigated using Voyager (magnetic field) data in the outer heliosphere (e.g. Burlaga, 1991; Burlaga, 2001) and using Helios (plasma) data in the inner heliosphere (e.g. Marsch et al., 1996).

Recently, to further quantify the multifractality, we have considered that generalized weighted Cantor set also in the context of turbulence cascade (Macek and Szczepaniak, 2008). We have argued that there is, in fact, need to use a two-scale cascade model. Here we investigate the resulting multifractal singularity spectrum depending on two scaling parameters and one probability measure parameter, demonstrating that a much better agreement has been obtained, especially for $q < 0$. We hope that this generalized new asymmetric multifractal model could shed light on the nature of turbulence and will be a useful tool for analysis of intermittent turbulence in various environments.

2. Two-scale Cantor set

A simple example of multifractals is the Cantor set with two scales $l_1 + l_2 \leqslant l$ is shown in Fig. 1. After n iterations we have $\binom{n}{k}$ intervals of width $l_k = l_1^k l_2^{n-k}$, where

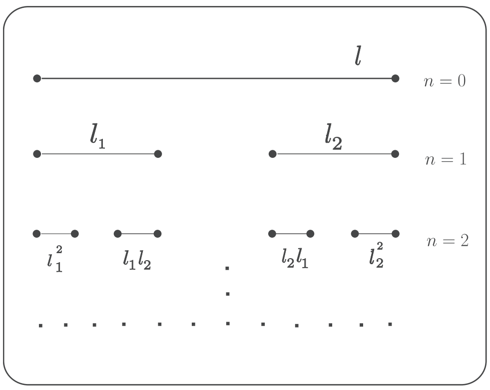

Fig. 1. Two-scale Cantor set.

$k = 1, \ldots, n$. The resulting set of 2^n closed intervals (more and more narrow segments visited with various probabilities) for $n \to \infty$ becomes the weighted two-scale Cantor set. One can find the two-scale Cantor set in many textbooks (e.g. Falconer, 1990; Ott, 1993), but it is still difficult to trace complexity of this strange attractor that exhibits multifractality in various real systems.

According to a standard scenario, each of cascading eddies is breaking down into two new ones, but not necessarily equal and twice smaller. In particular, space filling turbulence could be recovered for the system of size $l, l_1 + l_2 = l$ (normalized, $l = 1$). In the inertial region, $\eta \ll l_k \ll 1$, the energy is not allowed to be dissipated directly until the Kolmogorov scale η is reached. However, in this range at each n-th step of the binomial multiplicative process, the flux of kinetic energy density ε transferred to smaller eddies (energy transfer rate) could be divided into nonequal fractions p and $1 - p$. (cf. Macek and Szczepaniak, 2008, Fig. 1).

3. Solar wind data

We have extensively analyzed the Helios data using plasma parameters measured *in situ* in the inner heliosphere (Schwenn, 1990). The X-velocity (mainly radial) component of the plasma flow, v_x, has been already investigated by Macek (1998), Macek (2002), Macek (2003) and Macek and Redaelli (2000). The Alfvénic fluctuations with longer (two-day) samples have been studied by Macek (2006), Macek (2007), Macek et al. (2005) and Macek et al. (2006). Recently, Macek and Szczepaniak (2008) have selected even longer (four-day) time intervals of v_x samples in 1976 (each of 8531 data points, interpolated with sampling time of 40.5 s) for both slow and fast solar wind streams measured at various distances from the Sun. Here, we analyze the multifractal spectra obtained using the Helios 2 data. The fractal and multifractal scaling has also been tested using Ulysses observations (e.g. Horbury and Balogh A., 2001) and with ACE/WIND data (e.g. Hnat et al., 2003; Hnat et al., 2007; Kiyani et al., 2007).

4. Methods of data analysis

4.1. Generalized dimensions

The generalized dimensions are important characteristics of *complex* dynamical systems (e.g. Grassberger, 1983; Grassberger and Procaccia, 1983; Hentschel and Procaccia, 1983; Halsey et al., 1986). Since these dimensions are related to frequencies with which typical orbits in phase space visit different regions of the attractors, they provide information about dynamics of the systems (Ott, 1993). In the case of turbulence cascade these generalized measures are related to inhomogeneity with which the energy is distributed between different eddies (Meneveau and Sreenivasan, 1991). In this way they provide information about dynamics of multiplicative process of cascading eddies. Here high positive values of q emphasize regions of intense energy transfer rate, while negative values of q accentuate low-transfer rate regions.

Let us consider the generalized weighted Cantor set where the probability of visiting one segment of size l_1 is p (say, $p \leqslant 1/2$), and for the remaining segment of size l_2 is $1 - p$ in Fig. 1. For any q one obtains $D_q = \tau(q)/(q-1)$ by solving numerically the following transcendental equation (e.g. Ott, 1993)

$$\frac{p^q}{l_1^{\tau(q)}} + \frac{(1-p)^q}{l_2^{\tau(q)}} = 1. \quad (1)$$

The multifractal singularity spectrum $f(\alpha)$ as a function of a singularity strength α is also obtained from Eq. (1) by the following Legendre transformation

$$\alpha(q) = \frac{\mathrm{d}\tau(q)}{\mathrm{d}q} \quad (2)$$

$$f(\alpha) = q\alpha(q) - \tau(q). \quad (3)$$

4.2. Turbulence scaling

In the inertial range the standard q-order ($q > 0$) structure function is scaling as

$$S_u^q(l) = \langle |u(x+l) - u(x)|^q \rangle \propto l^{\xi(q)}, \quad (4)$$

where $u(x)$ and $u(x+l)$ are velocity components parallel to the longitudinal direction separated from a position x by a distance l. As is usual, the temporal scales can be interpreted as the spatial scales, $x = v_{sw}t$, where v_{sw} is the average solar wind speed (Taylor's hypothesis). The transfer rate of the energy flux, ε_l, is widely estimated by

$$\varepsilon(l) \sim \frac{|u(x+l) - u(x)|^3}{l}. \quad (5)$$

It can be argued that for $-\infty < q < \infty$ in some region the total probability measure should scale with the exponent $\tau(q) \equiv (q-1)D_q$ as

$$\sum_i \mu_i^q \sim l^{\tau(q)} \quad (6)$$

where $\mu_i = \varepsilon_l / \langle \epsilon_L \rangle$ is the probability measure of ith eddy in the d-dimensional physical space. Here, for simplicity the third moment of structure function of velocity fluctuations in Eq. (5) is used for estimation of this measure (Marsch et al., 1996). Recently, hydromagnetic generalization of this approximation for the Alfvénic fluctuations is considered by Sorriso-Valvo et al. (2007).

From Eqs. (4)–(6) we have (Tsang et al., 2005)

$$\tau(q) = d(q-1) + \xi(3q) - q\xi(3) \quad (7)$$

Admittedly, the structure function scaling exponent $\xi(q)$ is easier to measure experimentally than the spectrum of dimensions $D_q \equiv \tau(q)/(q-1)$ in Eq. (6), which is easier to interpret theoretically, see Eq. (1). Surely, as seen from Eq. (7) both have the same information about multifractality, at least for $q > 0$. However, because we are also interested in negative q, it is more convenient to use dimensions instead of structure functions.

5. Results and discussion

5.1. Dimensions for solar wind attractor and turbulence models

First, to estimate the generalized dimensions for the solar wind attractor we should calculate for a given continuous index q and embedding dimension m the so-called generalized correlation sum $\sum \mu_i^q(m,l)$ as a function of hyperspheres of radius l that cover the presumed attractor. For positive integer q, this can be interpreted as an average probability of finding q from N vectors in embedding space separated by a distance smaller than l (see Macek, 2007, Equation (3)). For large dimensions m and small distances l in the scaling region, according to Eq. (6), it can be argued that $\sum \mu_i^q(l) \propto l^{\tau(q)}$, where $\tau(q)$ is an approximation of the ideal limit $l \to 0$ of Eq. (1) (Grassberger and Procaccia, 1983). Hence, the slopes of the natural logarithm of $\sum \mu^q(m,l)$ versus $\ln l$ (normalized) provides

$$\tau_{q,m}(l) = \frac{\mathrm{d}[\ln \sum_{i=1}^{N} \mu_i^q(m,l)]}{\mathrm{d}(\ln l)}. \quad (8)$$

If a plateau exists in a scaling region, $l_{\min} < l < l_{\max}$, which does not depend on m for some $m > m_0$, this plateau can be identified as the requested generalized dimension. Finally, the average slope for $6 \leqslant m \leqslant 10$ on the logarithmic scales is taken as $D_q = \tau(q)/(q-1)$ (Macek, 1998; Macek, 2006; Macek, 2007; Macek et al., 2005; Macek et al., 2006). The results obtained using the moving average filter and singular-value decomposition linear filter for standard $q = 2$ are given by Macek et al. (2005, Fig. 2), and are compared with $q = -2$ in Fig. 2(a) and (b) of Macek (2006), correspondingly, while those obtained for somewhat shorter samples ($N = 4514$) have been discussed by Macek (1998) and by Macek and Redaelli (2000) using the nonlinear Schreiber filters. Next, the generalized dimensions D_q as a function of q (cf. Macek, 2007, Equation (3)) with the statistical errors of the average slopes obtained using weighted least squares fitting over the scaling range are shown in Figs. 3(a) and 4(a) of the paper by Macek (2007) and compared with one-scale and two-scale Cantor set model (cf. Macek et al., 2005, Fig. 3). In the later case the value of a parameter is $s = 0.47$ (only slightly smaller than $\frac{1}{2}$) that indicates that the complex solar wind system could only be weakly non-conservative (small dissipation), but still allowing for existence of some attractors of the system.

Second, the results for the generalized dimensions D_q as a function of q, calculated from the data and compared with those obtained using Eq. (1) for solar wind turbulence for the slow (a) and (c) and fast (b) and (d) solar wind streams at distances of 0.3 AU and 0.97 AU, correspondingly, are presented in Fig. 3(a)–(d) of the paper by Macek and Szczepaniak (2008).

5.2. Multifractal spectrum for turbulence

Here the results for the corresponding singularity spectra $f(\alpha)$ as a function of a singularity strength α are shown in Fig. 2(a)–(d). The values of $f(\alpha)$ given in Eqs. (2) and

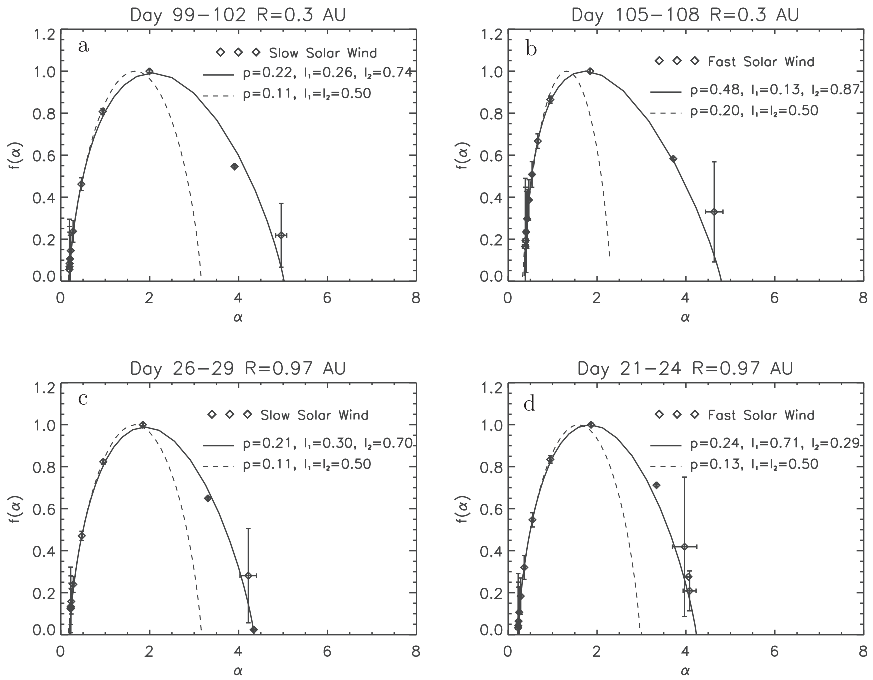

Fig. 2. The singularity spectrum $f(\alpha)$ as a function of a singularity strength α. The values obtained for one-dimensional turbulence are calculated for the generalized two-scale (continuous lines) model and the usual one-scale (dashed lines) p-model with parameters fitted to the multifractal spectrum using the v_x velocity components (diamonds) for the slow (a) and (c) and fast (b) and (d) solar wind streams at distances of 0.3 AU and 0.97 AU, correspondingly.

(3), for one-dimensional turbulence, $d = 1$, are calculated using the radial velocity components $u = v_x$ (in time domain). It is well known that for $q < 0$ we have some basic statistical problems (Macek, 2006; Macek, 2007). Nevertheless, in spite of large statistical errors in Fig. 2, especially for $q < 0$, the multifractal character of the measure can still clearly be discerned. Therefore one can confirm that both the spectrum of dimensions and singularity spectrum still exhibit the multifractal structure of the solar wind in the inner heliosphere (cf. Macek and Szczepaniak, 2008).

For $q \geqslant 0$ these results agree with the usual one-scale p-model fitted to the singularity spectra as obtained analytically using $l_1 = l_2 = 0.5$ in Eq. (1) and the corresponding value of the parameter $p = 0.11, 0.20, 0.11$, and 0.13 for the slow (a) and (c) and fast (b) and (d) solar wind streams at distances of 0.3 AU and 0.97 AU, correspondingly, as shown by dashed lines. On the contrary, for $q < 0$ (right part of the singularity spectrum in Fig. 2) the p-model cannot describe the observational results, as noted by Marsch et al. (1996). Here we show that the experimental values are consistent also with the singularity spectrum obtained numerically from Eqs. (1)–(3) for the weighted two-scale Cantor set using an asymmetric scaling, i.e., using unequal scales $l_1 \neq l_2$, as is depicted in Fig. 2(a)–(d) by continuous lines. By using the Helios 2 data we also confirm the universality of the shape of the multifractal spectrum shown in Fig. 2 for both slow and fast streams and various heliocentric distances, as noticed, e.g., by Burlaga (2001). In our view, the obtained shape of the multifractal spectrum results not only from the nonuniform probability of the energy transfer rate but mainly from the multiscale nature of the cascade.

Finally, we see that the multifractal spectrum of the solar wind is only roughly consistent with that for the multifractal measure of the self-similar weighted symmetric one-scale weighted Cantor set only for $q \geqslant 0$, as also seen from the standard structure function analysis. On the other hand, this universal spectrum is in a very good agreement with the two-scale asymmetric weighted Cantor set schematically shown in Fig. 1 for both positive and negative q. Obviously, taking two different scales for eddies in the cascade, one obtains a more general situation than in the usual p-model of Meneveau and Sreenivasan (1987) for fully developed turbulence, especially for an asymmetric scaling, $l_1 \neq _2$. Hence we hope that this generalized model will be a useful tool for analysis of intermittent turbulence in space plasmas.

6. Conclusions

In this way, we have supported our conjecture that trajectories describing the system in the inertial manifold of phase space asymptotically approach the attractor of low-dimension (Macek, 1998). We have shown that the multifractal spectrum of the solar wind attractor is consistent with that for the multifractal measure of the generalized two-scale weighted Cantor set. The values of the parameters fitted for $l_1 + l_2 \sim 1$ and $p \sim 10^{-1}$ for the slow wind demonstrate that the complex solar wind plasma could only be weakly non-conservative (small dissipation) and quantify its nonlinear dynamics; some parts of the attractor in phase space are visited at least one order of magnitudes more frequently than other parts as illustrated in (Macek, 1998, Fig. 5).

We have also studied the inhomogeneous rate of the transfer of the energy flux indicating multifractal and intermittent behaviour of solar wind turbulence in the inner heliosphere. In particular, we have demonstrated that for the model with two different scaling parameters a much better agreement with the real data is obtained, especially for $q < 0$. Basically, the generalized dimensions for solar wind are consistent with the generalized p-model for both positive and negative q, but rather with different scaling parameters for sizes of eddies, while the usual p-model can only reproduce the spectrum for $q \geqslant 0$. It appears that solar wind turbulence is rather space filling, even though, in general, the proposed generalized two-scale weighted Cantor set model should also be valid for non space filling turbulence. Therefore we propose this cascade model describing intermittent energy transfer for analysis of turbulence in various environments.

Thus these results show multifractal structure of the solar wind in the inner heliosphere. Hence we suggest that there exists an inertial manifold for the solar wind, in which the system has *multifractal* structure, and where noise is certainly not dominant. The multifractal structure, convected by the wind, might probably be related to the complex topology shown by the magnetic field at the source regions of the solar wind.

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References

Bruno, R., Carbone, V., Veltri, P., Pietropaolo, E., Bavassano, B. Identifying intermittency events in the solar wind. Planet. Space Sci. 49, 1201–1210, 2001.

Burlaga, L.F. Multifractal structure of the interplanetary magnetic field: Voyager 2 observations near 25 AU, 1987–1988. Geophys. Res. Lett. 18, 69–72, 1991.

Burlaga, L.F. Lognormal and multifractal distributions of the heliospheric magnetic field. J. Geophys. Res. 106, 15917–15927, doi:10.1029/2000JA000107, 2001, 2001.

Carbone, V. Cascade model for intermittency in fully developed magnetohydrodynamic turbulence. Phys. Rev. Lett. 71, 1546–1548, doi:10.1103/PhysRevLett.71.1546, 1993.

Falconer, K. Fractal Geometry: Mathematical Foundations and Applications. J. Wiley, New York, 1990.

Grassberger, P. Generalized dimensions of strange attractors. Phys. Lett. A 97, 227–230, doi: 10.1016/0375-9601(83)90753-3, 1983.

Grassberger, P., Procaccia, I. Measuring the strangeness of strange attractors. Physica D 9, 189–208, doi: 10.1016/0167-2789(83)90298-1, 1983.

Halsey, T.C., Jensen, M.H., Kadanoff, L.P., Procaccia, I., Shraiman, B.I. Fractal measures and their singularities: the characterization of strange sets. Phys. Rev. A 33, 1141–1151, doi:10.1103/PhysRevA.33.1141, 1986.

Hentschel, H.G.E., Procaccia, I. The infinite number of generalized dimensions of fractals and strange attractors. Physica D 8, 435–444, doi: 10.1016/0167-2789(83)90235-X, 1983.

Hnat, B., Chapman, S.C., Rowlands, G. Intermittency, scaling, and the Fokker–Planck approach to fluctuations of the solar wind bulk plasma parameters as seen by the WIND spacecraft. Phys. Rev. E 67, 056404, doi:10.1103/PhysRevE.67.056404, 2003.

Hnat, B., Chapman, S.C., Kiyani, K., Rowlands, G., Watkins, N.W. On the fractal nature of the magnetic field energy density in the solar wind. Geophys. Res. Lett. 34, L15108, doi:10.1029/2007GL029531, 2007.

Horbury, T.S., Balogh, A. Evolution of magnetic field fluctuations in high-speed solar wind streams: Ulysses and Helios observations. J. Geophys. Res. 106, 15929–15940, doi:10.1029/2000JA000108, 2001.

Kiyani, K., Chapman, S.C., Hnat, B., Nicol, R.M. Self-similar signature of the active solar corona within the inertial range of solar-wind turbulence Phys. Res. Lett. 98, 211101, doi:10.1103/PhysRevLett98.211101, 2007.

Macek, W.M. Testing for an attractor in the solar wind flow. Physica D 122, 254–264, doi: 10.1016/S0167-2789(98)00098-0, 1998.

Macek, W.M. Multifractality and chaos in the solar wind, in: Boccaletti, S., Gluckman, B.J., Kurths, J., Pecora, L.M., Spano, M.L. (Eds.), Experimental Chaos, vol. 622. American Institute of Physics, New York, pp. 74–79, doi: 10.1063/1.1487522, 2002.

Macek, W.M. The multifractal spectrum for the solar wind flow, in: Velli, M., Bruno, R., Malara, F. (Eds.), Solar Wind 10, 679. American Institute of Physics, New York, pp. 530–533, doi: 10.1063/1.1618651, 2003.

Macek, W.M. Modeling multifractality of the solar wind. Space Sci. Rev. 122, 329–337, doi:10.1007/s11214-006-8185-z, 2006.

Macek, W.M. Multifractality and intermittency in the solar wind. Nonlin. Processes Geophys. 14, 695–700, 2007. Available from: <http://www.nonlin-processes-geophys.net/14/695/2007/>.

Macek, W.M., Redaelli, S. Estimation of the entropy of the solar wind flow. Phys. Rev. E 62, 6496–6504, doi:10.1103/PhysRevE.62.6496, 2000.

Macek, W.M., Strumik, M. Testing for nonlinearity and low-dimensional dynamics in the slow solar wind. Adv. Space Res. 37, 1544–1549, doi:10.1016/j.asr.2005.07.038, 2006.

Macek, W.M., Szczepaniak, A. Generalized two-scale weighted Cantor set model for solar wind turbulence. Geophys. Res. Lett. 35, L02108, doi:10.1029/2007GL032263, 2008.

Macek, W.M., Bruno, R., Consolini, G. Generalized dimensions for fluctuations in the solar wind. Phys. Rev. E 72, 017202, doi:10.1103/PhysRevE.72.017202, 2005.

Macek, W.M., Bruno, R., Consolini, G. Testing for multifractality of the slow solar wind. Adv. Space Res. 37, 461–466, doi:10.1016/j.asr.2005.06.057, 2006.

Mandelbrot, B.B. Multifractal measures, especially for the geophysicist. Pure Appl. Geophys. 131, Birkhäuser Verlag, Basel, pp. 5–42, doi:10.1007/BF00874478, 1989.

Marsch, E., Tu, C.-Y. Non-Gaussian probability distributions of solar wind fluctuations. Ann. Geophys. 12, 1127–1138, 1994.

Marsch, E., Tu, C.-Y. Intermittency, non-Gaussian statistics and fractal scaling of MHD fluctuations in the solar wind. Nonlin. Processes Geophys. 4, 101–124, 1997. Available from: <http://www.nonlin-processes-geophys.net/4/101/1997/>.

Marsch, E., Tu, C.-Y., Rosenbauer, H. Multifractal scaling of the kinetic energy flux in solar wind turbulence. Ann. Geophys. 14, 259–269, 1996.

Meneveau, C., Sreenivasan, K.R. Simple multifractal cascade model for fully developed turbulence. Phys. Rev. Lett. 59, 1424–1427, doi:10.1103/PhysRevLett.59.1424, 1987.

Meneveau, C., Sreenivasan, K.R. The multifractal nature of turbulent energy dissipation. J. Fluid Mech. 224, 429–484, doi:10.1017/S0022112091001830, 1991.

Ott, E. Chaos in Dynamical Systems. Cambridge University Press, Cambridge, 1993.

Polygiannakis, J.M., Moussas, X. Fractal properties of the interplanetary magnetic field fluctuations: indications of chaotic dynamics. Astron. Astrophys. 283, 990–996, 1994a.

Polygiannakis, J.M., Moussas, X. On experimental evidence of chaotic dynamics over short time scales in solar wind and cometary data using nonlinear prediction techniques. Solar Phys. 151, 341–350, 1994b.

Schwenn, R. Large-scale structure of the interplanetary medium, in: Schwenn, R., Marsch, E. (Eds.), Physics of the Inner Heliosphere, vol. 20. Springer-Verlag, Berlin, pp. 99–181, 1990.

Sorriso-Valvo, L., Marino, R., Carbone, V., Lepreti, F., Veltri, P., Noullez, A., Bruno, R., Bavassano, B., Pietropaolo, E. Observation of inertial energy cascade in interplanetary space plasma. Phys. Rev. Lett. 99, 115001, doi:10.1103/PhysRevE.71.066313, 2007.

Strogatz, S.H. Nonlinear Dynamics and Chaos. Addison-Wesley, Reading, 1994.

Theiler, J., Eubank, S., Longtin, A., Galdrikian, B., Farmer, J.D. Testing for nonlinearity in time series: the method of surrogate data. Physica D 58, 77–94, doi: 10.1016/0167-2789(92)90102-S, 1992.

Tsang, Y.-K., Ott, E., Antonsen Jr., T.M., Guzdar, P.N. Intermittency in two-dimensional turbulence with drag. Phys. Rev. E 71, 066313, doi:10.1103/PhysRevLett.99.115001, 2005.