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Multiscale Multifractal Intermittent Turbulence in Space Plasmas

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We examine generalized dimensions and the corresponding multifractal singularity spectrum depending on one probability measure parameter and two scaling parameters, demonstrating that the multifractal scaling is often asymmetric. In particular, we analyze time series of velocities of the slow and fast speed streams of the solar wind plasma measured in situ by Advanced Composition Explorer spacecraft. We show that the universal shape of the multifractal spectrum results not only from the nonuniform probability of the energy transfer rate but rather from the multiscale nature of the cascade. It is worth noting that for the model with two different scaling parameters a better agreement with the solar wind data is obtained. Only in the case of the multiscale cascade one can reproduce the entire multifractal spectrum, especially for the negative index of the generalized dimensions. Therefore we argue that there is a need to use the multi-scale cascade model. Hence we propose this new more general model as a useful tool for analysis of intermittent turbulence in various environments.

Keywords: multifractals, turbulence, intermittency, space plasmas

1. Introduction

Multifractality is commonly related to a probability measure that may have different fractal dimensions on different parts of the support of this measure [1]. In this case the measure is multifractal. Here we propose a notion of multifractality based on an extended self-similarity that depends on scale. We consider the concept of the multiscale multifractality in the context of scaling properties of intermittent turbulence in astrophysical and space plasmas [2, 3]. To quantify scaling of this turbulence, we use a generalized weighted Cantor set with two different scales describing with various probabilities nonuniform intermittent multiplicative process of distribution of the kinetic energy between cascading eddies of various sizes [4, 5].

The question of multifractality is of great importance for space plasmas because it allows us to look at intermittent turbulence in the solar wind [6–12]. Starting from Richardson's scenario of turbulence, many authors try to recover the observed scaling exponents, using some simple and more advanced fractal and multifractal models of turbulence describing distribution of the energy flux between cascading eddies at various scales. In particular, the multifractal spectrum has been investigated using Voyager (magnetic field fluctuations) data in the outer heliosphere [6, 7] and using Helios (plasma) data in the inner heliosphere [11]. The multifractal scaling has also been investigated using Ulysses observations, e.g., [13] and with Advanced Composition Explorer (ACE) and WIND data, e.g., [14, 15].

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In general, the spectrum of generalized dimensions D_q as a function of a continuous index q, with a degree of multifractality $\Delta = D_{-\infty} - D_{\infty}$, quantifies multifractality of a given system [4, 5, 16]. A chaotic strange attractor has been identified in the solar wind data by Macek [17] and further examined by Macek and Redaelli [18]. We have also considered the D_q spectrum for the solar wind attractor using a multifractal model with a measure of the self-similar weighted Cantor set with one parameter describing uniform compression in phase space and another parameter for the probability measure of the attractor of the system. The spectrum of D_q is found to be consistent with the data, at least for positive index q [19–23]. However, the full spectrum is necessary to estimate the degree of multifractality. Notwithstanding of the well-known statistical problems with negative q [21], we have recently succeeded in estimating the entire spectrum for solar wind attractor using a generalized weighted Cantor set with two different scales describing nonuniform compression [4].

Therefore to further quantify turbulence, we have considered this generalized weighted Cantor set also in the context of turbulence cascade [5]. In this way we have argued that there is, in fact, need to use a multi-scale cascade model. Therefore we have already investigated the multifractal spectrum of dimensions depending on two scaling parameters and one probability measure parameter using Helios data and, in particular, we have demonstrated that intermittent pulses are stronger for asymmetric scaling and a much better agreement is obtained, especially for $q < 0$.

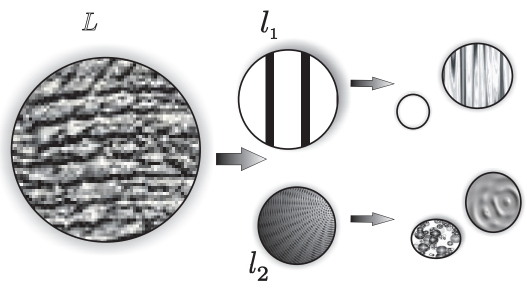

Fig. 1 Generalized two-scale weighted Cantor set model for solar wind turbulence, cf. [2, 4].

In this paper, we would like to test the degree of multifractality and asymmetry of the multifractal scaling for the wealth of data provided by another space mission. Namely, we further consider the question of scaling properties of intermittent turbulence using velocities of the slow and fast speed streams of the solar wind measured in situ by ACE during solar minimum and maximum at Earth's orbit, R = 1 AU. By using our cascade model we show that the degree of multifractality of the solar wind in the inner heliosphere is greater for fast solar wind velocity fluctuations than that for the slow solar wind. On the other hand, as the solar activity decreases the slow solar wind spectrum becomes more asymmetric. Thus we still hope that this generalized new asymmetric multifractal model could shed light on the nature of turbulence.

2. Two-Scale Cantor Set Model

At each stage of construction of the weighted two-scale Cantor set we basically have two scaling parameters l_1 and l_2, where $l_1 + l_2 \leq 1$ (normalized), and two different weights p_1 and p_2. To obtain the generalized dimensions $D_q \equiv \tau(q)/(q-1)$ for this interesting example of multifractals we use the following partition function at the n-th level of construction [24, 25]

$$\Gamma_n^q(l_1, l_2, p_1, p_2) = \left(\frac{p_1^q}{l_1^{\tau(q)}} + \frac{p_2^q}{l_2^{\tau(q)}} \right)^n = 1. \quad (1)$$

The resulting strange attractor of 2^n closed intervals (narrow segments with various widths and probabilities) for $n \to \infty$ is the generalized weighted two-scale Cantor set.

Here we consider a standard scenario of cascading eddies, each breaking down into two new ones, but not necessarily equal and twice smaller as schematically shown in Fig. 1, cf. [2, 4]. In particular, space filling turbulence could be recovered for $l_1 + l_2 = 1$. Naturally, in the inertial region of the system of size L, $\eta \ll l \ll L$, we do not allow the energy to be dissipated directly, assuming $p_1 + p_2 = 1$, until the Kolmogorov scale η is reached. However, in this range at each n-th step of the binomial multiplicative process, the flux of kinetic energy density ε transferred to smaller eddies (energy transfer rate) could be divided into nonequal fractions p and $1-p$. In particular, for non space-filling turbulence, $l_1 + l_2 < 1$ one still could have a multifractal cascade, even for unweighted (equal) energy transfer, $p = 0.5$. Only for $l_1 = l_2 = 0.5$ and $p = 0.5$ there is no multifractality.

3. Solar Wind Data

We have already analyzed the Helios 2 data using plasma parameters measured *in situ* in the inner heliosphere [4] for testing of the solar wind attractor. The X-velocity (mainly radial) component of the plasma flow, v_x, has been already investigated by Macek [17, 19, 20] and Macek and Redaelli [18]. The Alfvénic fluctuations with longer (two-day) samples have been studied by in Ref. [4, 21] and [22, 23]. To study the turbulence cascade Macek and Szczepaniak have selected four-day time intervals of v_x samples in 1976 (solar minimum) for both slow and fast solar wind streams measured at various distances from the Sun [5]. In this paper we analyze time series of velocities of the solar wind measured by ACE in the ecliptic plane near the libration point $L1$, i.e., approximately at a distance of $R = 1$ AU from the Sun. Here we have selected even longer (five-day) time intervals of v_x samples, each of 6750 data points, interpolated with sampling time of 64 s, for both slow and fast solar wind streams during solar minimum (2006) and maximum (2001).

4. Methods of Data Analysis

The generalized dimensions D_q as a function of index q [24–27] are important characteristics of *complex* dynamical systems; they quantify multifractality of a given system [16]. In the case of turbulence cascade these generalized measures are related to inhomogeneity with which the energy is distributed between different eddies [3]. In this way they provide information about dynamics of multiplicative process of cascading eddies. Here high positive values of q emphasize regions of intense energy transfer rate, while negative values of q accentuate low-transfer rate regions.

Let us consider the generalized weighted Cantor set, where the probability of providing energy for one eddy of size l_1 is p (say, $p \leq 1/2$), and for the other eddy of size l_2 is $1-p$ as depicted in Fig. 1. For any q one obtains $D_q = \tau(q)/(q-1)$ by solving numerically the following transcendental equation, e.g., [16]

$$\frac{p^q}{l_1^{\tau(q)}} + \frac{(1-p)^q}{l_2^{\tau(q)}} = 1. \quad (2)$$

In the inertial range the transfer rate of the energy flux $\varepsilon(l)$ is widely estimated by the third moment of structure function of velocity fluctuations, e.g., [11]

$$\varepsilon(l) \sim \frac{|u(x+l) - u(x)|^3}{l}, \quad (3)$$

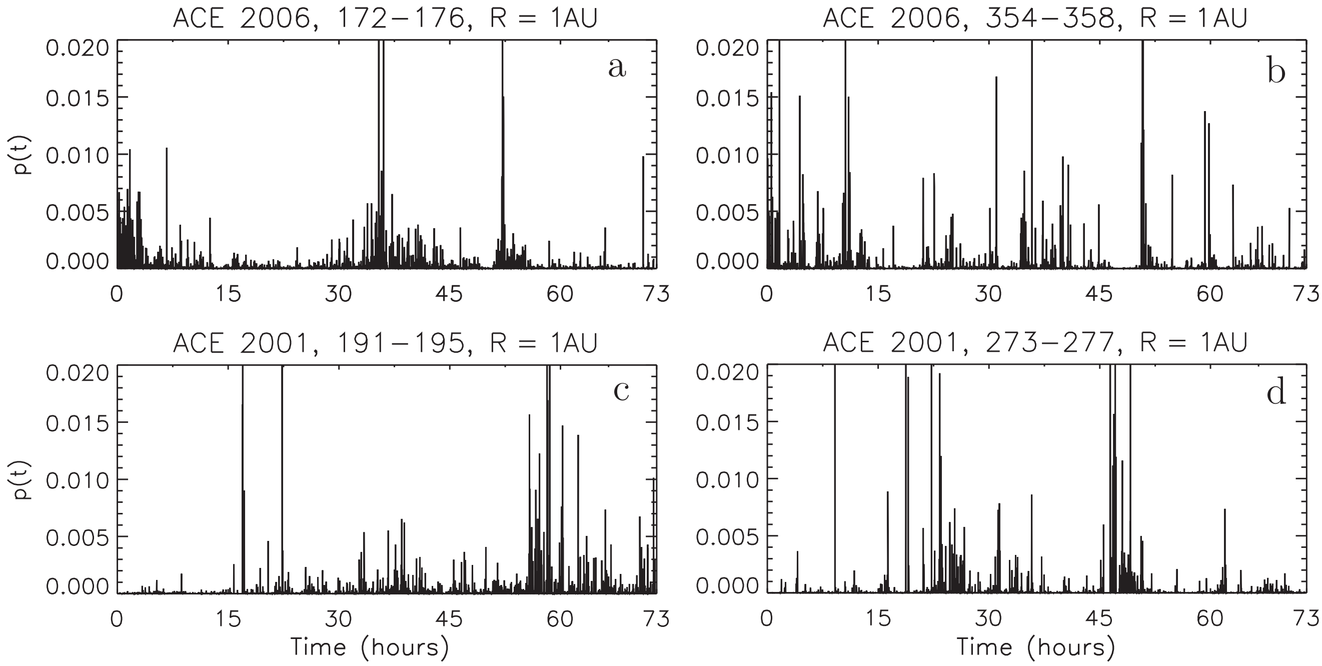

Fig. 2 The normalized transfer rate of the energy flux $p(t) = \varepsilon_i(t) \ / \ \sum \varepsilon_i(t)$ obtained using data of the v_x velocity components measured by ACE at 1 AU for the slow (a) and (c) and fast (b) and (d) solar wind during solar minimum (2006) and maximum (2001), correspondingly.

where $u(x)$ and $u(x+l)$ are velocity components parallel to the longitudinal direction separated from a position x by a distance l. Therefore to each ith eddy of size l in the turbulence cascade ($i = 1, \ldots, N = 2^n$) we associate a probability measure defined by

$$p_i(l) = \frac{\varepsilon_i(l)}{\sum_{i=1}^{N} \varepsilon_i(l)}. \tag{4}$$

This quantity can roughly be interpreted as a probability that the energy is transferred to an eddy of size $l = v_{\rm sw} t$. In Fig. 2 we show the multifractal measure $p(t) = \varepsilon_i(t) \ / \ \sum \varepsilon_i(t)$ given by Eqs. (3) and (4) and obtained using data of the velocity components $u = v_x$ (in time domain) as measured by ACE at 1 AU for the slow (a) and (c) and fast (b) and (d) solar wind during solar minimum (2006) and maximum (2001), correspondingly.

Now, one can further associate a generalized average probability measure of cascading eddies

$$\bar{\mu}(q,l) \equiv \sqrt[q-1]{\langle (p_i)^{q-1} \rangle_{\rm av}}, \tag{5}$$

and identify D_q as scaling of the measure with size l,

$$\bar{\mu}(q,l) \propto l^{D_q}. \tag{6}$$

Hence, the slopes of the logarithm of $\bar{\mu}(q,l)$ of Eq. (6) versus $\log l$ (normalized) provides

$$D_q = \lim_{l \to 0} \frac{\log \bar{\mu}(q,l)}{\log l}. \tag{7}$$

The singularity spectrum $f(\alpha) = q\alpha - \tau(q)$ as a function of $\alpha = \tau'(q)$ could also be obtained by using Legendre transformation [16, 21], or directly from the slopes or generalized measures. Using α_0, where $f(\alpha_0) = 1$, one can define a measure of asymmetry $A \equiv (\alpha_0 - \alpha_{\min})/(\alpha_{\max} - \alpha_0)$.

5. Results

The results for the generalized dimensions D_q as a function of q are shown in Fig. 3. The values of D_q given in Eq. (6), for one-dimensional turbulence, are again calculated using the radial velocity components $u = v_x$, cf. [22, Figure 3], and the corresponding results for the singularity spectra $f(\alpha)$ as a function of α are shown in Fig. 4 for the slow (a) and (c), and fast (b) and (d) solar wind streams at solar minimum and maximum, correspondingly. Both values of D_q and $f(\alpha)$ for one-dimensional turbulence have been computed directly from the data, by using the experimental velocity components.

In spite of statistical errors in Fig. 4 (a), b), (c) and (d), especially for $q < 0$, we see that the multifractal character of the measure can still clearly be discerned. Therefore one can confirm that the spectrum of dimensions still exhibits the multifractal structure of the solar wind in the inner heliosphere.

For $q \geq 0$ these results agree with the usual one-scale p-model fitted to the dimension spectra as obtained analytically using $l_1 = l_2 = 0.5$ in Eq. (2) and the corresponding value of the parameter $p \simeq 0.21$ and 0.20, 0.15 and 0.12 for the slow (a) and (c), and fast (b) and (d) solar wind streams at solar minimum and maximum, correspondingly, as shown by dashed

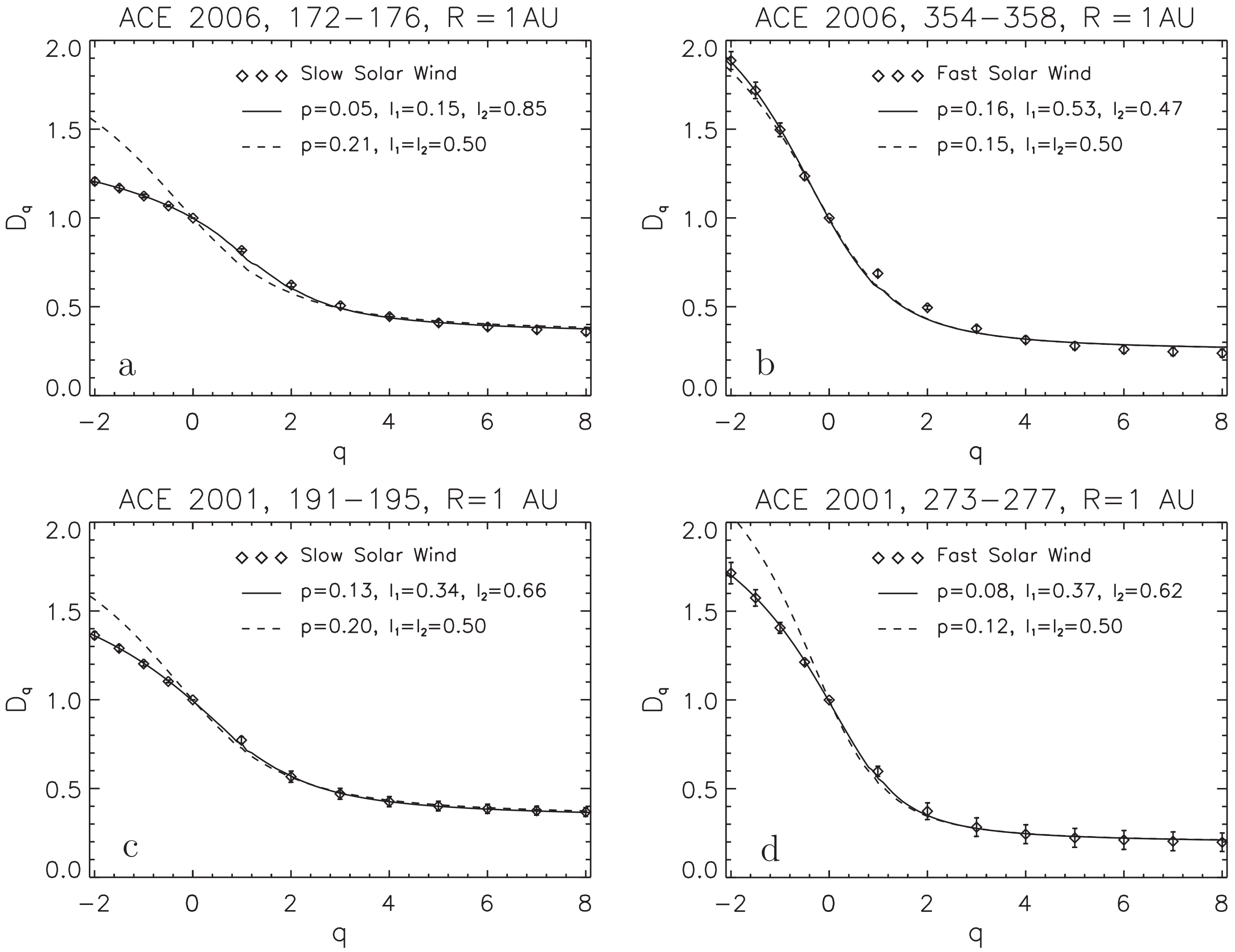

Fig. 3 The generalized dimensions D_q as a function of q. The values obtained for one-dimensional turbulence are calculated for the usual one-scale (dashed lines) p-model and the generalized two-scale (continuous lines) model with parameters fitted to the multifractal measure $\mu(q,l)$ obtained using data measured by ACE at 1 AU (diamonds) for the slow (a) and (c) and fast (b) and (d) solar wind during solar minimum (2006) and maximum (2001), correspondingly.

lines. On the contrary, for $q < 0$ the p-model cannot describe the observational results [11]. Here we show that the experimental values are consistent also with the generalized dimensions obtained numerically from Eqs. (5-7) for the weighted two-scale Cantor set using an asymmetric scaling, i.e., using unequal scales $l_1 \neq l_2$, as is shown in Figs. 3 and 4 (a), (b), (c), and (d) by continuous lines. We also confirm the universal shape of the multifractal spectrum, Fig. 4. In our view, this obtained shape of the multifractal spectrum results not only from the nonuniform probability of the energy transfer rate but mainly from the multiscale nature of the cascade.

It is well known that the fast wind is associated with coronal holes, while the slow wind mainly originates from the equatorial regions of the Sun. Consequently, the structure of the flow differs significantly for the slow and fast streams. Hence the fast wind is considered to be relatively uniform and stable, while the slow wind is more turbulent and quite variable in velocities, possibly owing to a strong velocity shear [28]. We see from Table 1 that the degree of multifractality Δ and asymmetry A of the solar wind in the inner heliosphere are different for slow ($\Delta = 1.2-1.6$) and fast ($\Delta = 2.3-2.6$) streams; the velocity fluctuations in the fast streams seem to be more multifractal than those for the slow solar wind (the generalized dimensions vary more with the index q). On the other

Table 1Degree of multifractality Δ and asymmetry A for solar wind data in the heliosphere

	Slow Solar Wind	Fast Solar Wind
Solar Min.	$\Delta = 1.22, A = 2.21$	$\Delta = 2.56, A = 0.95$
Solar Max.	$\Delta = 1.60, A = 1.33$	$\Delta = 2.31, A = 1.25$

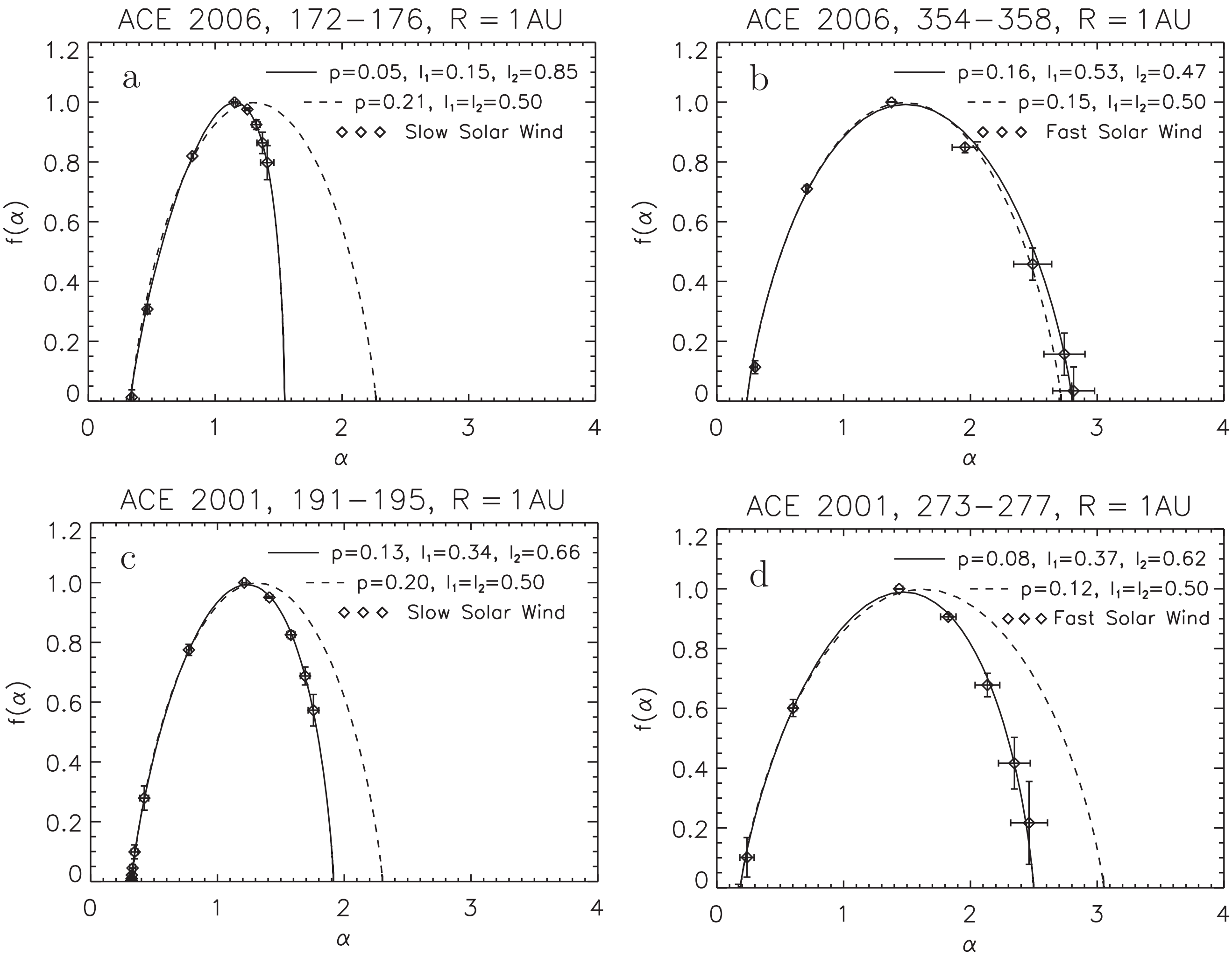

Fig. 4 The corresponding singularity spectrum $f(\alpha)$ as a function of α.

hand, it seems that in the slow streams the scaling is more asymmetric than that for the fast wind. In our view this could possibly reflect the large-scale scale velocity structure. Further, the degree of asymmetry of the dimension spectra for the slow wind is rather anticorrelated with the phase of the solar magnetic activity and only weakly correlated for the fast wind: (A decreases from 2.2 to 1.3) only the fast wind during solar minimum exhibits roughly symmetric scaling, $A \sim 1$, and one-scale Cantor set model applies.

We see that the multifractal spectrum of the solar wind is only roughly consistent with that for the multifractal measure of the self-similar weighted symmetric one-scale weighted Cantor set only for $q \geq 0$. On the other hand, this spectrum is in a very good agreement with two-scale asymmetric weighted Cantor set schematically shown in Fig. 1 for both positive and negative q. Obviously, taking two different scales for eddies in the cascade, one obtains a more general situation than in the usual p-model for fully developed turbulence [2], especially for an asymmetric scaling, $l_1 \neq l_2$. Hence we hope that this generalized model will be a useful tool for analysis of intermittent turbulence in space plasmas.

6. Conclusions

We have studied the inhomogeneous rate of the transfer of the energy flux indicating multifractal and intermittent behavior of solar wind turbulence in the inner heliosphere. In particular, we have demonstrated that for the model with two different scaling parameters a much better agreement with the real data is obtained, especially for $q < 0$. By investigating the ACE data we have shown that the degree of multifractality of the solar wind in the inner heliosphere is greater for fast solar wind velocity fluctuations than that for the slow solar wind; the generalized dimensions varies more with the index q. As the solar activity increases the slow solar wind becomes somewhat more multifractal, and the fast wind is slightly less multifractal. On the other hand, it seems that the degree of asymmetry of the dimension spectrum for the slow wind is rather anticorrelated with the phase of the solar activity.

Basically, the generalized dimensions for solar wind are consistent with the generalized p-model for both positive and negative q, but rather with different scaling parameters for sizes of eddies, while the usual p-model can only reproduce the spectrum for $q \geq 0$. In general, the proposed generalized multi-scale weighted Cantor set model should also be valid for non space filling turbulence. Therefore we propose this cascade model describing intermittent energy transfer for analysis of turbulence in various environments.

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