Re: lecture at the University of Calabria, May/June 2011

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Nonlinear and Fractal Analysis

Content and Level: The aim of the course is to give students an introduction to the new developments in nonlinear dynamics and fractals. Emphasis will be on the basic concepts of nonlinearity, (multi-)fractals, chaos, and strange attractors based on intuition rather than mathematical proofs. The specific exercises will also include applications to convection in a viscous fluid and intermittent turbulence in various environments. On successful completion of this course, students should understand and apply the fractal models to real systems and be able to evaluate the importance of nonlinearity and multifractality with possible applications to physics, astrophysics and space physics, chemistry, biology, and even economy.

Part I **Introduction to Nonlinear Dynamics, Fractals, and Chaos**

1. Introduction
   - Dynamical and Geometrical View of the World
   - Fractals
   - Stability of Linear Systems
2. Nonlinear Dynamics
   - Attracting and Stable Fixed Points
   - Nonlinear Systems: Pendulum
3. Fractals and Chaos
   - Strange Attractors and Deterministic Chaos
   - Bifurcations

4. Strange Attractors
    - Stretching and Folding Mechanism
    - Baker's Map
    - Logistic Map
    - Hénon Map
5. Conclusion: importance of nonlinearity and fractals

Part II **Chaotic Convection and Multifractal Turbulence**

1. Lorenz System
    - Rayleigh-Bénard Convection in a Viscous Fluid
    - Basic Equations and Approximations
    - Lorenz Model
    - A Generalized Lorenz Model for a Magnetized Fluid
    - New Strange Attractors
2. Intermittency
    - Bifurcations and Intermittency
    - Basic Types of Intermittency
3. Multifractals
    - Intermittent Turbulence
    - Weighted Two-Scale Cantors Set
    - Multifractals Analysis of Turbulence
4. Applications and Conclusions
    - Importance of Being Nonlinear
    - Importance of Multifractality

Bibliography

1. S. H. Strogatz, Nonlinear Dynamics and Chaos, Addison-Wesley, Reading, 1994.
2. E. Ott, Chaos in Dynamical Systems, Cambridge University Press, Cambridge, 1993.
3. H. G. Schuster, Deterministic Chaos: An Introduction, VCH Verlagsgesellschaft, Weinheim 1988.
4. K. Falconer, Fractal Geometry: Mathematical Foundations and Applications, Wiley, Chichester, England, 1990.
5. E. N. Lorenz, Deterministic Nonperiodic flow, J. Atmos. Sci. **20**, 130, 1963.
6. Macek W. M., Strumik, M., Model for hydromagnetic convection in a magnetized fluid, Physical Review E **82**, 027301, 2010.
7. W. M. Macek and A. Wawrzaszek, Evolution of asymmetric multifractal scaling of solar wind turbulence in the outer heliosphere, J. Geophys. Res., **114**, A03108, 2009.

Prerequisites: Some basic notions from ordinary differential equations, including elements of partial differential equations will be useful, but all materials necessary to follow the lecture will be provided.

Wiesław M. Macek is Ordinary Professor of Mathematics and Physics at Cardinal Stefan Wyszyński University in Warsaw, Poland, and at Space Research Centre of the Polish Academy of Sciences. He received his Ph. D. in theoretical physics from Warsaw University in 1976 and habilitation in space physics from the Faculty of Physics, Warsaw University, in 1988. He was a member of the Plasma Wave Science team on the Voyager mission to Neptune in 1989. His recent studies focus on nonlinear dynamics and fractal structure of the solar wind plasma.