Nonlinear dynamical systems

Multifractal Solar Wind

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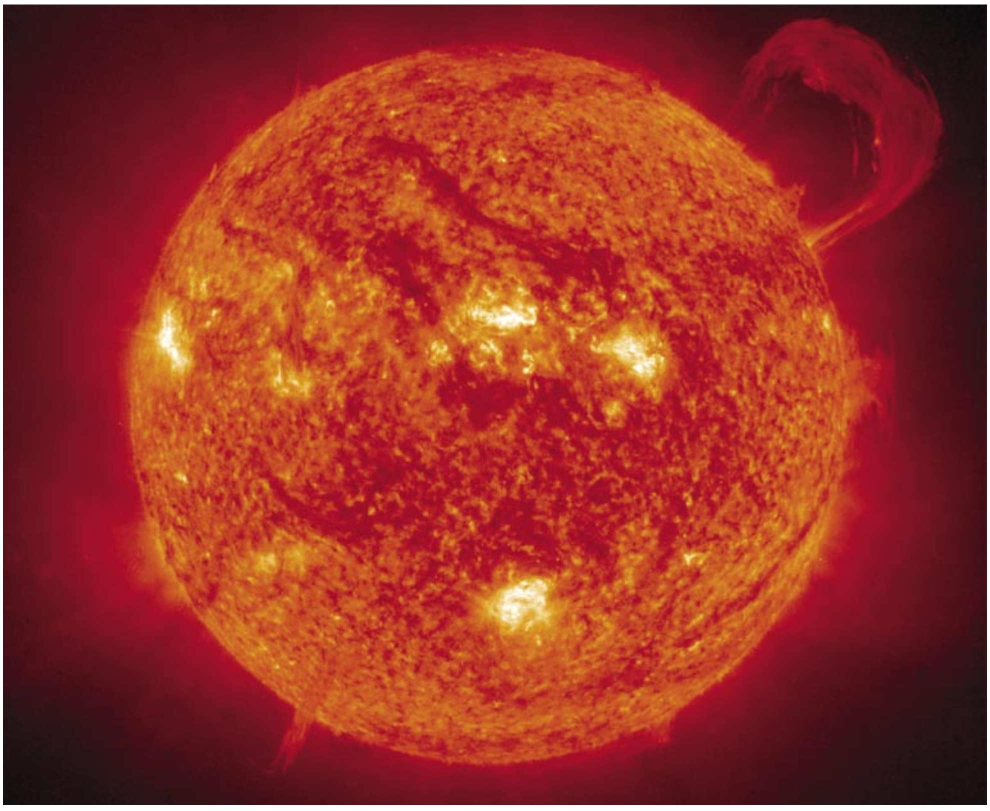
Within the complex dynamics of the solar wind's fluctuating plasma parameters, there is a detectable, hidden order described by a chaotic strange attractor which has a multifractal structure

The basic notions of fractal analysis and *nonlinear* dynamics appear in various contexts, and are not always understood unambiguously. We can say that a *fractal* is an irregular geometrical object, each of whose parts is (at least approximately) a reduced-size copy of the whole, i.e. fractals are generally self-similar at arbitrary scales, which is described in terms of a certain fractal dimension. A multifractal, in turn, is an object that demonstrates various self-similarities, described by a certain function called a multifractal spectrum of dimensions. A *multifractal* is therefore in a certain sense like a set of intertwined fractals. It is interesting that such a spectrum has certain universal properties.

Nonlinear dynamical systems are often highly sensitive to initial conditions resulting in chaotic motion. In practice, therefore, the behavior of such systems cannot be predicted in the long term, even though the laws of dynamics unambiguously determine its evolution. *Chaos* is thus a non-periodic long-term behavior in a deterministic system that exhibits sensitivity to initial conditions. Yet we are not entirely without hope here in terms of predictability, because in a dissipative system (with friction) the trajectories describing its evolution in the space of system



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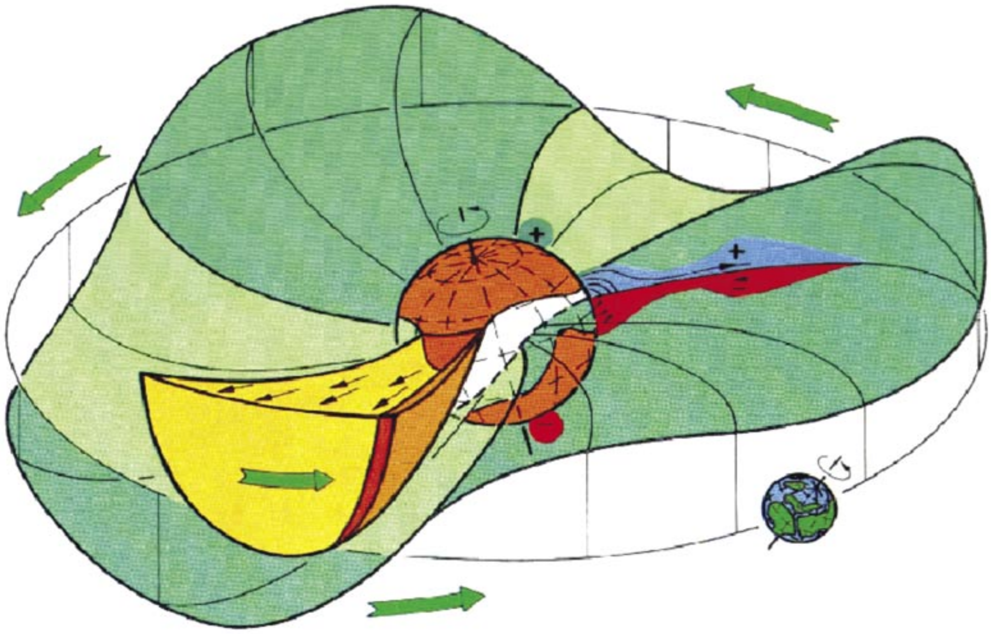
The Sun is a source of electromagnetic radiation in the visible spectrum, as well as radiation in the form of infrared, ultraviolet, radio, X-ray, and gamma waves. Aside from electromagnetic radiation, the Sun also emits radiation consisting of charged particles – mostly protons and electrons sent out with considerable energy, enabling them to overcome the Sun's gravitational pull



R. Schwenn, H. Rosenbauer

A schematic model of the solar wind "ballerina": the Sun's two hemispheres are separated by a neutral layer of a form reminiscent of a "ballerina's skirt," shown here. In the inner heliosphere the solar wind streams are of two forms called the slow (≈ 400 km s^{-1}) and fast (≈ 700 km s^{-1}) solar winds. The fast solar wind, associated with holes in the solar corona, is relatively uniform and stable, while the speed of the slow wind is variable

states asymptotically converge towards a certain invariant set, which is called an *attractor*; strange attractors are fractal sets (generally with a fractal dimension) which exhibit a hidden order within the chaos.

Solar Wind

From the upper solar atmosphere, charged particles are constantly radiating outward in all directions into interstellar space. Because of similarities to phenomena in the Earth's atmosphere, this interesting phenomenon was dubbed the "solar wind." Yet here, the role of the gas medium in the atmosphere is in fact played by ionized interplanetary gas or plasma (called the fourth state of matter) with a magnetic field "frozen into" it, which makes this physical system much more complex than ordinary air.

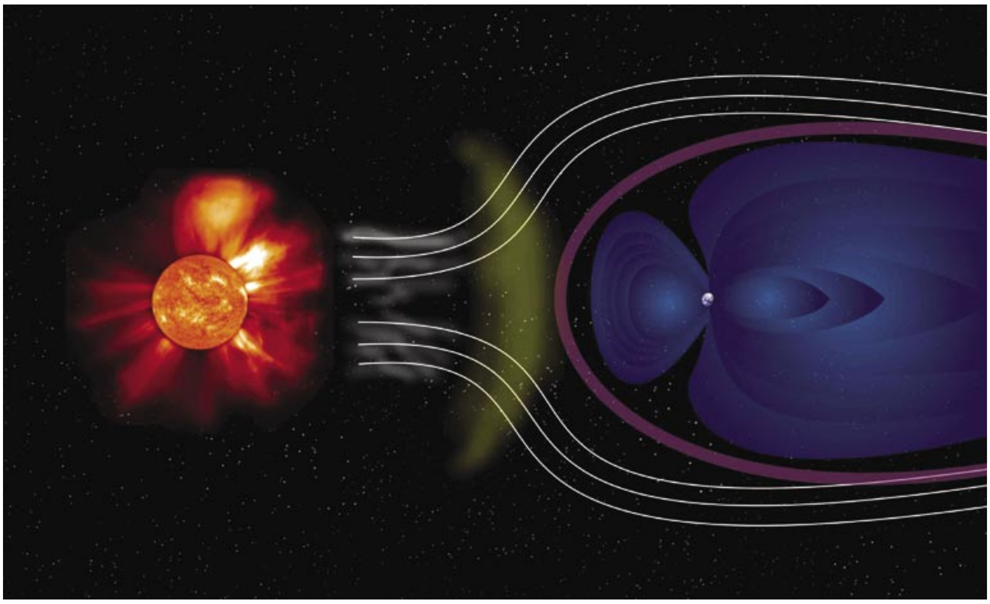
Supersonic streams of solar wind flowing outward from the Sun fill our solar system; this region of space is called the heliosphere. However, the interaction of the solar wind with the magnetic fields of the planets gives rise to what are called *magnetospheres*: areas of space which are dominated by the planets' magnetic fields. In the region of the heliosphere close to the Sun, the flow of the solar wind has two forms: the slow (≈ 400 km s^{-1}) and fast (≈ 700 km s^{-1}) streams. The fast wind, associated with holes in the solar corona, is relatively uniform and stable, while the speed of the slow solar wind is quite variable.

The nature of the fluctuations in solar wind plasma parameters is still very little understood. The slow solar wind most likely originates from non-linear processes in the solar corona. However, it appears that a certain kind of order does lie concealed within the irregular solar wind fluctuations, which can be described using new methods of non-linear time series analysis, based on fractal analysis and the theory of deterministic chaos. This involves the notion of a (multi-) fractal set, which is generally a strange attractor in a certain state space of a given dynamical system.

The strange attractor in the solar wind

In space plasma physics, the idea of chaotic motion on a strange attractor has been tested for the pulsations in radio emissions from the Sun, solar activity data, and magnetospheric dynamics. Following these applications, my co-workers and I have applied this notion to the analysis of solar wind plasma. Our investigations concentrate on analyzing solar wind time series, measured by the Helios spacecraft in the inner heliosphere, the region of space dominated by the solar wind. Analyzing the fluctuations in the speed of the solar wind stream obtained from space experiments, we have shown that the probability distributions of solar wind parameters differ significantly from the normal equilibrium distribution. Moreover, no dominant frequency can be identified within the spectrum of fluctuations. This points to the *non*-periodic behavior of this complex system. Moreover, by employing what we call a "false-closest-neighbors" method, we suggest that the deterministic component of solar wind plasma dynamics should be low-dimensional, and the irregular behavior of plasma parameter fluctuations may be modeled using a system of several ordinary differential equations.

Nonlinear dynamical systems



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We continue to hope that our analysis will to a certain extent enable us to predict the behavior of the solar wind, which exerts an important impact on phenomena occurring in the Earth's environment and even on its surface

The results we have obtained with the new method of topological embeddings suggest that the behavior of the solar wind can be approximately described by a low-dimensional *attractor* within a certain subspace of system states.

Recently, we have focused on the question of multifractality and the related phenomenon of intermittency (interrupted signal). They allow us to study the nature of the turbulent solar wind using the methods of nonlinear physics. Our analysis was based on both measurement data obtained from the Helios space probe and the modeling of analogous physical systems. The results of our work for the solar wind show that some parts of the attractor in the state space are visited much more frequently than other parts. We have also proposed a new simple analytical model for describing the multifractal structure of this complex plasma medium.

Conclusions

The most important result achieved in analyzing the observed parameters of solar wind plasma in the inner heliosphere is identifying its fractal structure and the multifractal model of solar wind fluctuation, which is of key significance in studying solar wind turbulence. Apparently, the fluctuations in the solar wind speed are not quite of a random nature, but result rather from deterministic nonlinear dynamics, which can be described by a small number of parameters, most likely by a strange attractor with a multifractal structure.

The confirmation of our findings will urge still further research, since if such an attractor really does exist, then a certain hidden order can be discerned within the complex dynamics of the solar wind. It is worth pointing out that such an attractor is an exceptionally important finding, opening up the possibility of approximately predicting the behavior of the solar wind, the main factor that controls planetary magnetospheres in our Solar System (including that of the Earth). Results obtained for cosmic plasma could also be important for the theory of turbulence, including their application to analogous physical objects, such as turbulence in fluids observed in laboratory or in geophysical media (the atmosphere, the plasma in the Earth's ionosphere, etc.) We hope that analysis of chaotic systems could lead us to a deeper understanding of their nature, and maybe even to predict their seemingly unpredictable behavior. ■

Further reading:

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